GRAMMAR SCHOOL ARITHMETIC

WATSON and WHITE

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GRAMMAR SCHOOL ARITHMETIC

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INTRODUCTION

THIS volume, the third of the series, is designed for use in the higher grammar grades.

It contains a brief and somewhat more mature treatment of topics covered by the first two books, and a thorough course in the more advanced subjects taught in the upper classes.

The aim has been to secure in pupils a high degree of facility and accuracy in computation, to develop the power of visualization, and to cultivate a habit of reliance upon independent thought, rather than upon rules and formulas, in obtaining results.

The attention of teachers is especially directed to the plan of developing the basal ideas of each new topic by means of oral exercises, thus insuring *appreciation of the new idea* in advance of the *conventional form of computation*.

The authors desire to acknowledge their obligation to Mr. Edward Southworth, Head Master of the Mather School, Boston, Mass., for helpful suggestions in the treatment of Mensuration; to Principal C. S. Gibson and Miss Mary Losacker of Seymour School, Syracuse, N.Y., for aid in the preparation of the sections pertaining to Interest; and to the many superintendents and others who have performed a most valuable service in reading and correcting the proofs.

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ARABIC NOTATION AND NUMERATION

1. *That which tells how many* is **number**; *e.g.* three, seven, five, two and one half.

2. *One* is a **unit**; *e.g.* one, one (dollar), one (book).

3. *A number that is applied to some particular thing or things* is called a **concrete number**; *e.g.* five (books), seven (dollars), ten (months).

4. *A number that is not applied to any particular thing or things* is called an **abstract number**; *e.g.* five, seven, eleven.

To the Teacher. — See Primary Arithmetic, pages 139 and 140.

5. *A number that is composed entirely of whole units* is an **integer**; *e.g.* six, eight, thirteen.

6. *One or more of the equal parts of a unit* is a **fraction**; *e.g.* $\frac{2}{3}$, $\frac{7}{8}$, $\frac{25}{100}$.

7. *The number above the line in a fraction* is the **numerator**; *the number below the line in a fraction* is the **denominator**; *e.g.* in the fractions $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{25}{100}$, the numerators are 2, 7, and 25. The denominators are 3, 8, and 100.

8. *The product of equal factors* is a **power.** (See § **47**); *e.g.*

4 is a power of 2 because $2 \times 2 = 4$
8 is a power of 2 because $2 \times 2 \times 2 = 8$
81 is a power of 3 because $3 \times 3 \times 3 \times 3 = 81$
100 is a power of 10 because $10 \times 10 = 100$

Name three other powers of 10.

9. *A fraction whose denominator is* 10 *or a power of* 10 is a **decimal fraction**; *e.g.* $\frac{7}{10}$, $\frac{25}{100}$, $\frac{438}{10000}$, .7, .25, .0438.

10. *Expressing numbers by means of figures or letters* is **notation**; *e.g.* 32, XXXII.

11. *Expressing numbers by means of figures* is **Arabic notation**; *e.g.* 349, 6872.351.

1, 2, 3, 4, 5, 6, 7, 8, *and* 9 are called **significant figures** *because they have values. The figure* 0, *called a* **cipher, naught,** *or* **zero,** *expresses no value.* It is used to give the significant figures their proper places in expressing numbers.

12. The value of each significant figure depends upon *the place which it occupies* when used with other figures in expressing a number.

The value of a figure in any place is *ten times as great* as it would be if it occupied the next place to the *right*, and *one tenth as great* as it would be if it occupied the next place to the *left.*

Since the value of a figure is increased tenfold as it is moved one place from right to left, and divided by ten as it is moved one place from left to right, Arabic notation is said to be based on a scale of ten; or, **the scale of Arabic notation** is a *decimal scale.*

The decimal scale extends through decimal fractions as well as integers, the scale of increase and decrease being uniform from the highest unit of the integer to the lowest unit of the decimal.

The names of the units occupying the different places are called the different **orders of units**; and each group of three orders of units constitutes a **period.**

The left hand period of an integer may contain only one or two figures, or orders of units; it is then called an **incomplete period.**

13. TABLE OF ARABIC NOTATION

Periods	Trillions' period			Billions' period			Millions' period			Thousands' period			Units' period								
Orders of Units	Hundred-trillions	Ten-trillions	Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
	1	2	7,	3	4	6,	2	0	8,	6	3	5,	4	0	9	.	2	3	9	1	0	7

Observe in the above table that

a. The decimal point (.) is placed between units' and tenths' places. Figures at the *left* of the decimal point express *integers*, and figures at the *right* of the decimal point express *decimal fractions.*

b. The different orders of units are numbered from the decimal point both *to the right* and *to the left.*

c. The values of the different orders of units *increase* uniformly *from right to left* and *decrease* uniformly *from left to right* in a tenfold ratio, throughout the integer and the decimal.

d. The name of each period is the same as that of the right-hand place in that period.

e. Commas are used to separate the periods, for convenience in reading.

14. *A number that is composed of an integer and a decimal* is called a **mixed decimal**; *e.g.* 2.5, 31.242, 600.00006.

15. *Naming the places of figures and reading numbers* is **numeration**; *e.g.*, to numerate the number .40236, we should say, tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths — forty thousand two hundred thirty-six, hundred-thousandths.

16. In reading numbers, the word *and* should not be used except *between the integer and the decimal* of a mixed decimal, or *between the integer and the fraction* of a mixed number; *e.g.* 30,245 is read, thirty thousand two hundred forty-five; .328 is read, three hundred twenty-eight thousandths; 30,245.328 is read, thirty thousand two hundred forty-five *and* three hundred twenty-eight thousandths.

17. *Read the following integers and write them in words:*

1. 42,930	6. 8,034,034	11. 400,000,040
2. 80,765	7. 3,001,001	12. 6,097,429
3. 49,060	8. 9,705,010	13. 913,074,060,812
4. 305,041	9. 389,046	14. 3,501,230,780,020
5. 200,030	10. 8,107,010	15. 600,400,300,001

18. *Read the following decimals and write them in words:*

1. .34	6. .8070	11. .20456
2. .751	7. .24305	12. .380751
3. .03	8. .9280	13. .0007
4. .705	9. .60834	14. .000007
5. .807	10. .90307	15. .603120

19. *Read the following mixed decimals and write them in words:*

1.	64.85	11.	9,500.5050	21.	900.900
2.	289.9	12.	384.20108	22.	.990
3.	407.07	13.	70,903.60050	23.	6.00006
4.	897.403	14.	8,000.800	24.	42.0402
5.	2,025.025	15.	8,000.00008	25.	100.00001
6.	83.0008	16.	.08008	26.	100.100
7.	4,920.0020	17.	.060010	27.	101.101
8.	370.0700	18.	.0010	28.	101.100
9.	9,876.540	19.	.1010	29.	10,010.1010
10.	300.00003	20.	400,004.00004	30.	100,000.100

20. *Express the following numbers in figures:*

1. Two hundred thousand, two hundred.
2. Twelve thousand, and two thousandths.
3. Eighty-eight thousand, and three hundredths.
4. One hundred, and one hundred thousandths.
5. One hundred thousand, and one hundred-thousandth.
6. Three thousand one hundred-thousandths.
7. Eight thousand, and eight thousandths.
8. Five billion, sixty thousand, two hundred.
9. Three hundred six million six.
10. Forty-eight thousand two hundred, and two hundred-thousandths.
11. Three hundred seventy-five thousand sixty, and four hundred ten thousandths.

12. Seventy thousand four hundred, and four hundred ten-thousandths.

13. Sixty thousand fifty, and sixty-nine ten-thousandths.

14. Ninety-one, and ninety-one thousandths.

15. Two thousand three hundred one, hundred-thousandths.

16. Five hundred eighteen, and five hundred eighteen ten-thousandths.

17. Thirty-nine thousand four millionths.

18. Two hundred two thousandths.

19. Two hundred, and two thousandths.

20. Two and two hundred thousandths.

21. Two and two hundred-thousandths.

22. Six hundred six thousand.

23. Six hundred six thousandths.

24. Six hundred, and six thousandths.

25. Six hundred, and six hundred thousandths.

26. Six hundred, and six hundred-thousandths.

ROMAN NOTATION

21. *Expressing numbers by means of letters* is **Roman notation.**

For many years the Roman system of notation was commonly used in Europe. The ancient Greeks also had a system of notation which employed the letters of the Greek alphabet. Both of these systems were awkward, and of little use in making computations.

The Arabic numerals were used first in India. The figure 0 was lacking until about the fifth century. Its introduction added greatly to the usefulness of the system.

Arabic notation was first used in Europe about the twelfth century, having been brought there by the Arabs. It is now the prevailing system of notation throughout the civilized world.

22. The Roman system of notation employs the following seven capital letters in expressing numbers:

I (1), V (5), X (10), L (50), C (100), D (500), M (1000).

In combining these letters, the following principles are observed:

a. Repeating a letter repeats its value; e.g.

$$X = 10, \quad XX = 20, \quad XXX = 30.$$

b. When a letter follows one of greater value, its value is added to the greater value; e.g. C = 100, L = 50, CL = 150.

c. When a letter precedes one of greater value, its value is subtracted from the greater value; e.g. C = 100, X = 10, XC = 90.

d. When a letter is placed between two letters of greater value, its value is subtracted from the sum of the two greater values; e.g. C = 100, X = 10, L = 50, CXL = 140.

e. A bar placed over a letter multiplies its value by 1000; *e.g.* XC = 90, $\overline{XC}$ = 90,000.

23. *Read the following numbers and express them in Arabic numerals:*

1. IX	8. MDCC	15. DCXL
2. XIII	9. $\overline{XVI}$	16. LXXVIII
3. XIX	10. MCMIX	17. XCV
4. CCC	11. MCMXI	18. XCIV
5. CDVII	12. CLIII	19. CCXCI
6. XCVI	13. $\overline{M}$	20. DLXXXI
7. CLXIX	14. CMIX	21. MCMXIX

24. *Express the following numbers in Roman numerals:*

1. 8	**5.** 86	**9.** 83	**13.** 64	**17.** 237
2. 18	**6.** 44	**10.** 99	**14.** 110	**18.** 550
3. 119	**7.** 55	**11.** 14	**15.** 208	**19.** 1555
4. 29	**8.** 136	**12.** 75	**16.** 400	**20.** 1911

ADDITION

25. **Addition** *is the process of uniting two or more numbers into one number; e.g.* $2 + 5 = 7$.

26. *The numbers added* are **addends**; *e.g.* $3 + 10 = 13$; 3 and 10 are the addends.

27. *The result of addition* is the **sum**; *e.g.* 8 books and 7 books are 15 books; 15 is the sum.

28. *The addends* and the *sum* are called the **terms of addition.**

29. The sign $+$ indicates addition and is read *plus.*

30. The sign $=$, called the *sign of equality*, is read *equals*, and indicates that the expression preceding it has the same value as the expression following it.

31. In column addition, we should learn to *read* a column of figures, catching the combinations of two figures at a glance, just as we read a book without stopping to spell the words.

In the following examples, add by combinations of two figures as indicated in the units' column of example 1, especially where the sum of two figures does not exceed 12.

Test each sum by adding in reverse order. Time yourself and see how quickly you can get correct answers.

1.	2.	3.	4.	5.
235 } 7	838	925	28	89
682 }	209	7463	39	22
326 } 12	761	729	476	917
796 }	489	5834	834	483
899	117	609	276	9876
468 } 10	343	182	9013	2345
722 }	536	4231	1862	1076
326 } 11	498	5687	918	8864
245 }	704	21	4705	173
462 } 10	428	4598	8196	8888
348 }	193	234	729	2222

32. *Numbers to be added should be written so that units of the same order stand in the same column.*

In writing decimals, this will be accomplished by making the decimal points stand in a column.

Dimes, cents, and mills are expressed decimally as tenths, hundredths, and thousandths of a dollar.

33. *In examples* **1–5**, *add and test, timing yourself.*

1.	2.	3.	4.	5.
$ 34.25	$ 3.82	$ 9.764	$ 48.39	$186.424
69.87	14.32	5.20	446.19	4.2468
801.06	90.125	49.0742	72.934	.9374
12.14	6.187	.894	693.126	102.0738
198.28	2.353	2.763	28.987	84.176
79.63	4.589	.058	6.104	9.334
918.47	81.236	.9278	92.193	19.2376
29.13	9.88	4.615	8.56	5.28
40.88	71.24	.8923	.79	80.342
60.82	3.257	.705	42.138	9.76
41.98	4.934	400.0006	8.973	3.582

6. Add four dollars and ninety-one cents, sixty-three dollars seventy-five cents and eight mills, twenty-seven dollars forty-two cents and two mills, three hundred seventy-eight dollars twenty-nine cents and seven mills, nine hundred forty-two dollars, six dollars and seventy-eight cents.

7. Find the sum of eighty-one and eighty-one thousandths, sixty-three and twenty-nine hundredths, two hundred fourteen and one hundred fifty-eight ten-thousandths, five hundred sixteen thousandths, twenty-nine and forty-four ten-thousandths, six hundred eighty-four ten-thousandths, ninety-six ten-thousandths, fifty-six ten-thousandths, seventy-six and eight tenths.

SUBTRACTION

34. **Subtraction** *is the process of finding the difference between two numbers; e.g.* $21 - 7 = 14$; 13 cents $-$ 5 cents $=$ 8 cents.

35. *The number from which we subtract* is the **minuend.** *The number subtracted* is the **subtrahend.** *The result of subtraction* is the **difference** or **remainder.**

36. The difference is always the number that must be added to the subtrahend to obtain the minuend; *e.g.* $17 - 9 = 8$. 17 is the minuend, 9 is the subtrahend, and 8 is the difference or remainder.

37. *The minuend*, *subtrahend*, and *remainder* are the **terms of subtraction.**

38. The sign $-$ indicates subtraction and is read *minus*, or *less*.

39. Numbers to be subtracted should be written so that units of any order in the subtrahend stand under units of the same order in the minuend.

40. The correctness of work in subtraction may be tested by adding the remainder and the subtrahend. If this gives the minuend, the work is correct.

41. **1.** From 7364 take 3875.

7364
3875
3489

Since 5 units cannot be taken from 4 units, we take 1 ten from 6 tens, which, united with 4 units, makes 14 units. 5 tens remain in the minuend. 5 units from 14 units leave 9 units.

In a similar manner, 7 tens from 15 tens leave 8 tens, 8 hundreds from 12 hundreds leave 4 hundreds, and 3 thousands from 6 thousands leave 3 thousands.

The entire remainder is 3489.

Subtract and test, timing yourself:

2. 3042 825	**3.** 63895 4287	**4.** 2961 1953	**5.** 27409 8129
6. 63204 9183	**7.** 28654 17946	**8.** 10090 1095	**9.** 26130 9231
10. 31024 2736	**11.** 82431 8243	**12.** 63205 8164	**13.** 90372 82365
14. 64351 27809	**15.** 30756 7912	**16.** 12005 11996	**17.** 3102 471
18. 28143 9204	**19.** 2000 199	**20.** 7000 6999	**21.** 202 193

Decimals to be subtracted should be written so that the decimal point of the subtrahend comes directly below that of the minuend. Why?

When the subtrahend contains more decimal places than the minuend, we may subtract as though ciphers were annexed to

the minuend to make as many decimal places in the minuend as in the subtrahend.

Annexing ciphers to a decimal affects its value how? Why?

42. *Written*

1. Subtract 39.2479 from 167.3.

$$\begin{array}{r} 167.3 \\ 39.2479 \\ \hline 128.0521 \end{array}$$ *Difference* or *Remainder*

Find the remainders, and test without re-writing the numbers:

2. 43527 − 389.19

3. \$68.42 − \$42.93

4. \$365. − \$46.12

5. \$89.10 − \$23.562

6. 30. − 4.7619

7. 563. − .9999

8. \$913. − \$.258

9. 63.9542 − 18.156

10. 864.23 − 1.0009

11. 909.091 − 89.0902

12. 384.79 − 93.6215

13. 29.810 − 13.7901

14. 6.8001 − 5.80013

15. \$40.78 − \$29.783

16. 8050.706 − 805.0706

17. 423.7 − 42.37

18. 68023.4 − 234

19. \$121.075 − \$12.10

20. 76513.28 − 7762.47103

21. 83.54 − .7592

ADDITION AND SUBTRACTION

43. *Oral*

1. Add 84 and 79

84 + 70 = 154

154 + 9 = 163

Say 84, 154, 163.

2. Subtract 76 from 91

91 − 70 = 21

21 − 6 = 15

Say 91, 21, 15.

3. $35 + 19$	9. $29 + 34$	15. $57 + 24$
4. $46 + 15$	10. $83 - 47$	16. $\$.86 - \$.38$
5. $\$.83 - \$.14$	11. $\$.79 + \$.24$	17. $\$.79 + \$.42$
6. $\$.37 + \$.48$	12. $\$1.21 + \$.43$	18. $\$1.20 - \$.84$
7. $\$.92 - \$.25$	13. $115 - 37$	19. $\$2.80 + \$.70$
8. $88 - 39$	14. $36 + 45$	20. $\$1.50 - \$.73$

MULTIPLICATION

44. **Multiplication** *is taking one number as many times as there are units in another; e.g.* 6 times $9 = 54$.

45. The *number multiplied* is the **multiplicand**; the *number by which we multiply* is the **multiplier**; the *result of multiplication* is the **product**; *e.g.* 12 times 20 are 240. 20 is the *multiplicand*, 12 is the *multiplier*, and 240 is the *product*. 20 and 12 are factors of 240.

46. *The multiplier*, *multiplicand*, and *product* are the **terms of multiplication.**

47. *Each of the numbers that are multiplied to produce a number* is a **factor** *of that number; e.g.* 2, 3, and 5 are factors of 30 because $2 \times 3 \times 5 = 30$.

48. The *multiplier and multiplicand* are **factors** of the *product.* The product is the same in whatever order the factors are taken; *e.g.* 6 times $7 = 42$, and 7 times $6 = 42$; $3 \times 5 \times 4 = 60$, and $4 \times 3 \times 5 = 60$.

49. The sign $\times$, placed between two numbers, indicates that one of them is to be multiplied by the other.

50. Multiplication of integers is a short method of *adding equal integers; e.g.* 4×5 may be obtained by adding four 5's, thus $5+5+5+5=20$, or by adding five 4's, thus $4+4+4+4+4=20$.

51. *Oral*

1.

3	8	5	7	1	9	0	6	11	4	10	12	2
8	12	6	2	11	5	7	10	3	1	4	9	0

Multiply each number in the upper row by every number in the lower row.

2. What is the effect of annexing a cipher to an integer? Two ciphers? Three ciphers? Four ciphers?

3. How may we multiply a number by 100,000?

4. $300 = 3 \times 100 \quad 15 \times 300 = 15 \times 3 \times 100 = ?$

5. Multiply 25 by 10; by 100; by 1000.

6. Multiply 368 by 10; by 100; by 1000.

7. Multiply 9 by 10,000; by 100,000; by 1,000,000.

8. Multiply 12 by 40; by 400; by 4000.

9. Multiply 36 by 200; by 2000; by 20,000.

10. Multiply 70 by 1000; by 50; by 800; by 4000.

52. *Written*

1. Multiply 5972 by 689.

$$\begin{array}{r} 5972 \\ 689 \\ \hline 53748 \\ 47776 \\ 35832 \\ \hline 4114708 \end{array}$$

In multiplying 5972 by 689 we multiply by 9, by 80, and by 600, and add the results (called partial products). The sum of the partial products is the product required.

We omit the ciphers at the right of the partial products after the first. The second partial product is 477,760. Read the third partial product.

2. Multiply 864 by 403.

$$\begin{array}{r} 864 \\ 403 \\ \hline 2592 \\ 3456 \\ \hline 348192 \end{array}$$

When the multiplier contains a cipher, a partial product is omitted. Why? The next partial product begins two places to the left. Read the partial products in example 2.

In examples **3–17**, *find the products, and read each partial product:*

3. 368×29	**8.** 7359×83	**13.** 4907×199
4. 4596×38	**9.** 9138×43	**14.** 2051×7892
5. 6874×63	**10.** 294×137	**15.** 345×4006
6. 1024×99	**11.** 809×809	**16.** 4239×618
7. 2809×83	**12.** 799×835	**17.** 9999×8507

DIVISION

53. **Division** *is the process of finding one of two factors, when the other factor and the product are given; e.g.* 35 is the product of 7 and 5; 35 divided by 7 equals 5; 35 divided by 5 equals 7.

54. *The number divided* is the **dividend.**

55. *The number by which we divide* is the **divisor.**

56. *The number obtained by division* is the **quotient.**

57. *When the divisor is not exactly contained in the dividend, the part of the dividend that is left* is called the **remainder.**

Name the terms used in division.

58. The sign $\div$ between two numbers indicates that the first is to be divided by the second. Division may also be indicated

by writing the dividend above, and the divisor below, a horizontal line ; *e.g.*

$35 \div 7$ or $\frac{35}{7}$ means 35 divided by 7.
$3 \div 11$ or $\frac{3}{11}$ means 3 divided by 11.

59. Division may be tested by multiplying the divisor and quotient together and adding the remainder, if there is one. If this result equals the dividend, the work is correct. Why ?

60. 1. Divide 981,504 by 213.

```
      4608   Quotient
213)981504
    852
    1295
    1278
     1704
     1704
```

213 is not contained in 9 or 98, but is contained in 981 four times. This is 4 thousands because 981 is thousands. The remainder is 129 (thousands). Bringing down 5 (hundred), we have 1295 (hundred), which contains 213 six (hundred) times, with a remainder of 17 (hundred). Bringing down the cipher, we have 170 (tens), which does not contain 213 any tens times. We write 0 in tens' place in the quotient, bring down 4, and obtain 8 units for the last figure of the quotient, with no remainder.

NOTE 1. — In the above example, we may obtain the quotient figures by using the left-hand figure of the divisor for a *guide figure*, thus, 2 in 9, four times; 2 in 12, six times; 2 in 17, eight times.

When the second figure of the divisor is 7, 8, or 9, we may add 1 to the left-hand figure for a guide figure; *e.g.* if the divisor is 286, it is nearly 300; therefore, we may use 3 for a guide figure instead of 2. When the second figure of the divisor is 5 or 6, we may take for the guide figures both the left-hand figure and the left-hand figure plus 1.

NOTE 2. — When the divisor is not greater than 12, the quotient should be obtained by *short division ;* that is, by expressing only the dividend, divisor, and quotient. The quotient may then be placed either above or below the dividend, according to convenience, thus,

Divisor 12)564677 *Dividend*
$47056\frac{5}{12}$ *Quotient*

or

$47056\frac{5}{12}$ *Quotient*
12)564677 *Dividend*

2. $2785 \div 5$
3. $3928 \div 6$
4. $2890 \div 7$
5. $2796 \div 6$
6. $61{,}933 \div 9$
7. $137{,}401 \div 11$
8. $68{,}347 \div 12$
9. $7{,}640{,}328 \div 12$
10. $29{,}346 \div 11$
11. $3249 \div 10$
12. $32{,}695 \div 57$
13. $33{,}874 \div 49$
14. $99{,}003 \div 25$
15. $45{,}914 \div 59$
16. $335{,}630 \div 62$
17. $491{,}289 \div 73$
18. $216{,}428 \div 84$
19. $412{,}582 \div 58$
20. $981{,}384 \div 75$
21. $912{,}946 \div 24$
22. $427{,}473 \div 97$
23. $9664 \div 16$
24. $13{,}734 \div 18$
25. $62{,}826 \div 74$
26. $2{,}098{,}119 \div 987$
27. $67{,}117{,}890 \div 98$
28. $29{,}067{,}642 \div 1032$
29. $65{,}980{,}064 \div 5004$

30. What number multiplied by 351 will give 347,692 for a product?

31. 1,993,164 is the product of 489 and what other number?

32. By what must 982 be multiplied to obtain 3,537,492?

MULTIPLICATION AND DIVISION OF DECIMALS

61. *Oral*

1. Moving a figure one place to the right affects its value how? Two places? Three places? Four places?

2. Pointing off one decimal place in a number is the same as moving all the figures of the number one place to the right. How does it affect the value of the number?

3. Pointing off two decimal places in a number affects its value how? Three places? Four places?

4. How many decimal places must we point off in a number to divide it by 10? by 1000? by 100? by 10000?

5. Divide 12,468 by 10; by 100; by 1000; by 10,000.

6. Divide 367.54 by 10; by 100; by 1000; by 10,000.

7. How may any integer be divided by 10? by 100? by 1000?

8. How may any decimal be divided by 10? by 100? by 1000?

62. *Written*

1. Multiply 3.456 by 2.47.

$$\begin{array}{r} 3.456 \\ 2.47 \\ \hline 24192 \\ 13824 \\ 6912 \\ \hline 8.53632 \end{array}$$

$$3.456 = 3456 \div 1000$$

$$2.47 = 247 \div 100$$

$$3.456 \times 2.47 = 3456 \times 247 \div 1000 \div 100$$

$$3456 \times 247 = 853632$$

$$853632 \div 1000 \div 100 = 8.53632$$

We divide 853,632 by 1000 and 100 by pointing off 3 + 2, or 5, decimal places.

Summary

To multiply decimals, multiply them as integers. Point off in the product as many decimal places as there are decimal places in both factors. If the number of figures in the product is less than the required number of decimal places, prefix ciphers.

2. 32.5×17
3. 426×5.9
4. 3.08×6.7
5. 6.015×3.1
6. $42{,}805 \times .6$
7. $4.039 \times .24$
8. $.875 \times 1.9$
9. $13.55 \times .037$
10. $.068 \times 5.81$
11. $.351 \times .42$
12. $.4907 \times .018$
13. $.029 \times 568$
14. $2.879 \times .015$
15. $.030 \times 5960$
16. $42.691 \times .08$

17. 30.04×3.400	**20.** 93.50×78.92	**23.** $.999 \times 1000$
18. $.9756 \times 84$	**21.** $9.10 \times .086$	**24.** $.888 \times 8.88$
19. $.0231 \times .098$	**22.** $4.375 \times .092$	**25.** 15.15×98.07

63. *Oral*

1. One factor has three decimal places, the other four. How many has the product?

2. The product has four decimal places, the multiplicand one. How many has the multiplier?

3. The product has six decimal places, the multiplier three. How many has the multiplicand?

4. The product has four decimal places. What could be the number of decimal places in each of the factors?

64. *Written*

1. Divide 27.3587 by **4.7.**

```
      5.821
4.7 | 27.3587
      235
      ----
       385
       376
       ---
        98
        94
        --
        47
        47
```

The quotient and divisor are factors of what?

The dividend is what of the divisor and quotient?

When the factors are given, how may the number of decimal places in the product be found?

When the product and one factor are known, how may the number of decimal places in the other factor be found?

Summary

To divide decimals, divide as with integers and point off in the quotient as many decimal places as there are in the dividend, minus the number of decimal places in the divisor.

If the dividend contains fewer decimal places than the divisor, annex ciphers to make the required number.

NOTE. — It has been found helpful to make a dot, before dividing, as many places to the right of the decimal point in the dividend as there are decimal places in the divisor, and on a line with the tops of the figures, making the decimal point in the quotient directly over this dot, thus:

$$\begin{array}{r} 5.821 \\ 4.7\,\overline{)\,27.3{\cdot}587} \end{array}$$

Divide and test:

2. 27.72 by 3.85
3. 5074.65 by 56.7
4. 10.5252 by 2.94
5. 6.79592 by .76
6. 111.34 by 293
7. 16.35 by .025
8. 205.3758 by 64.2
9. 102.6 by .27
10. 7644 by .84
11. 6793.2 by .999
12. 340.2 by .042
13. 34,177 by 14.3
14. 190.0892 by 20.3
15. 8.19 by 195
16. 35.434 by .014
17. 8674.975 by .025
18. 397 by .125
19. 273.273 by 63.7
20. 1.906438 by .634
21. 33.84387 by 3890.1

65. *Find the quotients correct to three decimal places:*

1. $439 \div 86$
2. $92 \div 407$
3. $9.91 \div 13$
4. $8645 \div 237$
5. $42.356 \div .029$
6. $\dfrac{86.924}{.39}$
7. $\dfrac{999}{3025}$
8. $\dfrac{46}{987}$
9. $\dfrac{32}{416}$
10. $\dfrac{89.1}{1900}$
11. $\dfrac{29.41}{5000}$
12. $\dfrac{42.8}{639.4}$
13. $\dfrac{99}{880}$
14. $\dfrac{36.71}{760}$
15. $\dfrac{287.5}{4100}$

INDICATED OPERATIONS

66. The **signs of aggregation** are:

a. **Parentheses**	( )	*c.* **Brackets**	[ ]
b. **Braces**	{ }	*d.* **Vinculum**	———

An expression written within, or included by, any of these signs is to be treated as a single number.

67. The operations indicated within a sign of aggregation *must be performed before* those operations indicated outside the sign; *e.g.*

$$40 \times (9-6) \div [2+4] =$$
$$40 \times \quad 3 \quad \div \quad 6 \quad = 20$$

68. When several successive operations are indicated without the use of signs of aggregation, the indicated *multiplication and division* must be performed *before* the indicated *addition and subtraction; e.g.*

$$40 \times 9 - 6 \div 2 + 4 =$$
$$360 \quad - \quad 3 \quad + 4 = 361$$

69. *Oral*

1. $4+3\times 2=?$

2. $(4+3)\times 2=?$

3. $4\times 3+2=?$

4. $4\times(3+2)=?$

5. $8+4\div 2=?$

6. $(8+4)\div 2=?$

7. $8\times 7+21\div 3=?$

8. $(8+4)+9\div 3=?$

9. $9\times 7-21\div 3=?$

10. $9\times(2+3)\div 3=?$

11. $18\div 6+3=?$

12. $18\div(6+3)=?$

13. $11\times 7-3\times 2=?$

14. $11\times(7-3)\times 2=?$

15. $14\div 2+5\times 8=?$

16. $14\div(2+5)\times 8=?$

17. $36\div 9-6\div 3=?$

18. $36\div(9-6)+3=?$

70. *Written*

Perform the operations indicated:

1. $25.13 \div (47.2 - 43.7)$
2. $2.85 \times [9.6 + 3.02 + .86]$
3. $2.03 \times 607.015 - 59.6034$
4. $2.03 \times \overline{607.015 - 59.6034}$
5. $487 + 598 + \{6.45 - (20.3 - 14.35)\}$
6. $41.983 - .87 \times 10.3 + .047$
7. $(41.983 - .87) \times [10.3 + .047]$
8. $2310 \div [10 \times .7] + 604 \times 3.50$
9. $378.34 - 58.7 + 649.83 \times \overline{64.8 - 6.48}$

71. *Indicate the operations and solve:*

1. The difference between 496.37 and 288.037, multiplied by the quotient of 183.75 divided by 2.5.

2. A grocer bought a load of potatoes containing 48 bushels, at 65 cents a bushel, and sold them at 80 cents a bushel. What was his profit?

3. The product of three numbers is 18.902. Two of the numbers are .02 and 130. Find the other.

4. A machinist earns $1080 a year. He pays $180 a year for rent, $306 for food, and $369 for other expenses. In how many years, at that rate, can he save $900?

5. What number divided by 20.8 will give the quotient 85 and the remainder 11.7?

6. A city lot worth $1200 and three carriages at $190 each were given in exchange for 30 acres of land. At what price per acre was the land valued?

7. A confectioner put 2151 pounds of candy into boxes holding 1 pound, 3 pounds, and 5 pounds, respectively, using the same number of boxes of each kind. How many boxes were used for all the candy?

8. What number must be added to the sum of 342.807, 231.96, and 324.7 to equal the difference between 2107.62 and and 1009.006?

9. A certain number was divided, and 20.45 was both quotient and divisor. What was the number divided?

10. Make and solve a problem that requires the product of two numbers to be subtracted from the product of two other numbers.

11. Make and solve a problem that requires the product of two numbers to be added to the product of two other numbers and the sum divided by a certain number.

12. Divide by 37 the result obtained by adding 111 to the product of 148 and 6090.

13. A merchant bought 345 pounds of wool of one man, 3067 pounds of another, 468 pounds of another, and 384 pounds of another, and sold $\frac{1}{2}$ of it at 27 cents a pound. What did he receive for the part sold?

14. Make and solve a problem that may be indicated thus: $\$10 - (\$.35 + \$2.20 + \$6.19 + \$.18)$.

TESTS OF DIVISIBILITY

72. *The figures used in Arabic notation* are called **digits.** Name the digits.

73. *A number that can be exactly divided by* 2 is an **even number;** *e.g.* 2, 4, 18.

74. *A number that cannot be exactly divided by* 2 is an **odd number**; *e.g.* 3, 7, 19.

75. A number is exactly divisible

a. By 2, *if the digit in units' place is* 0 *or even; e.g.* 70, since the units' digit is 0; 35,976, since the units' digit is even.

b. By 4, *if the digits in units' and tens' places are* 0's; or *if the number expressed by them is divisible by* 4; *e.g.* 3100, 3976. How do you know? How can we tell without actual trial that 2398 is not divisible by 4?

c. By 8, *if the digits in units', tens', and hundreds' places are* 0's, or *if the number expressed by them is divisible by* 8; *e.g.* 11,000 and 37,112. How do you know? Why not 76,518?

d. By 3, *if the sum of its digits is divisible by* 3; *e.g.* 24,762, since $2 + 4 + 7 + 6 + 2$, or 21, is divisible by 3.

e. By 9, *if the sum of its digits is divisible by* 9; *e.g.* 397,647, since $3 + 9 + 7 + 6 + 4 + 7$, or 36, is divisible by 9.

f. By 5, *if the units' digit is* 0 or 5; *e.g.* 80; 115.

g. By 25, *if the units' and tens' figures are* 0's, or *if the number expressed by them is divisible by* 25; *e.g.* 1900; 8375.

h. By 125, *if the units', tens', and hundreds' figures are* 0's, or *if the number expressed by them is divisible by* 125; *e.g.* 13,000; 71,750.

i. By 10 *or a power of* 10, *if it contains as many* 0's *at the right of its significant figures as there are* 0's *at the right of the* 1 *in the divisor; e.g.* 390 is divisible by 10; 390,000 is divisible by 10,000.

j. By 6, *if the number is even and the sum of its digits is divisible by* 3; *e.g.* 21,108.

76. *Oral*

1. *Test each of the following numbers for divisibility by* 2, 4, 8, 3, 6, 9, 5, 25, *and* 125:

a.	1440	*f.*	22,825	*k.*	108,819	*p.*	429,000
b.	4950	*g.*	54,901	*l.*	90,626	*q.*	6,485,479
c.	4875	*h.*	1,629,433	*m.*	35,015	*r.*	20,525,750
d.	36,090	*i.*	302,275	*n.*	833,950	*s.*	9,031,330
e.	711,000	*j.*	181,365	*o.*	1,530,000	*t.*	1,234,567

2. An even number will not exactly divide an odd number. Why?

3. What numbers can be exactly divided by 6? Can you tell what numbers may be exactly divided by 18?

4. If 10 will divide a given number, what other numbers will divide the same number?

5. If 8 will divide a given number, what other numbers will divide it?

6. If 125 will divide a given number, what other numbers will divide it?

IDEAS OF PROPORTION

77. *Oral*

1. 36 is how many times 12? 12 is what part of 36? If 12 oranges cost \$.35, 36 oranges will cost how many times \$.35? How much will they cost? How many oranges can be bought for \$.70?

2. 125 is what part of 500? If 500 sheets of paper cost 90 cents, 100 sheets will cost what part of 90 cents? 100 sheets will cost how much? At the same rate, how many sheets can be bought for 9 cents? For 45 cents?

3. A 3-pound basket of grapes cost 10 cents. At the same rate, what must be paid for a 12-pound basket? How many pounds can be bought for 50 cents?

4. An automobile travels 67 miles in 4 hours. At the same rate, how far will it travel in 8 hours? In what time will it travel $33\frac{1}{2}$ miles?

5. A Vermont farmer made 7 pounds of maple sugar from 23 gallons of sap. At that rate, how many gallons of sap were required for 35 pounds of sugar? How many pounds of sugar could be made from 92 gallons of sap?

6. A man is paid for his work at the rate of $17 for 44 hours' work. What does he receive for 11 hours' work? How long must he work to earn $8\frac{1}{2}$?

7. A Kansas farmer raised 518 bushels of wheat on 14 acres of land. That was an average of how many bushels on two acres? 259 bushels of this crop were raised on how many acres? How many acres would be required to produce 1036 bushels, at the same rate? How many bushels could be raised on 42 acres at the same rate?

8. It required $110 a week to buy food for 40 boarders at a certain boarding house. What would be the weekly cost of food for 160 boarders, at the same rate? How many persons could be fed for $11 per week?

9. If 300 quarts of milk cost $21, what will 300 gallons cost, at the same price per quart?

10. How many books at 32¢ each will cost as much as 405 books at 96¢ each?

11. In how many minutes will a steamer, going 100 rods a minute, go as far as a man will row in 28 minutes, if he rows 25 rods a minute?

FACTORS AND MULTIPLES

78. *A number that exactly contains another number* is a **multiple** *of that number; e.g.* 21 is a multiple of 7. It is also a multiple of 3.

79. *A factor that is an integer* is called an **integral factor**; *e.g.* 8 is an integral factor of 56.

80. *A number that is not the product of integral factors other than itself and* 1 is a **prime number**; *e.g.* 2, 3, 5, 7, 11, and 13.

81. *A number that is the product of integral factors other than itself and* 1 is a **composite number**; *e.g.* 16, 24, 35, 1000.

82. *A factor that is a prime number* is a **prime factor**; *e.g.* 13 is a prime factor of 26.

NOTE. — In finding the factors of a number it is customary to consider only integral factors.

83. *Oral*

1. Give the factors of 51; 45; 99; 87; 96; 69; 84; 91.
2. Name three factors of 80.
3. Name as many factors of 24 as you can.
4. Of what numbers are 3, 5, and 11 the prime factors?
5. Name four multiples of 8.
6. 132 is the product of 11 and what other factor?
7. Name all the prime numbers smaller than 132.
8. 98 is the product of three factors. Two of them are 2 and 7. What is the other?
9. Of what number are 2, 3, 5, and 7 the prime factors?
10. Give the prime factors of 35; 45; 81; 63; 57; 38; 48; 51; 108; 231; 121; 144.

11. 5, 2, and what other number are the prime factors of 70?

12. Give two factors of 30 that are not prime.

13. What even number is prime?

84. Rule for finding whether a Number is Prime or Composite.

1. *If the given number is odd, divide it by* 3.
2. *If* 3 *gives a remainder, divide the given number by* 5.
3. *Continue this process, using each prime number in order as a divisor, until an exact divisor is found, or until the divisor equals or exceeds the quotient. If no exact divisor is found until the divisor used equals or exceeds the quotient, the number is prime. Otherwise it is composite. Even numbers need not be tested. Why?*

85. 1. Applying the tests given on page 26 instead of actually dividing by 3 and 5, determine whether 191 is prime or composite.

$7\overline{)191}$ 27–2 rem. $11\overline{)191}$ 17–4 rem. $13\overline{)191}$ 14–9 rem. $17\overline{)191}$ 11–4 rem.

191 is not divisible by 3 or 5. (How do we know?) Since the divisor, 17, is greater than the quotient, 11, and no exact divisor has been found, 191 must be prime.

Find whether each of these numbers is prime or composite:

2. 123	**6.** 263	**10.** 197	**14.** 1618	**18.** 401
3. 253	**7.** 143	**11.** 217	**15.** 487	**19.** 593
4. 187	**8.** 721	**12.** 361	**16.** 781	**20.** 3950
5. 561	**9.** 407	**13.** 1005	**17.** 437	**21.** 1241

86. *Written*

1. Find the prime factors of 7020.

2	7020
2	3510
3	1755
3	585
3	195
5	65
	13

By what kind of numbers do we divide? Why?
Which divisors do we use first?
What besides the divisors is a prime factor?

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 13$ Prime factors, *Ans.*

Find the prime factors of:

2. 112	**8.** 145	**14.** 3087	**20.** 1682
3. 420	**9.** 129	**15.** 667	**21.** 561
4. 660	**10.** 625	**16.** 310	**22.** 1001
5. 1111	**11.** 4293	**17.** 399	**23.** 1225
6. 1055	**12.** 1425	**18.** 1287	**24.** 6822
7. 4626	**13.** 1414	**19.** 253	**25.** 7290

CANCELLATION

87. Dividing both dividend and divisor by the same number affects the quotient how?

$$\frac{462}{66} = \frac{\cancel{2} \times \cancel{3} \times 7 \times \cancel{11}}{\cancel{2} \times \cancel{3} \times \cancel{11}} = 7 \text{ } \textit{Quotient;} \text{ or } \begin{array}{r} 7 \\ \cancel{77} \\ \cancel{231} \\ \cancel{462} \\ \cancel{66} \\ \cancel{33} \\ \cancel{11} \\ 1 \end{array} = 7 \text{ } \textit{Quotient}$$

In either method we divide both dividend and divisor by 2, by 3, and by 11

Taking out the same factor from both dividend and divisor is **cancellation.**

88. *Solve by cancellation:*

1. Divide $36 \times 54 \times 49 \times 38 \times 50$ by $70 \times 18 \times 30$.
2. $(28 \times 152 \times 48) \div (14 \times 19 \times 24 \times 2 \times 8) = ?$
3. $(182 \times 5 \times 54) \div (13 \times 35 \times 6) = ?$
4. What is the quotient of $108 \times 48 \times 80$ divided by $27 \times 72 \times 40$?
5. Divide $125 \times 45 \times 7 \times 10$ by $49 \times 5 \times 2 \times 225$.
6. Divide $65 \times 51 \times 11 \times 9 \times 4$ by $17 \times 20 \times 12 \times 11 \times 26$.
7. Divide $25 \times 26 \times 72 \times 14$ by $78 \times 9 \times 120$.
8. How many bushels of potatoes at 80 cents a bushel must be given in exchange for 45 pounds of tea at 64 cents a pound?
9. $(240 \times 36 \times 385 \times 26) \div (12 \times 154 \times 65)$.
10. What prime factor besides 19 and 11 has 8569?

LEAST COMMON MULTIPLE

89. *Oral*

1. $3 \times 8 = ?$ 24 is what of 3? Of 8?
2. $4 \times 6 = ?$ 24 is what of 4? Of 6?
3. Name all the numbers of which 24 is a multiple.
4. Define multiple.

90. *A number that exactly contains two or more numbers* is a **common multiple** *of those numbers; e.g.* 12 is a common multiple of 2, 3, 4, and 6. 36 also is a common multiple of 2, 3, 4, and 6.

Can you name any other common multiple of 2, 3, 4, and 6?

91. *The smallest number that exactly contains two or more numbers* is their **least common multiple** (L. C. M.); *e.g.* 18 is the least common multiple of 3, 6, and 9. 36 is a common multiple of 3, 6, and 9. Why is it not the least common multiple?

92. *Oral*

Find the L. C. M. of:

1. 5 and 3
2. 2, 5, and 4
3. 4 and 10
4. 18 and 12
5. 20 and 6
6. 8 and 12
7. 5, 6, and 2
8. 1, 8, 6, and 4
9. 2, 3, and 11
10. 5, 4, and 4
11. 7, 4, and 2
12. 10, 15, and 4
13. 2, 4, 8, and 12
14. 12, 5, and 15
15. 7 and 12
16. 14 and 6
17. 2, 15, 6, and 5
18. 4, 18, 3, and 12

93. When the least common multiple is a large number, the following direct method is employed in finding it:

Let it be required to find the L. C. M. of 12, 15, and 18.

$$12 = 2 \times 2 \times 3$$
$$15 = 3 \times 5$$
$$18 = 2 \times 3 \times 3$$

What kind of factors have we found? A number, in order to contain 12, must have what prime factors? What prime factors must it have in order to contain 15? 18?

A number that contains 12, 15, and 18 must have how many factors 2? How many factors 3? How many factors 5?

What is the smallest number that has the factors 2, 2, 3, 3, and 5? What, then, is the L. C. M. of 12, 15, and 18?

The prime factors may be easily found in this way:

2	12	15	18
3	6	15	9
	2	5	3

By what kind of numbers do we divide?

$2 \times 3 \times 2 \times 5 \times 3 = 180$ L. C. M.

94. *Find the L. C. M.:*

1. 36, 54, 60	8. 315, 60, 140, 210	15. 70, 15, 30, 14
2. 18, 24, 36	9. 24, 84, 54, 360	16. 48, 240, 21
3. 48, 144, 180	10. 75, 20, 35, 120	17. 9, 36, 90, 63, 42
4. 7, 9, 54	11. 98, 21, 35, 315	18. 25, 15, 60, 50
5. 72, 40, 48	12. 72, 48, 96, 192	19. 13, 19, 17
6. 90, 24, 36	13. 120, 18, 20, 60	20. 2, 3, 4, 5, 6, 7, 9
7. 105, 210, 21, 28	14. 48, 24, 40, 30	21. 21, 56, 45, 70

GREATEST COMMON DIVISOR

95. *A number that will exactly divide two or more numbers* is a **common divisor** *of those numbers; e.g.* 5 is a common divisor of 30, 40, and 60.

96. *The largest number that will exactly divide two or more numbers* is their **greatest common divisor** (G. C. D.); *e.g.* 10 is the greatest common divisor of 30, 40, and 60.

NOTE. — A common divisor is sometimes called a *common factor*, and the greatest common divisor is sometimes called the *highest common factor*.

97. *Numbers that have no common divisor* are **prime to each other**; *e.g.* 13 and 15.

98. *Oral*

1. *Find the G. C. D. of:*

a. 18, 9, 12	*e.* 56, 24, 40	*i.* 90, 45, 60
b. 40, 30, 35	*f.* 70, 28, 42	*j.* 54, 27, 36
c. 14, 30, 16	*g.* 33, 22, 121	*k.* 60, 24, 36, 48
d. 36, 30, 18	*h.* 21, 54, 39	*l.* 96, 32, 48

2. Name two numbers of which 11 is a common divisor.

3. Name three numbers of which 12 is a common divisor.

4. Name two numbers which are prime to each other.

5. What is the greatest number that will exactly divide 84, 60, and 36?

6. Name two numbers of which 13 is the G. C. D.

7. Tell which of these pairs of numbers are prime to each other:

a. 12 and 49 *b.* 48 and 60 *c.* 38 and 63 *d.* 16 and 45

99. *Written*

1. Find the greatest common divisor of 336, 504, and 924.

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$$
$$504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$$
$$924 = 2 \times 2 \times 3 \times 7 \times 11$$

$2 \times 2 \times 3 \times 7 = 84$ G. C. D.

Factoring the numbers and selecting the common prime factors, we find them to be 2, 2, 3, and 7. Since all of them are factors of each of the given numbers, their product, 84, is the greatest common divisor required.

The common prime factors may easily be found in this way:

2	336	504	924
2	168	252	462
3	84	126	231
7	28	42	77
	4	6	11

$2 \cdot 2 \cdot 3 \cdot 7$ Common prime factors.

Find the G. C. D.:

2. 84, 126
3. 180, 210
4. 448, 168
5. 396, 468
6. 280, 60, 80
7. 320, 144, 560
8. 252, 96, 120, 24
9. 120, 168, 216
10. 90, 270, 160
11. 305, 60, 90
12. 180, 72, 81
13. 176, 121, 165
14. 378, 126, 189
15. 144, 243, 135
16. 364, 143, 312
17. 576, 400, 240
18. 168, 630, 616, 350
19. 1980, 945, 245

20. Find the greatest number that will exactly divide 567, 378, and 504.

21. Find all the common prime factors of 630, 720, and 540.

22. Find the product of all the common prime factors of 216, 432, and 720.

23. Find a number that is prime to 350.

24. Name three numbers of which 13 is the greatest common divisor.

FRACTIONS

100. *One or more of the equal parts of a unit* is a **fraction**; *e.g.* $\frac{1}{8}$; $\frac{3}{5}$; $\frac{2}{7}$; $\$\frac{5}{10}$.

101. *A fraction is always an expression of* **division.** For example, if 1 inch is divided into 8 equal parts, each part is $\frac{1}{8}$ of

an inch. If a line 7 inches long is divided into 8 equal parts, one part is $\frac{7}{8}$ of an inch long. That is, 1 inch $\div 8 = \frac{1}{8}$ inch, and 7 inches $\div 8 = \frac{7}{8}$ inch.

Take your rule and draw a line 1 inch long. Divide it into 4 equal parts. How long is one part? Draw a line 3 inches long. Divide it into 4 equal parts. Measure one of the parts. 3 inches $\div 4 = ?$

Draw a line 5 inches long. Divide it into 8 equal parts. Measure one of the parts. 5 inches $\div 8 = ?$ $3 \div 7 = ?$ $9 \div 11 = ?$

102. *The number above the line in a fraction* is the **numerator.** *It is always a* **dividend.** In the fractions $\frac{1}{3}$, $\frac{7}{9}$, $\frac{15}{5}$, $\frac{23}{12}$, the numerators are 1, 7, 15, and 23.

103. *The number below the line in a fraction* is the **denominator.** *It is always a* **divisor.** In the fractions $\frac{1}{3}$, $\frac{7}{9}$, $\frac{15}{5}$, $\frac{23}{12}$, the denominators are 3, 9, 5, and 12.

104. *The numerator and the denominator* are the **terms of a fraction**; *e.g.* the terms of $\frac{7}{11}$ are 7 and 11.

105. *The* **value of a fraction** *is the quotient obtained by dividing the numerator by the denominator.*

REDUCTION OF FRACTIONS

106. *Changing the form of a number without changing its value* is **reduction**; *e.g.* 8 pt. = 4 qt.; \$7 = 700 ct.; 7 ft. = $2\frac{1}{3}$ yd.; $\frac{12}{4} = 3$; $\frac{18}{24} = \frac{3}{4}$; $\frac{2}{5} = \frac{6}{15}$.

REDUCTION TO LOWEST TERMS

107. A fraction is in **lowest terms** *when the numerator and denominator are prime to each other; e.g.* $\frac{8}{19}$, $\frac{15}{23}$, $\frac{6}{35}$.

108. *Oral*

1. A fraction is always an expression of what operation?

2. The numerator of a fraction is which term in division? The denominator? The value of the fraction?

3. Dividing the dividend and the divisor by the same number affects the quotient how?

4. Dividing the numerator and the denominator of a fraction by the same number affects the value of the fraction how?

Summary

A fraction may be reduced to lowest terms by dividing its terms by their common factors, continuing the process until the terms are prime to each other; or, *by dividing both terms of the fraction by their greatest common divisor.*

109. *Written*

1. Reduce $\frac{135}{165}$ to lowest terms.

$$\frac{135}{165} = \frac{27}{33} = \frac{9}{11}.$$

We divide both terms by 5 and then by 3. If the greatest common divisor, 15, is used, only one division is necessary.

Reduce to lowest terms:

2. $\frac{108}{165}$	**6.** $\frac{470}{2350}$	**10.** $\frac{194}{1746}$	**14.** $\frac{315}{1575}$	**18.** $\frac{182}{234}$
3. $\frac{135}{405}$	**7.** $\frac{112}{1888}$	**11.** $\frac{1386}{4284}$	**15.** $\frac{51}{68}$	**19.** $\frac{581}{4067}$
4. $\frac{315}{665}$	**8.** $\frac{647}{1294}$	**12.** $\frac{231}{495}$	**16.** $\frac{19}{38}$	**20.** $\frac{275}{325}$
5. $\frac{132}{143}$	**9.** $\frac{384}{1152}$	**13.** $\frac{1368}{3420}$	**17.** $\frac{13}{91}$	**21.** $\frac{176}{221}$

22. Express in lowest terms $637 \div 833$.

23. Express in lowest terms the quotient of 288 divided by 504.

24. Express in lowest terms $\frac{2016}{3528}$.

REDUCTION OF IMPROPER FRACTIONS TO INTEGERS OR MIXED NUMBERS

110. *A fraction whose numerator is smaller than its denominator* is a **proper fraction**; *e.g.* $\frac{2}{9}$, $\frac{1}{15}$, $\frac{16}{17}$. The value of a proper fraction is always less than 1.

111. *A fraction whose numerator equals or exceeds its denominator* is an **improper fraction**, *e.g.* $\frac{5}{5}$, $\frac{9}{8}$, $\frac{24}{12}$. The value of an improper fraction compares how with 1?

112. *A number that is composed of an integer and a fraction* is a **mixed number**; *e.g.* $5\frac{3}{7}$, $10\frac{1}{3}$, $201\frac{6}{11}$, 18.25.

113. *Oral*

1. A fraction is an expression of what operation?

2. Define the value of a fraction.

3. The value of an improper fraction is always an integer or a mixed number. How may we find it?

Find the values of:

4. $\frac{5}{4}$	8. $\frac{10}{3}$	12. $\frac{39}{6}$	16. $\frac{82}{11}$	20. $\frac{67}{12}$	24. $\frac{64}{15}$
5. $\frac{6}{5}$	9. $\frac{13}{3}$	13. $\frac{48}{8}$	17. $\frac{94}{9}$	21. $\frac{36}{11}$	25. $\frac{56}{14}$
6. $\frac{8}{2}$	10. $\frac{17}{5}$	14. $\frac{51}{4}$	18. $\frac{144}{2}$	22. $\frac{84}{12}$	26. $\frac{121}{11}$
7. $\frac{11}{4}$	11. $\frac{25}{6}$	15. $\frac{73}{8}$	19. $\frac{235}{5}$	23. $\frac{39}{13}$	27. $\frac{136}{12}$

114. *Written*

1. $\frac{196}{4}$	**3.** $\frac{392}{14}$	**5.** $\frac{697}{32}$	**7.** $\frac{1612}{20}$	**9.** $\frac{830}{67}$
2. $\frac{456}{25}$	**4.** $\frac{717}{27}$	**6.** $\frac{785}{41}$	**8.** $\frac{989}{45}$	**10.** $\frac{3900}{129}$
11. $\frac{723}{18}$	**14.** $\frac{8603}{49}$	**17.** $\frac{993}{36}$		**20.** $\frac{5973}{66}$
12. $\frac{6437}{16}$	**15.** $\frac{879}{36}$	**18.** $\frac{902}{88}$		**21.** $\frac{7934}{79}$
13. $\frac{879}{37}$	**16.** $\frac{338}{18}$	**19.** $\frac{3522}{171}$		**22.** $\frac{20501}{124}$

REDUCTION OF INTEGERS AND MIXED NUMBERS TO IMPROPER FRACTIONS

115. *Written*

1. Reduce $38\frac{7}{9}$ to a fraction.

$38 = 38 \times 9$ ninths $= 342$ ninths.

342 ninths plus 7 ninths $= 349$ ninths.

The work may be expressed thus: $38\frac{7}{9} = \frac{349}{9}$ *Ans.*

$$\begin{array}{r} 9 \\ \hline 342 \\ 7 \\ \hline 349 \end{array}$$

Reduce to improper fractions:

2. $17\frac{2}{5}$	**9.** $26\frac{3}{15}$	**16.** $125\frac{1}{30}$	**23.** $217\frac{7}{18}$
3. $15\frac{3}{4}$	**10.** $45\frac{10}{11}$	**17.** $159\frac{3}{8}$	**24.** $248\frac{5}{6}$
4. $29\frac{13}{14}$	**11.** $57\frac{1}{9}$	**18.** $167\frac{5}{6}$	**25.** $459\frac{5}{16}$
5. $25\frac{5}{13}$	**12.** $25\frac{7}{9}$	**19.** $24\frac{20}{31}$	**26.** $160\frac{11}{134}$
6. $59\frac{7}{8}$	**13.** $35\frac{5}{11}$	**20.** $55\frac{35}{44}$	**27.** $383\frac{7}{16}$
7. $170\frac{3}{5}$	**14.** $57\frac{3}{14}$	**21.** $129\frac{4}{11}$	**28.** $646\frac{54}{61}$
8. $497\frac{7}{8}$	**15.** $61\frac{7}{8}$	**22.** $216\frac{18}{30}$	**29.** $559\frac{30}{48}$

30. In 560 there are how many 5ths?

31. Reduce 250 to 16ths.

32. Change $12\frac{5}{8}$ to 16ths.

33. Change 156 to a fraction whose denominator shall be 12.

LEAST COMMON DENOMINATOR

116. *Fractions whose denominators are alike have a* **common denominator**; *e.g.* 60 is a common denominator of $\frac{3}{60}$, $\frac{18}{60}$, and $\frac{27}{60}$.

117. *Fractions having the smallest possible common denominator have their* **least common denominator**; *e.g.* $\frac{1}{20}$, $\frac{6}{20}$, $\frac{9}{20}$.

118. *Oral*

1. We have found that when we add fractions having different denominators, we must first change them to fractions having the same denominator. What shall we call that denominator?

2. Since the common denominator must contain all the given denominators, it must be what of those denominators? (A number that exactly contains two or more other numbers is what?)

3. The *least common denominator*, then, must be which multiple of the given denominators?

4. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{2}{3}$ to fractions having the least common denominator.

119. *Written*

Change $\frac{7}{10}$, $\frac{8}{15}$, $\frac{16}{33}$, and $\frac{17}{30}$ to fractions having the least common denominator.

	7	8	16	17
2	10	15	33	30
3	5	15	33	15
5	5	5	11	5
	1	1	11	1

$2 \times 3 \times 5 \times 11 = 330$ L. C. M.

$330 \div 10 = 33 \quad \frac{7 \times 33}{10 \times 33} = \frac{231}{330}$

$330 \div 15 = 22 \quad \frac{8 \times 22}{15 \times 22} = \frac{176}{330}$

$330 \div 33 = 10 \quad \frac{16 \times 10}{33 \times 10} = \frac{160}{330}$

$330 \div 30 = 11 \quad \frac{17 \times 11}{30 \times 11} = \frac{187}{330}$

$\frac{231}{330}, \frac{176}{330}, \frac{160}{330}, \frac{187}{330}$ *Ans.*

Change to fractions having the least common denominator:

1. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
2. $\frac{2}{3}, \frac{3}{5}, \frac{7}{9}$
3. $\frac{7}{8}, \frac{3}{10}, \frac{1}{2}$
4. $\frac{6}{7}, \frac{9}{13}$
5. $\frac{5}{7}, \frac{8}{12}, \frac{13}{28}, \frac{3}{14}$
6. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{5}{8}$
7. $7, \frac{7}{8}, \frac{9}{10}, \frac{3}{5}$
8. $\frac{1}{2}, \frac{1}{4}, \frac{5}{6}, \frac{9}{12}$
9. $\frac{1}{3}, \frac{3}{4}, \frac{3}{5}, \frac{4}{9}, \frac{9}{10}$
10. $\frac{1}{2}, \frac{7}{8}, \frac{9}{10}, \frac{3}{4}, \frac{4}{5}$
11. $\frac{24}{35}, \frac{15}{91}, \frac{37}{65}$
12. $\frac{4}{10}, \frac{5}{8}, \frac{1}{4}, \frac{11}{12}$
13. $\frac{5}{6}, \frac{15}{34}, \frac{5}{12}, \frac{3}{17}$
14. $\frac{4}{9}, \frac{21}{26}, \frac{7}{18}, \frac{5}{13}$
15. $\frac{2}{7}, \frac{8}{21}, \frac{5}{13}, \frac{1}{63}, \frac{8}{11}$

ADDITION OF FRACTIONS AND MIXED NUMBERS

120. *A number is in its* **simplest form** *when it is in the form of an integer, or a proper fraction in its lowest terms, or a mixed number whose fractional part is in its lowest terms; e.g.* 18, $\frac{3}{7}$, and $5\frac{1}{9}$ are in simplest form; $\frac{26}{2}$, $\frac{21}{24}$, $\frac{46}{9}$, and $8\frac{3}{6}$ are not. Why?

Answers should always be expressed in simplest form.

121. *Written*

1. Add $\frac{7}{9}$, $\frac{9}{16}$, and $\frac{11}{12}$.

2	9	16	12
2	9	8	6
3	9	4	3
	3	4	1

$2 \times 2 \times 3 \times 3 \times 4 = 144$, L. C. D.

$\frac{7}{9} = \frac{112}{144}$

$\frac{9}{16} = \frac{81}{144}$

$\frac{11}{12} = \frac{132}{144}$

$\frac{325}{144} = 2\frac{37}{144}$ *Sum*

2. Add $10\frac{3}{5}$, $7\frac{5}{8}$, and $6\frac{3}{4}$.

$10\frac{3}{5} = 10\frac{24}{40}$
$7\frac{5}{8} = 7\frac{25}{40}$
$6\frac{3}{4} = 6\frac{30}{40}$
$23\frac{79}{40} = 24\frac{39}{40}$ *Sum*

We add the integers and fractions separately, and then unite the sums.

RULE. — *To add fractions, reduce them to their least common denominator, add the numerators, place the sum over the common denominator, and reduce the result to simplest form.*

When there are integers or mixed numbers, add the integers and the fractions separately, and unite the results.

Add:

3. $\frac{3}{5}$, $\frac{4}{7}$, $\frac{13}{21}$, $\frac{17}{15}$

4. $\frac{1}{3}$, $\frac{2}{9}$, $\frac{1}{15}$, $\frac{5}{6}$

5. $\frac{7}{8}$, $\frac{11}{12}$, $\frac{17}{18}$, $\frac{23}{24}$, $\frac{26}{27}$

6. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{13}$, $\frac{19}{21}$

7. $16\frac{1}{12}$, $24\frac{1}{18}$, $43\frac{11}{14}$

8. $\frac{1}{2}$, $\frac{2}{9}$, $\frac{3}{11}$, $\frac{7}{18}$, $\frac{17}{33}$

9. $7\frac{3}{4}$, $8\frac{2}{5}$, $\frac{5}{7}$, $\frac{3}{8}$

10. $12\frac{3}{4}$, $19\frac{5}{6}$, $28\frac{9}{16}$

11. $19\frac{5}{7}$, $18\frac{3}{8}$, $15\frac{1}{2}$, $12\frac{1}{14}$

12. $7\frac{2}{3}$, $9\frac{7}{15}$, $6\frac{3}{4}$, $4\frac{5}{8}$

13. During a storm, a tree was broken off $17\frac{3}{8}$ feet from the ground. The piece broken off was $41\frac{5}{6}$ feet long. How tall was the tree before it was broken?

14. The subtrahend is $89\frac{31}{42}$ and the remainder $49\frac{11}{56}$. What can you find? Find it.

15. A rectangular field is $509\frac{5}{18}$ feet long and $347\frac{1}{16}$ feet wide. How many feet of fence are required to inclose it?

SUBTRACTION OF FRACTIONS AND MIXED NUMBERS

122. Fractions must have a common denominator in order that one may be subtracted from the other.

123. *Written*

1. From $11\frac{1}{15}$ take $\frac{4}{9}$.

$11\frac{1}{15} = 11\frac{3}{45}$

$\frac{4}{9} = \frac{20}{45}$ How is 45 obtained?

$\frac{13}{45}$ *Difference*

In subtracting mixed numbers, if the fraction in the subtrahend is greater than that in the minuend, one integral unit of the minuend must be united with the fraction to form an improper fraction, before subtracting.

2. From $29\frac{1}{6}$ take $13\frac{7}{10}$.

$29\frac{1}{6} = 29\frac{5}{30} = 28\frac{35}{30}$ How do we obtain $\frac{35}{30}$?

$13\frac{7}{10} = 13\frac{21}{30} = 13\frac{21}{30}$

$15\frac{14}{30} = 15\frac{7}{15}$ *Difference*

3.	$12\frac{1}{3} - \frac{2}{5}$	**11.**	$\frac{17}{40} - \frac{3}{8}$	**19.**	$18\frac{7}{10} - 13\frac{7}{20}$
4.	$26\frac{1}{2} - 4\frac{2}{3}$	**12.**	$18\frac{1}{7} - 15\frac{1}{2}$	**20.**	$381\frac{3}{15} - 332\frac{7}{10}$
5.	$242\frac{3}{5} - 33\frac{5}{7}$	**13.**	$110\frac{1}{2} - 109\frac{7}{9}$	**21.**	$62\frac{5}{9} - 46\frac{5}{7}$
6.	$298\frac{3}{5} - 149\frac{2}{3}$	**14.**	$112\frac{1}{5} - 15\frac{1}{3}$	**22.**	$330\frac{1}{2} - 140\frac{3}{11}$
7.	$43\frac{7}{8} - 22\frac{8}{9}$	**15.**	$17\frac{2}{3} - 15\frac{4}{5}$	**23.**	$189\frac{7}{9} - 143\frac{1}{2}$
8.	$26\frac{2}{3} - 8\frac{5}{6}$	**16.**	$146\frac{3}{5} - 127\frac{7}{9}$	**24.**	$407\frac{1}{17} - 398\frac{1}{3}$
9.	$\frac{3}{7} - \frac{3}{8}$	**17.**	$167\frac{2}{3} - 76\frac{3}{4}$	**25.**	$90\frac{2}{5} - 48\frac{4}{7}$
10.	$1\frac{7}{8} - 1\frac{7}{12}$	**18.**	$42\frac{1}{3} - 36\frac{5}{7}$	**26.**	$81\frac{4}{11} - 37\frac{1}{2}$

MULTIPLICATION AND DIVISION COMBINED

124. *Written*

$(20 \div 4) \times (21 \div 7) = ?$

$20 \div 4 = 5 \quad 21 \div 7 = 3 \quad 5 \times 3 = 15$ *Ans.*

or,

$$\frac{20}{4} \times \frac{21}{7} = \frac{20 \times 21}{4 \times 7} = 15 \quad \textit{Ans.}$$

20 and 21 are dividends and 4 and 7 are divisors. The result is the same whether we make each division separately and then multiply the quotients, or divide the product of the dividends by the product of the divisors. In many cases the latter way is easier, because we may use cancellation; *e.g.*

a. $(20 \div 4) \times (21 \div 7) = \left(\frac{20}{4} \times \frac{21}{7}\right) = \frac{\overset{5}{\cancel{20}} \times \overset{3}{\cancel{21}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{7}}} = 15$ *Ans.*;

b. $(18 \div 7) \times (28 \div 24) \times (210 \div 15) = \frac{\overset{3}{\cancel{18}} \times \overset{7}{\cancel{28}} \times \overset{42}{\cancel{210}}}{7 \times \underset{6}{\cancel{24}} \times \underset{5}{\cancel{15}}} = 42$ *Ans.*

Find results:

1. $(22 \div 11) \times (12 \div 5) \times (25 \div 6) \times (25 \div 2)$
2. $(16 \div 4) \times (20 \div 6) \times (55 \div 10) \times (42 \div 11)$
3. $(52 \div 13) \times (35 \div 21) \times (12 \div 7) \times (21 \div 3)$
4. $\frac{28}{9} \times \frac{42}{4} \times \frac{36}{7} \times \frac{63}{14}$
5. $(36 \div 27) \times (35 \div 75) \times (25 \div 12) \times (12 \div 7)$
6. $(7 \div 49) \times (68 \div 7) \times (14 \div 8) \times (35 \div 17)$
7. $(40 \div 39) \times (52 \div 10) \times (34 \div 13) \times (125 \div 10)$
8. $(70 \div 35) \times (26 \div 20) \times (68 \div 13) \times (125 \div 35)$
9. $(70 \div 17) \times (68 \div 24) \times (35 \div 7)$
10. $\frac{49}{25} \times \frac{75}{42} \times \frac{108}{26} \times \frac{98}{15}$
11. $\frac{51}{60} \times \frac{49}{56} \times \frac{24}{34} \times \frac{17}{5} \times \frac{20}{3}$
12. Multiply the quotient of 79 divided by 24 by the quotient of 168 divided by 79.

MULTIPLICATION OF FRACTIONS

125. *Any integer may be* **expressed as a fraction** *by writing it as a numerator with* 1 *for a denominator; e.g.* 5 is the same as $\frac{5}{1}$; 19 is the same as $\frac{19}{1}$; $\frac{2}{3} \times 7 \times \frac{15}{42}$ is the same as $\frac{2}{3} \times \frac{7}{1} \times \frac{15}{42}$.

126. *The word* **of**, *between fractions, means the same as the* **sign of multiplication**; *e.g.* $\frac{2}{3}$ of $\frac{8}{9} = \frac{2}{3} \times \frac{8}{9}$; $\frac{2}{3}$ of $4 \times \frac{7}{16} = \frac{2}{3} \times \frac{4}{1} \times \frac{7}{16}$; $\frac{1}{2} \div \frac{1}{3}$ of $\frac{2}{5} = \frac{1}{2} \div (\frac{1}{3} \times \frac{2}{5})$.

127. *An indicated multiplication of two or more fractions* is called a **compound fraction**; *e.g.* $\frac{2}{7} \times \frac{8}{9}$; $\frac{3}{16} \times \frac{12}{21} \times \frac{24}{25}$; $\frac{5}{7}$ of $\frac{3}{8}$.

128. *Written*

1. Find the product of $\frac{2}{3}$, $\frac{5}{7}$, and $\frac{9}{16}$.

Each of these fractions indicates what operation?

Since all the numerators are dividends and all the denominators are divisors, we may find the result by dividing the product of the numerators by the product of the denominators, using cancellation:

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{5}{7} \times \frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}} = \frac{15}{56} \text{ Ans.}$$

Find the products:

2. $\frac{3}{4} \times \frac{4}{5}$	**8.** $\frac{5}{6}$ of $\frac{9}{10}$ of $\frac{4}{5}$	**14.** $\frac{7}{15} \times \frac{2}{3} \times 7 \times \frac{15}{42}$
3. $\frac{7}{10} \times \frac{5}{8}$	**9.** $\frac{7}{8} \times \frac{7}{12} \times \frac{6}{7}$	**15.** $\frac{6}{7}$ of $\frac{2}{3}$ of $\frac{8}{9} \times 14$
4. $\frac{2}{3}$ of $\frac{7}{9}$ of $\frac{9}{14}$	**10.** $\frac{9}{16} \times \frac{8}{9} \times \frac{3}{4}$	**16.** $\frac{7}{8} \times \frac{5}{11} \times \frac{16}{15} \times 22$
5. $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{12}{20}$	**11.** $\frac{14}{25} \times \frac{10}{7} \times \frac{8}{9}$	**17.** $\frac{7}{9} \times 2 \times \frac{3}{8}$ of $\frac{7}{12}$
6. $\frac{4}{9} \times \frac{7}{19} \times \frac{9}{14}$	**12.** $\frac{16}{17} \times 34 \times \frac{5}{8}$	**18.** $\frac{8}{10} \times \frac{3}{17}$ of $34 \times \frac{5}{16}$
7. $\frac{5}{6}$ of $\frac{1}{2}$ of $\frac{2}{3}$	**13.** $\frac{3}{5}$ of $\frac{2}{9}$ of 15	**19.** $\frac{13}{15} \times \frac{9}{26} \times \frac{8}{27}$

129. *Mixed numbers may be reduced to improper fractions and then multiplied;* thus,

$$1\tfrac{2}{3} \times 8\tfrac{1}{2} \times \tfrac{5}{17} \times 4 =$$

$$\frac{5}{3} \times \frac{\overset{1}{\cancel{17}}}{\underset{1}{\cancel{2}}} \times \frac{5}{\underset{1}{\cancel{17}}} \times \frac{\overset{2}{\cancel{4}}}{1} = \frac{50}{3} = 16\tfrac{2}{3} \text{ Ans.}$$

Written

1. $6\frac{3}{4} \times 4\frac{1}{2}$
2. $12\frac{9}{11} \times 7\frac{1}{3}$
3. $77\frac{2}{3} \times 3$
4. $85\frac{4}{5} \times 47\frac{8}{9}$
5. $78\frac{1}{2} \times 17\frac{3}{5}$
6. Find $\frac{6}{11}$ of $\frac{5}{8}$ of $\frac{11}{12}$ of $8\frac{1}{5}$
7. $8\frac{1}{2} \times 8\frac{2}{3}$
8. $15\frac{2}{3} \times 8\frac{6}{7} \times \frac{5}{47}$
9. $5\frac{3}{11} \times 5\frac{2}{3} \times \frac{5}{8}$
10. $\frac{7}{15} \times 3\frac{1}{2} \times 9\frac{1}{7} \times 6\frac{5}{6}$
11. Multiply $\frac{7}{8}$ by $10\frac{1}{2}$ by $\frac{2}{3}$ by $\frac{3}{7}$ by $6\frac{2}{3}$.
12. Multiply: *a*. $25\frac{7}{9}$ by $24\frac{2}{7}$. *b*. 116 by $\frac{7}{8}$.
13. $15\frac{3}{4} \times 12\frac{4}{5} \times 20$
14. $17\frac{1}{2} \times 15\frac{4}{9} \times \frac{3}{7}$
15. $9\frac{1}{4} \times \frac{4}{37} \times 2\frac{1}{5}$
16. $6\frac{1}{4} \times \frac{11}{25} \times \frac{4}{11}$
17. $\frac{3}{4} \times 4 \times 5\frac{1}{2}$
18. $\frac{7}{20} \times 80 \times 5\frac{1}{2}$

130. In multiplying a large mixed number by an integer, time may be saved by multiplying the whole number and the fraction separately, then adding the products, thus:

Written

1. $845\frac{5}{11} \times 8 = ?$

$845\frac{5}{11}$

8

$3\frac{7}{11}$ $\quad \frac{5}{11} \times 8 = 3\frac{7}{11}$

6760 $\quad 845 \times 8 = 6760$

$6763\frac{7}{11}$ $\quad 6760 + 3\frac{7}{11} = 6763\frac{7}{11}$ *Ans.*

2. 897×5
3. $2081\frac{1}{15} \times 6$
4. $628\frac{3}{7} \times 15$
5. $830\frac{5}{18} \times 18$
6. $2037\frac{1}{16} \times 28$
7. $3547\frac{6}{7} \times 100$
8. $2301\frac{7}{20} \times 200$
9. $381\frac{5}{18} \times 27$
10. $3079\frac{5}{6} \times 15$
11. $413\frac{2}{15} \times 20$
12. $6283\frac{7}{9} \times 18$
13. $3100\frac{2}{15} \times 35$
14. $2050\frac{6}{13} \times 52$
15. $8310\frac{1}{9} \times 51$
16. $2806\frac{17}{20} \times 90$

DIVISION OF FRACTIONS

131. Divide $\frac{35}{72}$ by $\frac{5}{8}$.

Since $\frac{35}{72}$ is a product and $\frac{5}{8}$ is one of its factors, we may state the question thus:

$$\frac{35}{72} = \frac{5}{8} \times ? \quad \text{or} \quad \frac{35}{72} = \frac{5 \times ?}{8 \times ?}$$

In order to find the required factor we must divide the numerator 35 by 5, and the denominator 72 by 8, thus:

$$\frac{35 \div 5}{72 \div 8} = \frac{7}{9}.$$

That is exactly what we should do if the question were:

$$\frac{35}{72} \times \frac{8}{5} = ? \qquad \frac{\overset{7}{\cancel{35}}}{\underset{9}{\cancel{72}}} \times \frac{\cancel{8}}{\cancel{5}} = \frac{7}{9}.$$

The latter method is the more convenient, especially when the numerator of the divisor is not exactly contained in the numerator of the dividend, or the denominator of the divisor in the denominator of the dividend.

Therefore, *to divide by a fraction we interchange the terms of the divisor and multiply.*

132. *Written*

1. Divide $4\frac{2}{3}$ by $5\frac{3}{5}$.

Solution: $4\frac{2}{3} \div 5\frac{3}{5} = \frac{14}{3} \div \frac{28}{5} = \frac{\cancel{14}}{3} \times \frac{5}{\underset{2}{\cancel{28}}} = \frac{5}{6}$ *Ans.*

How do we treat mixed numbers before dividing?

2. Divide 47 by $6\frac{1}{2}$.

Solution: $47 \div 6\frac{1}{2} = \frac{47}{1} \div \frac{13}{2} = \frac{47}{1} \times \frac{2}{13} = \frac{94}{13} = 7\frac{3}{13}$ *Ans.*

How do we treat integers?

3. $\frac{8}{11} \div \frac{4}{33}$	**9.** $3\frac{1}{2} \div \frac{14}{17}$	**15.** $18 \div \frac{7}{10}$	**21.** $2\frac{3}{4} \div 1\frac{1}{16}$
4. $\frac{5}{8} \div \frac{3}{4}$	**10.** $6\frac{1}{4} \div \frac{9}{20}$	**16.** $15 \div \frac{5}{6}$	**22.** $4\frac{1}{3} \div 3\frac{1}{2}$
5. $\frac{7}{12} \div \frac{5}{6}$	**11.** $2\frac{3}{11} \div 5\frac{3}{4}$	**17.** $\frac{7}{9} \div 14$	**23.** $7\frac{1}{9} \div 6\frac{1}{7}$
6. $\frac{7}{18} \div \frac{3}{4}$	**12.** $\frac{3}{10} \div 4\frac{1}{5}$	**18.** $\frac{16}{24} \div 8$	**24.** $\frac{59}{27} \div \frac{14}{27}$
7. $\frac{14}{27} \div \frac{2}{3}$	**13.** $\frac{14}{15} \div 6\frac{1}{2}$	**19.** $2\frac{3}{4} \div 5\frac{1}{2}$	**25.** $\frac{21}{55} \div \frac{7}{11}$
8. $\frac{14}{15} \div \frac{7}{10}$	**14.** $2 \div \frac{8}{9}$	**20.** $7\frac{1}{8} \div 1\frac{1}{2}$	**26.** $12\frac{2}{9} \div 4\frac{10}{27}$

27. By what must $\frac{24}{15}$ be multiplied to make $\frac{36}{77}$?

28. One factor of $\frac{34}{40}$ is $\frac{17}{80}$. What is the other?

29. *a.* $\frac{15}{28} \times ? = \frac{9}{32}$. *b.* $? \times \frac{22}{39} = \frac{4}{15}$.

30. *a.* $\frac{91}{99} = 1\frac{17}{36} \times ?$ *b.* $\frac{49}{209} = \frac{84}{95} \times ?$

133. *A fraction whose terms are integers* is a **simple fraction**; *e.g.* $\frac{11}{12}$ is a simple fraction.

134. *A fraction that has a fraction in either or both of its terms* is a **complex fraction**; *e.g.* $\frac{3}{8\frac{2}{5}}$, $\frac{\frac{2}{3}}{16}$, $\frac{5\frac{1}{2}}{25}$, $\frac{3\frac{2}{3}}{7\frac{4}{9}}$, and $\frac{\frac{2}{3} \div 9}{1\frac{3}{8} - \frac{2}{5}}$ are complex fractions. Read each fraction.

A complex fraction is merely an indicated division of fractions, made by writing the dividend above a line and the divisor below the line, just as a simple fraction is an indicated division of integers; therefore,

A complex fraction may be reduced to a simple fraction by dividing the expression above the line by the expression below the line.

135. *Written*

1. Reduce $\frac{7}{8\frac{2}{3}}$ to a simple fraction.

$$\frac{7}{8\frac{2}{3}} = 7 \div 8\tfrac{2}{3} = \frac{7}{1} \div \frac{26}{3} = \frac{7}{1} \times \frac{3}{26} = \frac{21}{26} \textit{ Ans.}$$

2. Reduce $\frac{\frac{5}{17}}{40}$ to a simple fraction.

$$\frac{\frac{5}{17}}{40} = \frac{5}{17} \div 40 = \frac{\cancel{5}}{17} \times \frac{1}{\cancel{40}_{8}} = \frac{1}{136} \textit{ Ans.}$$

3. Reduce $\frac{7\frac{5}{8}}{2\frac{13}{20}}$ to its simplest form.

$$\frac{7\frac{5}{8}}{2\frac{13}{20}} = 7\tfrac{5}{8} \div 2\tfrac{13}{20} = \frac{61}{8} \div \frac{53}{20} = \frac{61}{\cancel{8}_{2}} \times \frac{\cancel{20}^{5}}{53} = \frac{305}{106} = 2\tfrac{93}{106} \textit{ Ans.}$$

In examples **4–13** *change the given complex fractions to simplest form:*

4. $\frac{7\frac{1}{9}}{\frac{16}{27}}$ **6.** $\frac{25\frac{3}{4}}{6}$ **8.** $\frac{17}{12\frac{4}{5}}$ **10.** $\frac{7\frac{1}{2}}{\frac{3}{4} \times 12\frac{1}{2}}$ **12.** $\frac{\frac{1}{4} \text{ of } \frac{5}{8}}{\frac{2}{3} \text{ of } \frac{5}{6}}$

5. $\frac{8\frac{1}{2}}{\frac{4}{5}}$ **7.** $\frac{\frac{14}{18}}{15}$ **9.** $\frac{\frac{5}{8}}{\frac{11}{16}}$ **11.** $\frac{2\frac{1}{4} \div \frac{3}{5}}{4\frac{1}{12} - \frac{5}{6}}$ **13.** $\frac{\frac{3}{4} \text{ of } 5\frac{1}{2}}{\frac{7}{8}}$

14. If $\frac{3}{5}$ of an acre of land is worth $\$72\frac{3}{10}$, what is the value of 3 acres at the same rate?

15. There are $5\frac{1}{2}$ yards in a rod. How many rods in $140\frac{1}{4}$ yards?

16. At $\$6\frac{3}{4}$ a ton how many tons of coal can be bought for $\$77\frac{5}{8}$?

136. In division, if the divisor contains a common fraction that cannot easily be reduced to a decimal, it is sometimes helpful to multiply both dividend and divisor by the denominator of the fraction, thus making both dividend and divisor integers, or simple decimals; *e.g.:*

$$.021\tfrac{1}{3})\overline{.416}$$

Multiplying both dividend and divisor by 3, and then dividing,

19.5 *Quotient*

$$.064)\overline{1.248\cdot0}$$

Written

137. *In the following examples find the quotients correct to two decimal places:*

1. $8.48 \div 19\frac{7}{8}$	**5.** $28.9 \div 7\frac{1}{7}$	**9.** $7.9\frac{3}{8} \div 4\frac{7}{9}$
2. $3.56 \div 41\frac{2}{3}$	**6.** $30.05 \div .17\frac{6}{7}$	**10.** $9.375 \div .16\frac{2}{3}$
3. $9.305 \div 9\frac{5}{6}$	**7.** $8.3 \div .07\frac{2}{3}$	**11.** $3.23 \div 1.21\frac{1}{3}$
4. $35.3125 \div 12\frac{3}{7}$	**8.** $.0135 \div .02\frac{10}{11}$	**12.** $.484 \div .5\frac{1}{3}$

COMPARATIVE STUDY OF DECIMALS AND COMMON FRACTIONS

138. *A fraction that is expressed by writing the numerator above and the denominator below a line* is a **common fraction;** *e.g.* $\frac{5}{7}$, $\frac{49}{81}$. (See § **9** for definition of decimal fractions.)

All decimal fractions may be expressed as common fractions without reducing them; *e.g.* $.0104 = \frac{104}{10000}$. What common fractions can be expressed as decimals without reducing them?

139. *When a decimal fraction is expressed without its denominator, by using the decimal point, it is said to be expressed in the* **decimal form.**

Oral

$.7 = \frac{7}{10}$, or 7 divided by 10

$.305 = \frac{305}{1000}$, or 305 divided by 1000

$.58\frac{1}{3} = \frac{58\frac{1}{3}}{100}$, or $58\frac{1}{3}$ divided by 100

In like manner tell the meanings of the following decimals:

1. .18	6. $.189\frac{1}{2}$	11. $.29\frac{1}{8}$	16. $.005\frac{1}{2}$
2. .41	7. $.16\frac{5}{9}$	12. $.007\frac{2}{13}$	17. .0034
3. .216	8. $.239\frac{2}{3}$	13. $.03\frac{2}{3}$	18. .165
4. .879	9. $.548\frac{6}{11}$	14. $.5134\frac{5}{8}$	19. $.000171\frac{1}{2}$
5. .200	10. $.73\frac{1}{2}$	15. $.4070\frac{1}{8}$	20. $.000\frac{3}{7}$

140. *A decimal may be reduced to a common fraction in simplest form by expressing it as a common fraction and reducing to lowest terms: e.g.* $.85 = \frac{85}{100} = \frac{17}{25}$; $13.8 = 13\frac{8}{10} = 13\frac{4}{5}$; $.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{\cancel{50}}{3} \times \frac{1}{\cancel{100}\ 2} = \frac{1}{6}$.

141. *Written. Reduce the following decimals to common fractions or mixed numbers in simplest form:*

1. .28	9. .875	17. .0054	25. $.003\frac{3}{4}$
2. .125	10. .375	18. .250	26. $.125\frac{4}{5}$
3. .235	11. .55	19. .1375	27. $.87\frac{1}{2}$
4. .75	12. .0025	20. $.04\frac{3}{8}$	28. $.66\frac{2}{3}$
5. .164	13. .56	21. $.12\frac{1}{2}$	29. $.136\frac{7}{8}$
6. .82	14. .68	22. $.62\frac{1}{2}$	30. 116.25
7. .138	15. 16.075	23. $.06\frac{1}{4}$	31. $2.33\frac{1}{3}$
8. .425	16. .0125	24. $.018\frac{1}{8}$	32. $.03\frac{1}{7}$

33. $22.62\frac{1}{2}$	38. $.07\frac{3}{11}$	43. $.162\frac{1}{2}$	47. $179.00\frac{5}{9}$
34. $7.087\frac{1}{2}$	39. $.126\frac{2}{3}$	44. $40.40\frac{5}{7}$	48. $8.00\frac{25}{36}$
35. $6.13\frac{1}{2}$	40. $.12\frac{6}{7}$	45. $61.41\frac{1}{9}$	49. $890.90\frac{4}{9}$
36. $58.06\frac{1}{4}$	41. $.166\frac{2}{3}$	46. $42.1\frac{5}{6}$	50. $8.000\frac{40}{41}$
37. $49.6\frac{7}{8}$	42. $.19\frac{7}{12}$		

142. Since a fraction is an expression of division, *a common fraction may be reduced to a decimal by dividing its numerator by its denominator.*

Before dividing, place a decimal point after the dividend. Annex ciphers as they are needed ; *e.g.*

$$\frac{7}{16} = 7.0000 \div 16 = .4375$$

$$39\frac{7}{16} = 39.4375$$

143. *Written. Reduce to decimals:*

1. $\frac{2}{5}$	11. $3\frac{1}{16}$	21. $\frac{38}{640}$	31. $19\frac{19}{125}$
2. $\frac{3}{5}$	12. $\frac{3}{80}$	22. $\frac{11}{250}$	32. $\frac{1}{625}$
3. $\frac{4}{5}$	13. $\frac{3}{20}$	23. $\frac{75}{200}$	33. $\frac{7}{125}$
4. $\frac{3}{4}$	14. $\frac{4}{25}$	24. $2\frac{3}{32}$	34. $12\frac{6}{625}$
5. $\frac{1}{8}$	15. $\frac{54}{36}$	25. $13\frac{7}{20}$	35. $14\frac{29}{80}$
6. $\frac{3}{8}$	16. $\frac{47}{64}$	26. $19\frac{7}{32}$	36. $9\frac{9}{1250}$
7. $\frac{7}{8}$	17. $\frac{43}{80}$	27. $12\frac{9}{25}$	37. $13\frac{1}{40}$
8. $\frac{7}{16}$	18. $\frac{17}{20}$	28. $\frac{11}{500}$	38. $2\frac{21}{250}$
9. $\frac{12}{40}$	19. $\frac{9}{160}$	29. $\frac{23}{400}$	39. $5\frac{8}{32}$
10. $\frac{5}{32}$	20. $\frac{56}{125}$	30. $\frac{87}{125}$	40. $\frac{23}{3200}$

144. *A fraction in lowest terms whose denominator contains other prime factors than 2 and 5 cannot be reduced to an exact entire decimal; e.g.* $\frac{2}{3}$, $\frac{5}{9}$, $\frac{15}{22}$, $\frac{8}{11}$, $\frac{19}{26}$, $\frac{41}{35}$.

Such a fraction may be reduced to a decimal of nearly the same value by carrying the division to a certain number of decimal places, thus :

Reduce $\frac{19}{26}$ to a decimal of four places.

$.7307\frac{9}{13}$ *Ans.*
$26.\overline{)19.0000}$

.7307 is almost equal to $\frac{19}{26}$.
The exact value of $\frac{19}{26}$ is $.7307\frac{9}{13}$.
The result may be expressed, .7307 + .

Written. Reduce to decimals of three places :

1. $\frac{2}{3}$	**7.** $\frac{24}{31}$	**13.** $8\frac{4}{15}$	**19.** $62\frac{10}{29}$
2. $\frac{6}{7}$	**8.** $\frac{7}{48}$	**14.** $33\frac{1}{9}$	**20.** $\frac{236}{388}$
3. $\frac{3}{11}$	**9.** $\frac{13}{17}$	**15.** $681\frac{4}{11}$	**21.** $1\frac{85}{275}$
4. $\frac{5}{9}$	**10.** $\frac{243}{19}$	**16.** $53\frac{8}{17}$	**22.** $143\frac{5}{42}$
5. $\frac{4}{9}$	**11.** $\frac{3}{240}$	**17.** $\frac{12}{23}$	**23.** $216\frac{9}{71}$
6. $\frac{12}{13}$	**12.** $5\frac{2}{9}$	**18.** $\frac{9}{61}$	**24.** $\frac{356}{301}$

A COMMON FRACTION AT THE END OF A DECIMAL

145. $.21\frac{1}{2} = .2 + (\frac{1}{2}$ of $\frac{1}{10}$, or $\frac{1}{20}$, or .05).
$.2 + .05 = .25$.

In a similar manner we may show that,

$.27\frac{1}{2} = .275$, $.3841\frac{1}{2} = .3845$, etc.
Also, that $.21\frac{1}{4} = .225$, $.341\frac{1}{4} = .3425$, etc.
Also, that $.83\frac{3}{4} = .875$, $.063\frac{3}{4} = .0675$, etc.
Also, that $.91\frac{1}{8} = .9125$, $.073\frac{3}{8} = .07375$, etc.

Oral. Express as entire decimals :

1.	*a.* $.81\frac{1}{2}$	*b.* $\$.471\frac{1}{2}$	*c.* $.5601\frac{1}{2}$	*d.* $271\frac{1}{2}$	*e.* $.041\frac{1}{2}$
2.	*a.* $.91\frac{1}{4}$	*b.* $3.81\frac{1}{4}$	*c.* $\$9.001\frac{1}{4}$	*d.* $\$.0391\frac{1}{4}$	*e.* $.01451\frac{1}{4}$
3.	*a.* $.023\frac{3}{4}$	*b.* $21.13\frac{3}{4}$	*c.* $\$21.063\frac{3}{4}$	*d.* $.00333\frac{3}{4}$	*e.* $.00903\frac{3}{4}$

4. *a.* $.17\frac{1}{4}$ *b.* $6.9\frac{1}{2}$ *c.* $80.03\frac{1}{2}$ *d.* $212\frac{3}{4}$ *e.* $61.9\frac{1}{4}$

5. *a.* $642\frac{1}{2}$ *b.* $63.97\frac{1}{2}$ *c.* $15.0\frac{3}{4}$ *d.* $24.00\frac{1}{2}$ *e.* $29.00\frac{1}{4}$

6. *a.* $1.40\frac{3}{4}$ *b.* $2.25\frac{3}{4}$ *c.* $10.0\frac{1}{2}$ *d.* $25\frac{3}{4}$ *e.* $4.000\frac{1}{2}$

ALIQUOT PARTS

146. *One of the equal parts of a number* is an **aliquot part** *of that number; e.g.* 8 oz. is an aliquot part of 16 oz. because 8 oz. is $\frac{1}{2}$ of 16 oz.; $16\frac{2}{3}$ cents is an aliquot part of 100 cents because $16\frac{2}{3}$ cents $= \frac{1}{6}$ of 100 cents.

Find the number of cents in $\$\frac{1}{2}$; $\$\frac{1}{3}$; $\$\frac{1}{4}$; $\$\frac{1}{5}$; $\$\frac{1}{6}$; $\$\frac{1}{7}$; $\$\frac{1}{8}$; $\$\frac{1}{10}$; $\$\frac{1}{12}$; $\$\frac{1}{20}$.

The answers you have given are all what kind of parts of a dollar?

Prove the correctness of the following table:

PARTS OF A DOLLAR

5 cents $= \$\frac{1}{20}$	$33\frac{1}{3}$ cents $= \$\frac{1}{3}$
$6\frac{1}{4}$ cents $= \$\frac{1}{16}$	$37\frac{1}{2}$ cents $= \$\frac{3}{8}$
$8\frac{1}{3}$ cents $= \$\frac{1}{12}$	50 cents $= \$\frac{1}{2}$
10 cents $= \$\frac{1}{10}$	$62\frac{1}{2}$ cents $= \$\frac{5}{8}$
$12\frac{1}{2}$ cents $= \$\frac{1}{8}$	$66\frac{2}{3}$ cents $= \$\frac{2}{3}$
$16\frac{2}{3}$ cents $= \$\frac{1}{6}$	75 cents $= \$\frac{3}{4}$
25 cents $= \$\frac{1}{4}$	$87\frac{1}{2}$ cents $= \$\frac{7}{8}$

This table should be committed to memory like the multiplication table, because its use will shorten many problems; *e.g.* 33 books, at $\$.16\frac{2}{3}$, each, will cost $33 \times \$\frac{1}{6} = \$5\frac{1}{2}$.

When handkerchiefs are $12\frac{1}{2}\,¢$ apiece, \$3 will buy as many handkerchiefs as $\$3 \div \$\frac{1}{8}$, or $\$3 \times \frac{8}{1} = 24$ handkerchiefs. *Ans.*

147. *Oral*

1. $\$.14\frac{2}{7}$ = what part of a dollar?

2. $\frac{2}{7}$ of a dollar are how many cents? $\frac{3}{7}$? $\frac{4}{7}$? $\frac{5}{7}$? $\frac{6}{7}$?

3. 20 cents are what part of a dollar? 40 cents? 60 cents? 80 cents? Which of these is an aliquot part of \$1?

4. Mention three aliquot parts of 12; two aliquot parts of 10; five aliquot parts of 64.

5. Give four numbers of which $8\frac{1}{3}$ is an aliquot part.

6. What is the cost of 28 pineapples when they are bought at the rate of $\$.14\frac{2}{7}$ apiece?

7. At $\$.33\frac{1}{3}$ a pound how many pounds of butter will \$5 buy?

8. A man bought five dozen cans of corn at the rate of $8\frac{1}{3}$ cents apiece. What did they cost?

148. *Written*

1. Find the cost of the following:

a. 166 pounds of pork at $12\frac{1}{2}$ cents a pound.
b. 248 lb. of veal at $16\frac{2}{3}$ cents a pound.
c. 148 boxes of strawberries at 25 cents a box.
d. 250 lb. of butter at $37\frac{1}{2}$ cents a pound.
e. 150 lb. of honey at 25 cents a pound.
f. 640 bars of soap at $6\frac{1}{4}$ cents a bar.
g. 960 dozen of eggs at $\$.16\frac{2}{3}$ a dozen.
h. 32 yd. of dress goods at $\$.33\frac{1}{3}$ a yard.
i. 328 grammar school arithmetics at $\$.62\frac{1}{2}$ apiece.
j. 656 steel shovels at $\$.87\frac{1}{2}$ each.

2. At $\$.33\frac{1}{3}$ a yard, how many yards of linen can be bought for \$150?

3. How many bushels of barley can be bought for \$624, at \$.75 a bushel?

4. At $\$.66\frac{2}{3}$ each, how many pocket knives can be purchased for \$64?

5. When butter is 25 cents a pound, how many pounds can be bought for \$650?

6. How many articles, at $14\frac{2}{7}$ cents each, can be purchased for \$154?

7. At $87\frac{1}{2}$ cents each, how many books can be bought for \$1456?

8. How many boxes of berries can be bought for \$250, at $16\frac{2}{3}$ cents a box?

9. At $\$.62\frac{1}{2}$ each, how many pairs of gloves can be bought for \$120?

10. A man bought potatoes at $\$.62\frac{1}{2}$ a bushel and sold them at $\$.87\frac{1}{2}$ a bushel. His profit was \$160. How many bushels were sold?

11. A dealer spent \$120 for chickens, and the same amount for ducks. The chickens cost him $16\frac{2}{3}$ cents, and the ducks $12\frac{1}{2}$ cents a pound. How many more pounds of ducks than chickens did he purchase?

SPECIAL CASES IN MULTIPLICATION

149. I. To multiply a number by 10 or a power of 10.

Each removal of a figure one place to the left multiplies its value by 10.

Therefore, if the multiplicand is an integer, annex as many ciphers as there are ciphers in the multiplier; if the multiplicand is a decimal, move the decimal point as many places to the right as there are ciphers in the multiplier.

This is the same as moving all the figures to the left.

II. To multiply a number by 25.

$$25 = 100 \div 4$$

Therefore, multiply the given number by 100 and divide the product by 4. (Apply Case I in multiplying by 100.)

III. To multiply a number by 125.

$$125 = 1000 \div 8$$

Therefore, multiply the given number by 1000 and divide the product by 8. (Apply Case I in multiplying by 1000.)

IV. To multiply a number by:

a. $.33\frac{1}{3}$, multiply the given number by $\frac{1}{3}$.

b. .25, multiply the given number by $\frac{1}{4}$.

c. $.16\frac{2}{3}$, multiply the given number by $\frac{1}{6}$.

d. $.14\frac{2}{7}$, multiply the given number by $\frac{1}{7}$.

e. .125, multiply the given number by $\frac{1}{8}$.

V. To multiply a number by 99.

$$99 = 100 - 1$$

Therefore, multiply the given number by 100 and subtract the multiplicand from the product thus obtained.

How can we multiply a number by 999?

VI. To multiply by a number having one or more ciphers at the right.

Multiply by the significant figures of the multiplier, and annex to the product thus obtained, as many ciphers as there are in the multiplier. Explain.

SPECIAL CASES IN DIVISION

150. I. To divide a number by 10 or a power of 10.

Each removal of a figure one place to the right divides its value by 10.

Therefore, if the dividend is an integer, point off as many decimal places as there are ciphers in the divisor; if the dividend is a decimal, move the decimal point as many places to the left as there are ciphers in the divisor.

This is the same as moving all the figures to the right.

II. To divide a number by 25.

$$25 = \frac{100}{4}$$

A number divided by $\frac{100}{4}$ equals the number multiplied by $\frac{4}{100}$, or the number multiplied by 4 and divided by 100.

Therefore, multiply the given number by 4, and divide the product by 100. (Apply Case I in dividing by 100.)

III. To divide a number by 125.

Multiply the given number by 8, and divide the product by 1000. Explain.

IV. To divide a number:

a. By $33\frac{1}{3}$, point off two decimal places and multiply by 3.

b. By $16\frac{2}{3}$, point off two decimal places and multiply by 6.

c. By $14\frac{2}{7}$, point off two decimal places and multiply by 7.

d. By $.33\frac{1}{3}$, divide by $\frac{1}{3}$. By $.16\frac{2}{3}$? By .125? By $.14\frac{2}{7}$? By 2.5? By $.11\frac{1}{9}$?

V. To divide by a number with one or more ciphers at the right.

Point off in the dividend as many decimal places as there are ciphers at the right of the divisor, then divide by the remaining figures.

Explain.

In the following examples, find results by the methods given in sections **149** *and* **150**.

151. *Oral*

1. 34×10
2. $305 \div 100$
3. 13×200
4. 24×25
5. $.00374 \times 1000$
6. 36×25
7. 406×100
8. $830 \div 10$
9. $18 \times 10{,}000$
10. $1750 \div 10{,}000$
11. $48{,}000 \div 1200$
12. $360 \times .33\frac{1}{3}$
13. $48 \times .125$
14. $875 \div 10{,}000$
15. $700 \div 25$
16. 32×125
17. $32 \times .125$
18. $14{,}000 \div 700$
19. $8100 \div 900$
20. $28 \times .14\frac{2}{7}$
21. $42 \times .14\frac{2}{7}$
22. $72 \times .16\frac{2}{3}$
23. $11 \div .16\frac{2}{3}$
24. $7 \div .125$
25. $35 \times .14\frac{2}{7}$
26. $72 \times .125$
27. $13 \div .16\frac{2}{3}$
28. $180 \times .16\frac{2}{3}$
29. $560 \times .25$
30. 19×3000
31. $27 \times .33\frac{1}{3}$
32. $.16\frac{2}{3} \times 78$
33. 48×125
34. $31 \div 25$
35. $.24 \times 16\frac{2}{3}$
36. $2.8 \div 70$
37. $.56 \times .125$
38. $.025 \times 3000$
39. $21 \times .14\frac{2}{7}$
40. 1.6×25
41. $.8 \div 25$
42. $80 \times .125$
43. $.008 \times 1100$
44. $.5 \times 500$
45. $.7 \div .125$

152. *Written*

1. $3.85 \times 15{,}000$
2. 572×99
3. $9.07 \div 25$
4. $63.47 \div .16\frac{2}{3}$
5. $83.750 \div .125$
6. $1263 \div 14\frac{2}{7}$
7. $864 \times .125$
8. $9654 \div .125$
9. $4.17 \times .33\frac{1}{3}$
10. $6350 \div 25$
11. $47.832 \div 125$
12. $83{,}496 \div 4000$
13. $8397 \div 900$
14. $87{,}416 \times .14\frac{2}{7}$
15. $5.364 \times .125$
16. 2397×99
17. 453×999
18. $88.9 \times 17{,}000$
19. $62{,}408 \times .125$
20. $80{,}172 \times .33\frac{1}{3}$
21. 5.07×125
22. $635 \div 25$
23. $4.302 \div .16\frac{2}{3}$
24. $23.8 \div 125$
25. $23.8 \div .125$

ACCOUNTS AND BILLS

153. *Individuals or groups of individuals transacting business with one another* are called **parties** *to the transactions.*

154. *A record of the business transactions between two parties* is an **account.**

Merchants and others transacting any considerable amount of business have sets of books in which accounts are kept.

There are various methods of recording transactions as they occur, and arranging them in the different books to suit the needs of the business; but it is the general custom to copy all accounts, finally, in a ledger, which shows in clear, concise form the complete account of each person, firm, or company with whom business is transacted.

In the ledger, each person's account is headed by his name. Money paid, services rendered, and goods *sold to* him are entered in the *left-hand* or **debit** side of the account.

Money, services, and goods *received from* him are entered in the *right-hand* or **credit** side of the account.

Accounts are balanced at regular intervals by footing the debit side and the credit side, and subtracting the smaller amount from the greater. The difference, called the **balance**, is then entered on the side having the smaller amount. This makes the two sides equal, or balance, each other.

Horizontal lines are then drawn below the footings, and the balance is brought forward to begin the account for a new period.

The following form represents the ledger account of Adolph Schiller, for October and November, at a hardware store. The number in the column at the left of dollars refers to the page of the day book (the book in which each day's transactions are recorded as they occur) in which the item was first entered.

Dr. *Adolph Schiller* Cr.

1907						1907					
Oct.	7	Nails	6	$ 5	75	Oct.	20	Locks	49	$ 1	75
	11	Doors	32	18	50		28	Cash	54	50	
	19	Door trimmings	48	7	48		31	Balance		41	48
	25	Windows	51	61	50						
				93	23					93	23
1907											
Nov.	1	Bal. brought for'd		41	48	Nov.	8	Cash	58	20	
	10	White lead	60	7	40		15	Labor	63	2	50
	17	Shovel	65		75						

Note. — Many bookkeepers omit from the ledger the words describing the articles bought and sold, as *nails*, *locks*, etc., leaving those columns blank. This practice is increasing.

Copy Mr. Schiller's account for November; balance it, and make the proper entry to begin the account for December.

At the time of balancing an account, it is customary to send to the debtor a copy of the account for the period for which the balance is made. This is called a **bill** or **statement.** Many business houses send monthly statements to their customers. Some business houses send a bill, or **invoice,** as it is called, with each list of goods sold.

155. *The party who sells the goods* is the **creditor**; *the party who purchases the goods* is the **debtor.**

In common usage, the term *debtor* means *any one who owes a debt*, and the term *creditor* means *any one to whom a debt is owed.*

156. A bill may be defined as follows:

A formal statement of a debtor's account, or of goods sold, services rendered, or cash paid, made out by the creditor and presented to the debtor, is a **bill.**

A bill should always contain these things:

1. The time and place of making out the bill.

2. The debtor's name and address.

3. The creditor's name and address.

4. A list of the items — that is, the goods sold, money paid, or services rendered, with the amount of each item.

5. The date of each transaction, if any of them occur at any other time than that of making out the bill.

6. The amount, or footing, of the bill.

157. When a bill is paid, the creditor *receipts* the bill by writing at the bottom, "Received Payment," followed by the date, and his own name. This shows that the bill has been paid. The debtor keeps the receipted bill. Why?

Sometimes a clerk, an agent, or a bookkeeper of the creditor receives the money for payment of a bill. He should then write the creditor's name under the words "Received Payment," and under the creditor's name, his own name or initials.

158. The following forms illustrate some of the ways in which bills are made out:

FORM I

Boston, October 1, 1908.

Mrs. John Doe
1421 West Street,
Boston, Mass.

Bought of **R. H. Stearns & Company**

140 TREMONT STREET BOSTON

	SEPT.					
98	7	1 GLOVES		4.00	4.00	
128		1 1/4 VEILING'	.25	.31		
		1 1/4 "	.50	.63	.94	
					4.94	

FORM 2

SHEET IRON PIPING A SPECIALTY — SHEET METAL WORK OF EVERY DESCRIPTION

ORDER NO. 51673 — PITTSBURG, PA. Aug. 16, 1908

Mr. John R. McKavney — 2528 Penn Ave.

BOUGHT OF **GRANT C. NOBBS,**

SHEET METAL WORK AND HARDWARE

PENN PERFECT FURNACES. TIN ROOFING. STOVES AND HOUSEFURNISHING GOODS.

SALESROOM AND WAREHOUSE 2623 AND 2625 PENN AVE. BOTH PHONES OFFICE AND WORKS 2520 AND 2522 SMALLMAN ST.

TERMS 30 days

$\frac{1}{2}$	Doz. 8" Hinge Hasps	@ 1.05		53		
	40 – 10%					29
10	Gr. $\frac{3}{4}$ – 7 Screws	@ .90	9	00		
	$87\frac{1}{2}$ – 5%				1	08
$\frac{1}{2}$	Doz. #338 Half Hatchets	@ 6.00			3	00
3	" #1 Sledge Handles	@ 1.10			3	30
2	Kegs Common Nails	@ 2.10			4	20
1	Doz. Rules #68	@ .95				95
2	Stanley Planes	@ .94			1	88
$\frac{3}{4}$	Doz. Niagara Handled Axes	@ 6.75			5	06
					19	76

159. *Oral*

1. Name the debtor and the creditor in Form 1. In Form 2. In Form 3. In Form 4.

2. Which of the forms contain both debit and credit items?

3. Which of the forms contain items for which bills have been previously sent?

4. In Form 5, what is the amount of the credit items? Of the debit items? What is the balance?

160. *Written*

1. Make out the bill sent to Mr. Schiller (see page 62) on Dec. 1, 1907, supplying names, dates, and addresses. The

FORM 3

A. J. REACH CO.

MAKERS OF FINE SPORTING GOODS

Terms Strictly
Net 30 Days or 2% 10 Days

PHILADELPHIA 8/30/07

YOUR ORDER 8/27/07
Sold to Simmons Hardware Co.,
OUR ORDER 8/30/07

SHIPPED VIA St. Louis, Mo.
1 Case Weight 150 lbs. Star Union Line

SHIPPED							
20		Dz 0 Balls	15.00	300	00		
1		Dz 5A Catcher's Mitts	84.00	84	00		
6/12		Dz 3 First Baseman's Mitts	48.00	24	00		
9/12		Dz OC Fielder's Gloves	30.00	22	50		
						430	50

RECEIVED PAYMENT
9 mo. 10 day, 1907
A. J. REACH CO.
per J. K. MacDonald

first debit item should be, "Account rendered, $41.48," because that was the balance shown on the bill which he received Nov. 1.

Receipt the bill as though you were cashier for the creditor.

Make out and foot bills of the following items, supplying dates and addresses; receipt them, either as creditor, or as the creditor's agent:

2. Bought by W. J. McDermott from Bentley and Settle,

20 bbl. patent flour, $5 per bbl; 2000 lb. granulated sugar, $5.15 per hundredweight; 300 lb. Java coffee, 22¢ per pound; 250 lb. maple sugar, 14¢ per pound.

McDermott has paid $125 in money.

FORM 4

STATEMENT

Philadelphia, Pa., March 1, 1907

A. J. Reach Company

TULIP AND PALMER STREETS

Norvell Shapleigh Hdwe. Co.
St. Louis, Mo.

TERMS;---NET CASH 30 DAYS

		Amount Rendered				
Feb.	1	To Mdse., as per bill rendered	31	91		
	19		115	50		
	22		127	20		
	25		733	39		
					1 008	00
		Cr.				
	12	By Mdse.	2	63		
	15	" Cash	29	28		
	26	" "	100	00		
					131	91
					876	09

3. A. Walrath sold to Donald Anderson,

5 lb. rice at 9 ¢.
4 dozen eggs at 21 ¢.
2 brooms at 35 ¢.
18 lb. chicken at 22 ¢.
2 bu. new potatoes at 35¢ per peck.
8 lb. tomatoes at 13¢ per pound.

FORM 5

Dey Brothers & Co

Dry Goods, Millinery, Dress Making, Carpets, Upholstery, &c.

SOUTH SALINA & JEFFERSON STS.

Syracuse, N.Y., Feb. 1 1908

Folio 645 Sold to Mrs. J. M. Wilson

Terms Cash 105 Page Ave

THIS BILL WILL BE CHECKED BY US AS PERFECTLY CORRECT UNLESS REPORTED OTHERWISE WITHIN TEN DAYS.

Jan		Acc. Rend.			6 53
	9	1 Corset	14	56	
		1 Lunch Cloth		1 75	2 31
	13	36 Lace	5	1 80	
		75 D	15	1 75	3 05
					11 89
	1907	Cr.			
Dec	28	1 Book			1 00
					10 89

PAID MAR 3 1908 DEY BROTHERS & CO

4. D. M. Edwards sold to Henry Fenner,

June 1, Account rendered, $15.
June 26, 1 pattern $.15.
$2\frac{1}{2}$ yd. dimity, $12\frac{1}{2}$¢ per yd.
July 1, 4 yd. chiffon, 50¢ per yd.
2 doz. buttons, 15¢ per doz.
$4\frac{1}{2}$ yd. braid at $.33.

Credit

July 6, 2 yd. chiffon at 50¢.
1 pr. gloves, $1.25.
Cash, $7.75.

5. Debtor, Miss Margaret Maddox;
Creditor, H. G. Stone & Son.

Account rendered, \$12.35.
24 yd. lace at 25¢.
2 spools twist at \$.05.
3 yd. lace at \$1.25.
$6\frac{3}{4}$ yd. net at $\$.62\frac{1}{4}$.
$6\frac{3}{4}$ yd. linen at $\$.62\frac{1}{2}$.

Credit

Cash, \$10.

REVIEW AND PRACTICE

161. *Oral*

1. Read 300.00300; 2000.002; 860.0860; CXIV; XLIV; MCMIX.

2. Find the change from \$1 for \$.28; \$.36; \$.71; \$.81; \$.53; \$.66; \$.17.

3. 75 is how many times 25? If 25 crates of oranges cost \$90, what will 75 crates cost at the same price per crate?

4. Add in the easiest way 28 and 45; 63 and 89; 16 and 87.

5. Name six aliquot parts of \$1.

6. Using aliquot parts, find

a. The cost of 32 packages of hominy at $12\frac{1}{2}$¢ a package.

b. The quantity of dates that \$10 will buy at $6\frac{1}{4}$¢ per pound.

c. The number of sheets of sandpaper that can be bought for \$2, at $8\frac{1}{3}$¢ per dozen sheets.

7. Annexing four ciphers to an integer affects its value how?

8. Name two composite numbers that are prime to each other.

9. What is the smallest number that will exactly contain 18 and 27?

10. What is the greatest number that will exactly divide 60, 15, and 90?

11. Name the prime numbers between 80 and 115.

12. The product of two numbers is 20,000. One of the numbers is 50. What is the other number?

13. $\frac{3}{4}$ is the product of 5 and what other number?

14. 1200 is the product of 30, 4, and what other number?

15. $1.824 = 1824 \div ?$

16. $.0375 = 375 \div ?$

17. $93 - 3 \times 11 + 200 \div 5 = ?$

18. $(93 - 3) \times (38 - 28) \div (5 \times 18) = ?$

19. Reduce to simplest form: $\frac{39}{52}$; $\frac{19}{13}$; $\frac{85}{17}$; $\frac{24}{18}$; $\frac{39}{15}$; $\frac{44}{12}$.

20. Reduce $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{3}{4}$ to fractions having a common denominator.

21. From $18\frac{1}{4}$ take $7\frac{2}{3}$.

22. Tell which of the following fractions cannot be reduced to exact decimals, and why: $\frac{7}{11}$, $\frac{7}{8}$, $\frac{15}{75}$, $\frac{13}{125}$, $\frac{9}{20}$, $\frac{18}{35}$.

23. Multiply 31 by 99.

24. Divide 7000 by 25.

25. How can you tell, without actual trial, that 742 will not exactly divide 1,834,659?

26. How may we know, without actual trial,

a. That 8 will not exactly divide 4,379,624,700 ?

b. That 5 will not exactly divide 3,079,623 ?

c. That 9 will exactly divide 2,405,376 ?

d. That 25 will exactly divide 397,400 ?

27. There are seven decimal places in a product and three decimal places in one of its two factors. How many decimal places are there in the other factor ?

28. The numerator of a fraction is which term in division ? The denominator ? The value of a fraction ?

29. A certain number containing five decimal places is the product of three factors. One of its factors contains two decimal places, and another factor three decimal places. The remaining factor contains how many decimal places ?

30. Make a problem which can be solved by the use of aliquot parts.

31. What number is the product of all the common prime factors of 84 and 132 ?

32. One of the school buildings in a certain city was heated by 150 tons of coal, costing 810 dollars. At the same price per ton, what was the cost of the coal for a school that required 75 tons ?

33. $3 \times 19 - 7 + 150 \div 2 = ?$

34. The cost of a number of horses is a product. The number of horses is one factor. What is the other factor ?

35. 480 is six times what number ? Which of these numbers is a product ? The number to be found is what term ?

36. 32 is .16 of what number ? 32 is which term in multiplication ? Which terms are .16 and the number to be found ?

37. The yearly wages of 36 men in a factory amount to $28,800. At the average wages, what do 12 men receive?

38. A farmer shipped 32 cans of milk to the city in one week, each can containing 40 quarts. How many gallons did he ship?

39. $.33\frac{1}{3}$ of $\$18 = ?$ $\$6 = .33\frac{1}{3}$ of what? $6 are how many hundredths of $18?

40. A seamstress buys a sewing machine for $55. If she pays $25 at the time of purchase, and $5 every month thereafter, in how many months will she finish paying for the machine?

41. How may we tell at a glance that 6 will not exactly divide 176,435? That 6 will exactly divide 933,012?

42. $\frac{2}{9}$ of the length of a trench is 60 feet. What is $\frac{8}{9}$ of its length?

43. 24 will exactly divide a certain number. Name six other numbers that will exactly divide that number.

44. 4.5 yards of lace cost $2.70. What is the cost of 1.5 yards of the same lace? Of 1 yard?

45. $1946 \div 19.46 = ?$

46. Read 2.00500; 300,083.383; .62550; 62,500.00050.

47. Read CDLXXV; CCCXCIII; MCXLIV; CIII; CXIV; XCVII.

48. *a.* $\frac{7}{8}$ of $\frac{4}{7} = ?$

b. $\frac{1}{2}$ is $\frac{7}{8}$ of what number?

c. What part of $\frac{4}{7}$ is $\frac{1}{2}$?

49. At a fruit stand, peaches are marked "4 for 5 cents." What does the dealer receive for 36 peaches?

50. Divide: *a.* 2496 by 10,000; *b.* 36.16 by .04; *c.* 13 by 125; *d.* 5600 by 400.

51. Determine which of the following numbers are prime and which are composite:

91; 97; 111; 203; 37,564,296; 131; 141; 113; 109.

52. How may we test the accuracy of our work in addition? In subtraction? In multiplication? In division?

162. *Written*

In examples **1–5** *find the sums and test by adding in a different order. Time yourself.*

1.	2.	3.	4.	5.
385.21	15.182	92.75	837.	99.37
46.83	619.83	689.98	.96	48.69
795.467	50.70	7.42	43.82	372.918
18.23	912.183	9.87	4.79	72.75
963.542	28.764	48.136	10.68	4.681
795.087	783.908	7.091	5.30	.37
32.145	58.392	36.98	12.98	.984
819.768	75.64	74.132	4.672	98.307
73.242	9.728	8.007	.89	8.137
53.718	12.34	2.19	3.765	4.90
910.763	90.806	63.981	48.92	25.36
42.86	9.173	3.42	7.96	7.008
8.51	20.304	7.895	12.834	.93
793.916	58.79	9.86	.098	24.135
213.804	9.309	57.713	1.39	.86
67.51	864.23	8.88	4.06	7.19

In examples **6–15** *subtract and test your work, timing yourself:*

6.	7.	8.
38700.5	$7934.68	$7000.53
498.499	279.69	909.44

9. $801010.02 + 1900.92

10. $3400.75 + 2896.075

11. $51000.001 + 1900.92

12. 28037.6 + 280.799

13. 240.50 + 39.49

14. 23037.644 + 280.799

15. Find the difference between 24007.901 and 980.89.

In examples **16–21** *find results, and test your work by the reverse operation:*

16. 3.07×51.8

17. $7968 \div 5.38$

18. 92.007×380

19. 8.05×39.8

20. $2133.854 \div 5.08$

21. $83412 \div 6000$

22. Multiply 837 by 12, and test your work by addition.

In examples **23–40** *perform the indicated operations in the shortest way:*

23. 287×125

24. $876 \div 25$

25. 563×99

26. 481×2500

27. $3074 \div 125$

28. 4.207×25

29. $90{,}876 \times .33\frac{1}{3}$

30. $8642 \div 16\frac{2}{3}$

31. $50.74 \div 125$

32. 39.72×99

33. $47.012 \times .25$

34. $88.7 \div .14\frac{2}{7}$

35. 48.35×7000

36. 859×1.25

37. $2100 \div 70{,}000$

38. $548 \times 33\frac{1}{3}$

39. $7867 \div 16\frac{2}{3}$

40. $6570 \times .25$

41. A miller ground .25 of a load of corn into meal, and cracked .35 of the load for chicken feed. There remained 360 bushels. The carload consisted of how many bushels?

42. A man who owned .375 of a ship sold $\frac{1}{3}$ of his share for $24,000. What was the entire value of the ship?

43. Express in words, and in Roman numerals, the number of the present year.

44. A music dealer marked a piano at \$750 and sold it for $.83\frac{1}{3}$ of the marked price. How much did he receive for it?

45. A man owns three houses. He rents the first for \$276 a year, the second for \$450, and the third for $\frac{2}{3}$ as much as he receives for the first two. How much rent does he receive in 5 years?

46. A monthly magazine averages 92 pages of advertisements each month. It receives \$276,000 a year for advertisements. What is the average cost of one page of advertisements for one month in this magazine?

47. The steamship *Lusitania*, during one trip, consumed 50 tons of coal per hour. At this rate, how many tons will she consume on a voyage lasting four days and twenty hours?

48. *a.* Mr. Rogers uses $\frac{5}{7}$ of his yearly income for household expenses and $\frac{2}{5}$ of the remainder for his son's tuition. What fraction of his income is left?

b. If \$660 are left, how much does the son's tuition cost?

49. What must be added to $83\frac{7}{26}$ to obtain $121\frac{11}{156}$?

50. A custom miller used to take $\frac{1}{8}$ of the grain as toll to pay him for grinding the remainder of it. He took $376\frac{1}{2}$ lb. for grinding a load of wheat. If a bushel of wheat weighs 60 lb., how many bushels did the load of wheat contain?

51. Make and solve:

a. A problem that requires addition and subtraction.

b. A problem that requires addition and multiplication.

c. A problem that requires multiplication and division.

52. Make out, foot, and receipt a bill containing three debit items and one credit item.

53. Make and solve a problem that requires you to find the least common multiple.

ARTICLES SOLD BY THE THOUSAND, HUNDRED, OR HUNDREDWEIGHT

163. *Written*

1. What is the cost of 8975 bricks at $7 per M.? (M. stands for 1000.)

8975 = 8.975 M.

Since 1 M. costs $7, 8.975 M. cost 8.975 × $7, or $—— *Ans.*

2. What must be paid for 980 soapstone pencils at $.30 per C.? (C. stands for 100.)

980 = 9.80 C.

Since 1 C. costs $.30, 9.80 C. will cost 9.80 × $.30, or $—— *Ans.*

3. Find the cost of 1550 lb. of new buckwheat flour at $2.50 per hundredweight (100 pounds).

1550 lb. = 15.50 hundredweight.

Since 1 cwt. costs $2.50, 15.50 cwt. cost 15.50 × $2.50, or $—— *Ans.*

NOTE. — In final results, a fraction of a cent, equal to or greater than ½ cent, is counted a whole cent. A fraction which is less than ½ cent is dropped.

4. *Find the cost of each of the following items:*

a. 83,900 bricks at $7.80 per M.

b. 8950 lb. sugar at $4.95 per C.

c. 1550 asparagus roots at $.95 per C.

d. 10,000 laths at $.45 per C.

e. 12,500 paper butter trays at $.40 per M.

f. 25 barrels of granulated sal soda, each barrel containing 325 lb., at $.90 per hundredweight.

g. 25,600 cakes of naphtha soap at $3.25 per C.

h. 1700 cubic feet of gas at $.95 per M.

DENOMINATE NUMBERS

164. *A number that is composed of units of weight or measure is a* **denominate** *number; e.g.* 5 lb., 7 rd., 6 hr. 3 min. 45 sec.

165. *The name of a unit of weight or measure* is a **denomination**; *e.g.* ounce, square foot, minute.

166. *A denominate number that is expressed in two or more denominations* is a **compound** *number; e.g.* 1 yd. 2 ft. 7 in.; 2 lb. 14 oz.

167. *A number that is expressed in but one kind of units* is a **simple number**; *e.g.* 3 days, 8 cents, 19 pounds, 125.

168. TABLE OF LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

Oil, vinegar, molasses, and other liquids are shipped in barrels or casks of various sizes. But for the purpose of indicating the capacities of vats, tanks, reservoirs, etc., $31\frac{1}{2}$ gallons are called a *barrel* (bbl.) and 63 gallons a *hogshead* (hhd.).

169. TABLE OF DRY MEASURE

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

170. TABLE OF AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).
2240 pounds = 1 long ton.
100 pounds = 1 hundredweight (cwt.).

The term *hundredweight* is used less than formerly, although its value (100 lb.) is still taken as a unit in quoting freight rates and prices of various articles, when the quantity used makes this a convenient unit of weight.

The *long ton* is used in wholesaling certain mining products. The ton of 2000 lb. is sometimes called a *short ton*.

171. TABLE OF TROY WEIGHT

24 grains (gr.) = 1 pennyweight (pwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

These weights are used in weighing gold, silver, and some jewels. To get an idea of the weight of a grain, think of the weight of a grain of wheat or rice.

172. TABLE OF APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ℥).

This table is used by druggists and physicians in compounding medicines; but medicines are bought and sold by Avoirdupois weight, except in quantities smaller than one ounce.

173. Druggists use a term *fluid ounce*, which is not a measure of weight, but of capacity, and is equal to $\frac{1}{16}$ of a pint. Thus, a 2-ounce bottle is a bottle that holds $\frac{1}{8}$ of a pint of any liquid regardless of its weight.

174. TABLE OF LINEAR MEASURE

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.).
320 rods = 1 mile (mi.).

175. TABLE OF SURVEYORS' LONG MEASURE

7.92 inches = 1 link (li.).
100 links = 1 chain (ch.).
80 chains = 1 mile (mi.).

This table, formerly used by surveyors in measuring land, should be learned and remembered, because descriptions of land in the public records of deeds and mortgages are largely made in the denominations of this measure.

176. TABLE OF SURFACE MEASURE

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).

177. TABLE OF SURVEYORS' SQUARE MEASURE

625 square links = 1 square rod.
16 square rods = 1 square chain.
10 square chains = 1 acre.

This table, like that of Surveyor's Linear Measure, is used in public records, chiefly.

178. TABLE OF VOLUME MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

179. TABLE OF COUNTING

12 = 1 dozen (doz.).
12 doz. = 1 gross.
20 = 1 score.

180. TABLE OF TIME

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (hr.).
24 hours	= 1 day (da.).
7 days	= 1 week (wk.).
$52\frac{1}{7}$ weeks	= 1 common year (yr.).
$52\frac{2}{7}$ weeks	= 1 leap year.
365 days	= 1 common year.
366 days	= 1 leap year.

Ten years are called a *decade*, and one hundred years make a *century*, but these terms are not used in arithmetical calculations.

The four thirty-day months may be remembered easily by the following old rhyme:

"Thirty days hath September,
April, June, and November."

February has 28 days, with 29 in leap year. The other months have 31 days.

The exact length of the solar year, that is, the time of one revolution of the earth around the sun, is 365 days 5 hours 48 minutes and 46 seconds, or nearly $365\frac{1}{4}$ days. By adding one day to the 365 every fourth year, too much time is added. This is corrected by counting every centennial year as a common year, except when its number is divisible by 400. The year 1900, therefore, was not a leap year, although its number was divisible by 4.

181. TABLE OF PAPER MEASURE

24 sheets = 1 quire.
20 quires = 1 ream.

The terms *bundle* (2 reams) and *bale* (5 bundles) are seldom used. The denomination *quire* is used mostly in measuring the finer grades of writing paper. Wrapping paper is sold by the pound or by the thousand sheets. Many kinds of paper are sold in packages of five hundred or one thousand sheets. Packages of five hundred sheets are sometimes called *reams*.

182. TABLE OF UNITED STATES MONEY

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.

The gold coins of the United States are the \$5, \$10, and \$20 pieces, once called the half eagle, eagle, and double eagle. Gold dollar coins are not in general circulation, although a few of them have been made.

The silver coins are the dollar, half dollar, quarter dollar, and dime. Silver half dimes are no longer coined. Most five-cent pieces are made of nickel. Most 1-cent pieces are made of bronze, though some nickel and copper cents are in circulation.

The mill is not coined.

183. TABLE OF ENGLISH MONEY

4 farthings (far.) = 1 penny (*d.*).
12 pence = 1 shilling (*s.*).
20 shillings = 1 pound (£).

Farthings are not coined, and are commonly written as fractions of a penny.

184. TABLE OF FRENCH MONEY

100 centimes = 1 franc.

185. TABLE OF GERMAN MONEY

100 pfennigs = 1 mark.

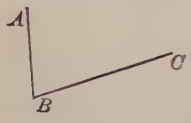
186. The denominations of Canadian money are like those of the United States.

187. TABLE OF ARC AND ANGLE MEASURE

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
An arc of 360° = 1 circumference.

188. *The difference in direction of two lines that meet* is an **angle**; *e.g.*

ANGLES



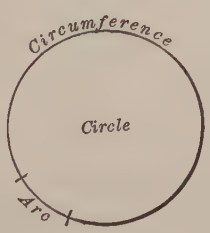
189. *The lines that meet to form an angle* are the **sides** *of the angle.*

Lines are read by means of letters placed at their extremities. Angles are read by means of letters placed at the extremities of their sides.

In the angle ABC the lines AB and BC are the sides.

190. *The sum of all the angles that can be formed around a point in a plane is* 360°.

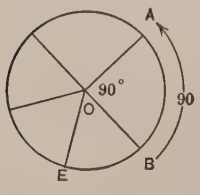
191. *A plane figure bounded by a curved line, every point of which is equally distant from a point within called the center*, is a **circle.**



192. *The boundary line of a circle* is its **circumference.**

193. *Any part of a circumference* is an **arc.**

194. *The number of degrees in an arc is always the same as the number of degrees in the angle at the center, whose sides meet the extremities of the arc*, thus:



The angle AOB is $\frac{1}{4}$ the sum of all the angles at the center, or 90°. The arc AB is $\frac{1}{4}$ of the circumference, or 90°. Can you tell the number of degrees in the arc BE? In the angle EOB?

Angles are measured by a protractor, an instrument made of metal, with degrees marked and numbered as shown below.

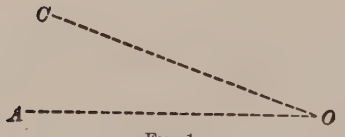


Fig. 1

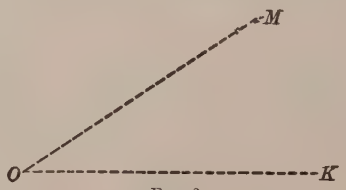


Fig. 2

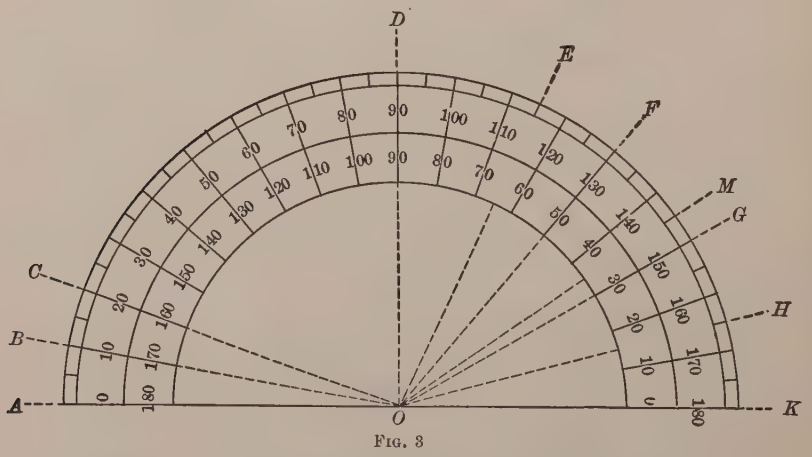


Fig. 3

To measure the angle AOC, the protractor is placed as indicated, so that the center, O, of the protractor, coincides with

the vertex, O, of the angle, and the sides AO and CO take the positions indicated. The scale on the protractor indicates that the angle AOC is an angle of 20°.

Notice that the length of the sides does not affect the size of the angle. They may be prolonged indefinitely without changing the angle.

State the number of degrees in each of the following angles as indicated by the protractor in Fig. 3:

a. Angle AOB	*e.* Angle MOK	*i.* Angle DOF
b. Angle AOD	*f.* Angle MOH	*j.* Angle COD
c. Angle AOE	*g.* Angle EOK	*k.* Angle BOF
d. Angle HOK	*h.* Angle COE	*l.* Angle COH

195. *An angle of* 90° is a **right** *angle.*

196. *An angle that is greater than a right angle* is an **obtuse** *angle.*

197. *An angle that is less than a right angle* is an **acute** *angle.*

In Fig. 3, what kind of angle is angle AOC? Angle AOD? Angle AOB? Angle AOF? Angle DOE?

198. MISCELLANEOUS DENOMINATIONS

6 feet = 1 **fathom**, used in measuring the depth of the water at sea.

40 rods = 1 **furlong.**

4 inches = 1 **hand**, used in measuring the height of horses.

1.15 common or **statute** miles = 1 **nautical mile**, or **knot**, used in measuring distances at sea and the speed of vessels. The nautical mile is assumed at 6086.07 feet, or 1.152664 statute miles, by the United States Coast Survey. For ordinary purposes of computation, however, 1.15 is sufficiently exact.

3 nautical miles = 1 **league**.

640 acres, or one square mile, = 1 **section** of land.

3.2 grains (approximately) = 1 **carat**, used in indicating the weight of diamonds and other gems. The term *carat* is also used in indicating the fineness of gold. 14-carat gold, or gold that is 14 carats fine, is metal of which $\frac{14}{24}$ is pure gold, and $\frac{10}{24}$ is alloy (that is, harder metal mixed with the gold to make it more durable). The word is sometimes spelled *karat* and jewelers use the abbreviation *k* in rings and other gold articles. What is the meaning of 20-*k* gold? Of 18-*k* gold? Of 10-*k* gold?

The term **perch** is sometimes used to indicate (*a*) one rod in length, or (*b*) one square rod of land, or (*c*) a quantity of stone or masonry 1 rod long, $1\frac{1}{2}$ feet wide, and 1 foot thick, containing $24\frac{3}{4}$ cubic feet.

One hundred square feet of painting or roofing are called a **square**.

199. TABLE OF EQUIVALENTS

1 gallon	= 231 cubic inches.
1 bushel	= 2150.42 cubic inches.
1 pound Avoir.	= 7000 grains.
1 pound Troy	= 5760 grains.
1 pound Apoth.	= 5760 grains.
£1 (Gt. Britain)	= $4.8665.
1 franc (France)	= $.193.
1 franc (Belgium)	= $.193.
1 lira (Italy)	= $.193.
1 mark (Germany)	= $.238.
1 yen (Japan)	= $.498.
1 ruble (Russia)	= $.515.

The grain is the same in the three weights, Avoirdupois, Troy, and Apothecaries'. It is obtained by taking a certain fraction (very nearly $\frac{1}{2\,5\,3}$) of the weight of a cubic inch of distilled water at its greatest density (39.2° nearly).

REDUCTION OF DENOMINATE NUMBERS

200. *Changing numbers to larger denominations* is **reduction ascending.**

201. *Changing numbers to smaller denominations* is **reduction descending.**

202. *Oral*

1. Find the number of cubic inches in one quart, liquid measure.

2. A fountain contains four barrels of water. How many gallons does it contain? How many hogsheads?

3. A grocer bought green peas at two dollars a bushel, and retailed them at ten cents a quart. What did he gain on three bushels?

4. What is a grocer's profit on half a ton of coffee bought at $15 per hundredweight and sold at 20 cents a pound?

5. A gold dollar weighs 25.8 grains. How many gold dollars will weigh 25.8 pennyweights?

6. A druggist bought 10 lb. (Avoir.) of oxalic acid. How many grains did he buy?

7. A can contained 20 ounces (Apoth.) of quinine sulphate. How many pounds (Apoth.) did it contain? How many drams? How many scruples? How many grains?

8. A field is 20 rods wide. How many feet wide is it? How many yards?

9. The perimeter of a square yard is how many inches?

10. A 100-acre farm contains how many square chains?

11. A cubic yard of earth is sometimes called a load. How many cubic feet of earth are there in twenty such loads?

12. Name the leap years from 1890 to 1920 inclusive.

13. What is the exact number of days from 2 o'clock P.M., Jan. 19, 1904, to 2 o'clock P.M., April 1, 1904?

14. A gross of $\frac{1}{2}$-pound cans of baking powder will fill how many cases, each holding 48 cans? How much will the baking powder weigh?

15. How many clothespins are there in a box containing 10 gross?

16. Four gallons of ammonia water will fill how many 4-ounce bottles?

17. How many quires of paper will a lady use in writing thirty letters if she uses two sheets of paper for each letter?

18. The earth makes one complete rotation every 24 hours. How many degrees does it turn in 1 hour?

19. A wheel in a factory makes 240 revolutions per minute. How many revolutions does it make in one second of time? Through how many degrees does it revolve in $\frac{1}{8}$ of a second?

20. A wheel has eight spokes that make equal angles at the center. How many degrees are there in each of the angles? Two of these angles together form what kind of angle?

21. A crown is an English coin equal to five shillings. A sovereign is a gold coin whose value is £1. Mr. Denham has in his purse a sovereign, two crowns, one half crown, a shilling, and a sixpence. All the money in the purse is equal to how many shillings?

22. 38,476 centimes are equal to how many francs and centimes?

23. 46 francs are equal to how many centimes?

24. 86.75 marks are equal to how many pfennigs?

25. At the rate of 20 pfennigs apiece, how many oranges can be bought for 4 marks?

26. Without a rule, draw a line 5 feet long on the blackboard. Measure and correct it.

27. *Estimate, then measure:*

a. The number of feet from the front door of your schoolhouse to the sidewalk.

b. The width of the sidewalk.

c. The width of the street.

d. The dimensions of the schoolroom windows.

e. The dimensions of the schoolroom doors.

f. Other things about the school.

203. 1. Reduce 22 A. 7 sq. yd. to square feet.

22
160
3520 Number of sq. rd. in 22 A. (22×160).
$30\frac{1}{4}$
880 ($3520 \times \frac{1}{4}$)
105600 (3520×30)
106480 Number of sq. yd. in 22 A.
7
106487 Number of sq. yd. in 22 A. 7 sq. yd.
9
958383 Number of sq. ft. in 22 A. 7 sq. yd.

2. Reduce 392,429 sec. to larger denominations.

60 | 392429 sec.
60 | 6540 min. 29 sec.
24 | 109 hr.
4 da. 13 hr.

4 da. 13 hr. 29 sec. *Ans.*

NOTE. — Compound numbers, other than those expressing time, or arc and angle measure, are seldom expressed in more than two denominations. Extended reductions are rarely needed.

In actual business, the work of reduction is performed by short and direct processes. For example, surveyors, in measuring land, use a metallic tape from fifty to one hundred feet in length, marked off in feet and tenths of a foot, or a chain with links one foot in length, marked in tenths. With this they obtain the dimensions of a piece of land in feet and tenths of a foot, and the area in square feet and hundredths of a square foot. The area in square feet divided by 43,560 (the number of square feet in one acre) gives the number of acres.

Feet are reduced to miles by dividing by 5280 instead of dividing successively by the numbers in the scale of linear measure.

Bushels are reduced to quarts by multiplying directly by 32.

In all measurements and computations, decimals are more generally used than formerly, taking the place of common fractions and the smaller units of denominate numbers.

In the following examples, use short and direct processes where possible.

204. *Written*

1. *Reduce:*

a. 14 wk. 3 da. to hours.

b. 5 T. 7 cwt. to pounds.

c. 4900 min. to higher denominations.
d. 7 mi. to inches.
e. 18 bbl. 13 gal. to pints.
f. 193,479 cu. in. to higher denominations.
g. 498,342 sec. to higher denominations.
h. 800,000 oz. to tons.
i. 86,240 pwt. to pounds.
j. 9 oz. Apoth. to grains.
k. 84,763*d.* to pounds, shillings, and pence.
l. £5 10*s.* 11*d.* to pence.
m. 48° 50′ 19″ to seconds.
n. 12 common years to minutes.
o. 190,113 in. to higher denominations.
p. 5040 pt. to hogsheads.
q. 3 yr. 7 mo. 21 da. to minutes. (Use 30 da. for one month.)
r. 4391 da. to years and days. (Use 365 days for a year.)
s. 17 A. 30 sq. rd. to square feet.
t. 5 cu. yd. to cubic inches.
u. 118,096 sq. yd. to higher denominations.
v. 834,769 cu. in. to higher denominations.

2. 12,480 in. are what part of a mile?

This problem may be solved in two ways:

a. $\frac{12480}{1} \times \frac{1}{12} \times \frac{2}{33} \times \frac{1}{5280} = ?$
b. $\frac{12480}{63360} = ?$ (There are 63,360 in. in a mile.)

3. $\frac{59}{288}$ lb. Troy = how many grains?
$\frac{59}{288} \times \frac{12}{1} \times \frac{20}{1} \times \frac{24}{1} = ?$
In what other way could this problem be solved?
(1 lb. Troy = how many grains?)

4. Change $\frac{5}{6}$ cu. yd. to cubic inches.
5. 270 sec. are what part of a day?

6. Change $\frac{7}{120}$ oz. Apoth. to grains.

7. 108 A. are what part of a square mile?

8. 5 rd. 7 ft. 6 in. are what fraction of a mile?

9. 18 lead pencils are what part of a gross?

10. $\frac{17}{540}$ cu. yd. = how many cubic inches?

11. *Find by reduction:*

a. The number of grains in 1 lb. Apoth.

b. The number of grains in a Troy pound.

c. The number of square feet in 1 acre.

d. The number of inches in a mile.

e. The number of grains in 1 oz. Troy.

f. The number of grains in 1 oz. Apoth.

12. *Using the table of equivalents, page* 84, *find:*

a. The number of cubic inches in one quart, liquid measure.

b. The number of cubic inches in one quart, dry measure.

c. The number of grains in one ounce, Apothecaries' weight.

d. The number of grains in one ounce, Avoirdupois weight.

e. The number of pounds Troy that are equivalent to one pound Avoirdupois.

13. *Find, to the nearest thousandth:*

a. The number of cubic feet that are equivalent to one bushel.

b. The number of francs that are equivalent to one dollar.

c. The number of cents that are equivalent to four marks.

d. The number of cents that are equivalent to one shilling.

e. The difference in size between a dry pint and a liquid pint.

f. The number of pounds Troy that are equivalent to 10 lb. Avoirdupois.

g. The number of bushels that a 40-gallon cask will hold.

14. How many 5-grain tablets can be made from $7\frac{1}{2}$ lb. (Avoir.) of potassium chlorate?

15. *Make and solve:*

a. A problem that requires reduction descending in linear measure.

b. A problem that requires reduction ascending in English money.

c. A problem that requires reduction of a denominate fraction to an integer of smaller denomination.

d. A problem that requires reduction of an integer to a fraction of higher denomination.

e. A problem that requires reduction descending in square measure.

f. A problem that requires reduction ascending in square measure.

ADDITION AND SUBTRACTION OF COMPOUND NUMBERS

205. *Written*

Add 7 lb. 8 oz., 15 lb. 14 oz., 23 lb. 15 oz.

Lb.	Oz.	
7	8	15 oz. + 14 oz. + 8 oz. = 37 oz. = 2 lb. 5 oz.
15	14	2 lb. + 23 lb. + 15 lb. + 7 lb. = 47 lb.
23	15	47 lb. 5 oz. *Ans.*
47	5	

Add:

1. 59 ft. 8 in., 47 ft. 11 in., 9 ft. 9 in.
2. 63 A. 16 sq. rd., 49 A. 53 sq. rd.
3. 5 hr. 5 min. 30 sec., 8 hr. 43 min. 47 sec.
4. 18 gal. 3 qt., 25 gal. 1 qt., 16 gal. 2 qt.
5. 41° 19′ 35″, 22° 50′ 29″, 133° 4′ 50″.
6. 16 T. 480 lb., 17 T. 730 lb., 19 T. 900 lb.
7. 25 yd. 2 ft., 6 yd. 1 ft., 8 yd. 2 ft.

8. 16 pk. 7 qt., 13 pk. 5 qt., 12 pk. 6 qt.
9. 27 cu. yd. 18 cu. ft., 42 cu. yd. 19 cu. ft.
10. 26 yr. 7 mo. 8 da., 17 yr. 8 mo. 9 da.
11. 8 lb. 7 oz., 16 lb. 14 oz., 19 lb. 10 oz.
12. 1 bu. 3 pk., 19 bu. 2 pk., 5 bu. 1 pk.
13. 12 wk. 13 da., 1 wk. 1 da., 25 wk. 6 da.
14. 27 hr. 38 min. 21 sec., 25 hr. 47 min. 29 sec.
15. 25 yr. 200 da., 27 yr. 321 da., 28 yr. 179 da.

206. *Written*

From 18 yr. 7 mo. 14 da.
take 6 yr. 8 mo. 26 da.
11 yr. 10 mo. 18 da. *Difference*

7 mo. 14 da. = 6 mo. 44 da.
18 yr. 6 mo. = 17 yr. 18 mo. (Why do we make these reductions?)
18 yr. 7 mo. 14 da. = 17 yr. 18 mo. 44 da.
17 yr. 18 mo. 44 da. − 6 yr. 8 mo. 26 da. = 11 yr. 10 mo. 18 da.

Subtract:

1. 18 yr. 3 mo. 14 da. − 10 yr. 1 mo. 18 da.
2. 19 yr. 2 mo. 1 da. − 3 yr. 8 mo. 27 da.
3. 29 yr. 2 mo. 8 da. − 24 yr. 8 mo. 8 da.
4. 42 yr. 26 da. − 28 yr. 2 mo. 12 da.
5. 14 yr. 2 mo. 28 da. − 1 yr. 9 mo. 12 da.
6. 17 yr. 4 mo. 8 da. − 3 yr. 6 mo. 7 da.
7. 29 yr. 9 mo. 13 da. − 8 yr. 5 mo. 18 da.
8. 4 yr. 3 mo. − 9 mo. 15 da.

9. How many years, months, and days are there from May 30, 1907, to Dec. 5, 1909?

1909 yr. 12 mo. 5 da.
1907 yr. 5 mo. 30 da.

NOTE. — December is the twelfth month and May the fifth. Count 30 da. for a month.

Find the time from:

10. July 29, 1837, to Mar. 26, 1888.
11. Aug. 20, 1841, to Nov. 15, 1908.
12. Dec. 17, 1840, to Feb. 18, 1896.
13. May 14, 1850, to Jan. 12, 1860.
14. Oct. 29, 1764, to Aug. 23, 1860.
15. Jan. 22, 1880, to June 15, 1903.
16. July 20, 1819, to Jan. 2, 1893.
17. May 8, 1899, to Feb. 12, 1908.
18. Feb. 11, 1901, to Jan. 31, 1906.

Subtract:

19. 15 hr. 52 min. 34 sec.
11 hr. 50 min. 50 sec.

20. 222° 41′ 15″
60° 20′ 35″

21. 43 hr. 44 min. 26 sec.
30 hr. 24 min. 48 sec.

22. 47 ft. 6 in.
33 ft. 10 in.

23. 57 gal. 1 qt.
48 gal. 3 qt.

24. 19° 31′
6° 41′

25. 38 bu. 1 pk.
27 bu. 3 pk.

26. 210 A. 86 sq. rd.
94 A. 106 sq. rd.

27. 46 lb. 5 oz.
25 lb. 12 oz.

28. 98 ft. 2 in.
67 ft. 9 in.

29. 52′ 13″
45′ 28″

30. 5 min. 47 sec.
2 min. 48 sec.

31. War was declared between the United States and Spain on the 25th day of April, 1898, and hostilities ceased on the 13th day of August in the same year. How long did the war last?

32. How much time has elapsed since April 14, 1861?

EXACT DIFFERENCES BETWEEN DATES

207. *Written*

1. What is the exact number of days between Dec. 16, 1895, and March 12, 1896?

Dec. 15
Jan. 31
Feb. 29
March 12
87 days *Ans.*

There are 15 days in December after the 16th. January has 31 days, February 29 (leap year), and March 12, making 87 days. Always count the last day.

Find the exact time between:

2. June 16, 1886, and April 7, 1887.
3. Nov. 21, 1898, and Dec. 14, 1898.
4. Jan. 26, 1907, and Dec. 21, 1907.
5. July 14, 1898, and Aug. 12, 1898.
6. Jan. 23, 1897, and June 4, 1897.
7. Sept. 19, 1899, and Feb. 16, 1900.
8. Nov. 28, 1905, and Oct. 26, 1906.

MULTIPLICATION AND DIVISION OF COMPOUND NUMBERS

208. **1.** Multiply 5 hr. 21 min. by 7.

5 hr. 21 min.
7
1 da. 13 hr. 27 min. *Product*

7×21 min. $= 147$ min. $= 2$ hr. 27 min.
7×5 hr. $= 35$ hr.
35 hr. + 2 hr. = 37 hr. = 1 da. 13 hr.

Multiply:

2. 6 lb. 7 oz. Avoir. by 8
3. 12. gal. 1 qt. by 9
4. 7 ft. 5 in. by 6
5. 12 A. 50 sq. rd. by 12
6. 2 hr. 15 min. 30 sec. by 15
7. 7° 40′ 18″ by 5
8. 42 min. 17 sec. by 15
9. 3 hr. 19 sec. by 15
10. 5 hr. 17 min. 19 sec. by 15
11. 18 min. 13 sec. by 15
12. Divide 41° 28′ 45″ by 15

$15\overline{)41° 28′ 45″}$
$2° 45′ 55″$ *Quotient*

$41° \div 15 = 2°$ and $11°$ rem.
$11° = 660′$. $660′ + 28′ = 688′$.
$688′ \div 15 = 45′$ and $13′$ rem.
$13′ = 780″$. $780″ + 45″ = 825″$.
$825″ \div 15 = 55″$.

Divide by 15 *and test your work:*

13. 40° 20′
14. 17° 18′ 15″
15. 1° 29′
16. 39′ 45″
17. 27° 30″
18. 14° 15″
19. 38° 1′
20. 7′ 30″
21. 41° 42′
22. 1° 11′
23. 40° 2′ 30″
24. 8° 40′ 45″
25. 59′
26. 17°
27. 1° 1′ 30″
28. 11° 19′

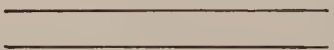
MEASUREMENTS

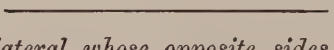
AREAS OF PARALLELOGRAMS

209. *A plane figure bounded by four straight lines* is a **quadrilateral**; *e.g.*

QUADRILATERALS

210. *Lines that are the same distance apart throughout their whole length* are **parallel lines**; *e.g.*





112. *A quadrilateral whose opposite sides are parallel* is a **parallelogram.** Which of the above figures are parallelograms?

212. *A parallelogram that has four right angles* is a **rectangle.** Which of the above figures are rectangles?

213. *Two lines that meet to form a right angle* are **perpendicular to each other.**

214. *The side on which a figure is supposed to rest* is its **base.**

215. *The perpendicular distance from the highest point of a figure to the base, or to the base extended,* is its **altitude**; *e.g.*

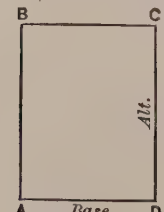


FIG. 1

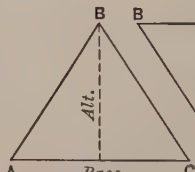


FIG. 2

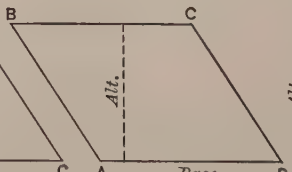


FIG. 3

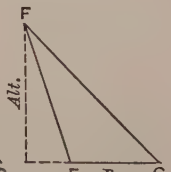


FIG. 4

216. *Figures are read by means of letters placed at their angles.* Thus, Fig. 1 is read, "Oblong *ABCD*." Fig. 2 is read, "Triangle *ABC*." Read Fig. 3. The base of Fig. 2 is *AC*. The altitude of Fig. 1 is *DC* or *AB*.

217. **The area of a rectangle** *is the product of its base and altitude expressed in the same denomination.*

NOTE 1. — In computing the area or volume of a figure, the given dimensions, if expressed in different denominations, should first be changed to the same denomination. Why?

NOTE 2. — In giving dimensions, the sign (′) is sometimes used to indicate feet, and the sign (″) to indicate inches; *e.g.* $16' = 16$ feet; $9'' = 9$ inches.

218. *Written*

1. A rectangular field is 60 rd. long and 28 rd. wide. How many acres does it contain?

2. What is the cost of paving an alley 570 ft. long and 23.7 ft. wide, at $2.15 per square yard?

3. The surveyor found my vacant lot to be 8 rd. long and 67.5 ft. wide. What fraction of an acre does it contain?

4. Along a city street, where the lots are all 12 rd. deep, how many feet wide must a lot be to contain $\frac{1}{4}$ of an acre?

219. *Oral*

1. In Fig. 5, how does the part *K* compare with the part *M*?

2. The area of the parallelogram *ABCD* compares how with the area of the parallelogram *EFCD*?

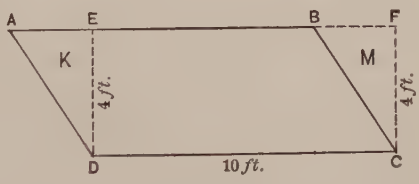


FIG. 5

3. What is the base of each of these parallelograms?

4. What is the altitude? What is the area?

5. How is the area of a rectangle found?

6. If the base of a rectangle is its length, the altitude is what?

7. If we know the base and altitude of a rectangle, how may we find the area?

8. Since any parallelogram may be made into a rectangle of the same base and altitude, how may we find the area of a parallelogram?

220. *The* **area of a parallelogram** *is equal to the product of its base and altitude expressed in the same denomination.*

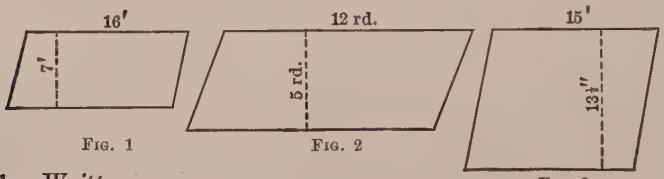


FIG. 1 FIG. 2 FIG. 3

221. *Written*

1. Figure 1 represents what part of a square rod?

2. Figure 2 represents what part of an acre?

3. Figure 3 represents what part of a square rod?

4. The area of a parallelogram is 52 square rods. Its base is 132 ft. What is its altitude?

5. The altitude of a parallelogram is 37 in.; its area is 74 sq. ft. Find its base.

AREAS OF TRIANGLES

222. *A plane figure bounded by three straight lines* is a **triangle**; *e.g.*

223. *Oral*

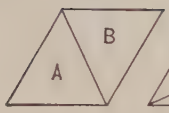


FIG. 1

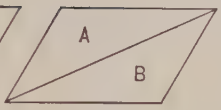


FIG. 2

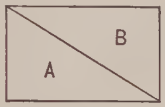


FIG. 3

FIG. 4

1. Figures 1, 2, 3, and 4 are what kind of figures? What kind of figures are A and B?

2. In each of the above figures how does A compare with B?

3. In each of the above figures, how do the base of the triangle and the base of the parallelogram compare?

How do the altitude of the parallelogram and of the triangle compare?

4. How is the area of the parallelogram found? Of the triangle?

224. *The* **area of a triangle** *is equal to one half the product of its base and altitude expressed in the same denomination.*

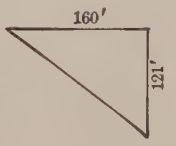
225. *Oral*

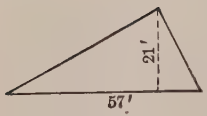
Find the areas of triangles having dimensions as follows:

	BASE	ALTITUDE		BASE	ALTITUDE
1.	7 ft.	4 ft.	6.	1 mi.	100 rd.
2.	1 yd.	1 yd.	7.	3 ft.	20 in.
3.	5 in.	20 in.	8.	5 yd.	5 ft.
4.	1 yd.	1 ft.	9.	1 ft. 6 in.	8 ft.
5.	80 rd.	20 rd.	10.	640 rd.	1 mi.

226. *Written*

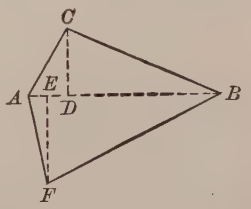
1. This figure represents a plot of ground inclosed by three streets. What part of an acre does it contain?





2. This figure represents a piece of cement floor at a railroad station. Find its cost at $1.08 a square yard.

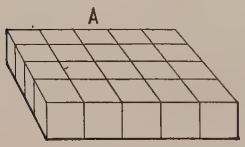
3. In this figure, $AB = 54$ in., $CD = 18$ in., $EF = 27$ in. Find the area of $ACBF$ in square yards.

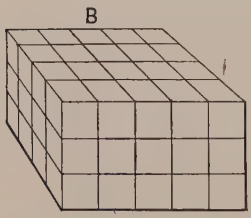


MEASUREMENT OF RECTANGULAR SOLIDS

227. *A solid bounded by six rectangles* is a **rectangular solid;** *e.g.* a chalk box. Give other examples.

228. *A solid bounded by six squares* is a **cube.** Define *cubic inch*, *cubic foot*, and *cubic yard.*





229. *The* **contents** *or* **volume** *of a rectangular solid is the number of cubic units* (cu. in., cu. ft., cu. yd.) *which it contains, and is equal to the product of its three dimensions.*

Figure *A* may represent $5 \times 4 \times 1$ cu. in., cu. ft., or cu. yd.

Figure *B* may represent $5 \times 4 \times 3$ cu. in., cu. ft., or cu. yd.

230. *Oral*

1. Explain how we determine that there are 27 cu. ft. in 1 cu. yd.

2. Explain how we determine that there are 1728 cu. in. in 1 cu. ft.

3. A candy box is 6 in. square and 2 in. deep. What is its volume?

4. A rectangular piece of wood 4 in. square and 10 in. long contains how many cubic inches?

5. A piece of timber 10 in. square and 1 yd. long contains how many cubic inches? It must be how long to contain 1200 cu. in.? 1350 cu. in.? 1 cu. ft.?

6. A box is 10 in. wide and 10 in. deep, inside measure. What must be its length in order that it may hold 1 bushel?

7. The bottom of a rectangular tin can measures $5\frac{1}{2}$ in. by 6 in. How deep must it be to hold 1 gallon?

A Cord

231. *Oral*

1. A pile of 4-foot wood, 8 ft. long and 4 ft. high, contains how many cubic feet?

2. A pile of 2-foot wood, 8 ft. long and 4 ft. high, contains how many cubic feet?

3. A pile of 1-foot wood, 8 ft. long and 4 ft. high, contains how many cubic feet?

4. If wood is cut into sticks that are 1 ft. 6 in. long, how many cubic feet are there in a pile 8 ft. long and 4 ft. high?

232. Originally, a cord of wood consisted of 128 cu. ft., or the equivalent of a pile of 4-foot wood, 8 ft. long and 4 ft. high. It is a growing custom, however, to consider as a cord

any quantity of wood that is equivalent to a pile 8 ft. long and 4 ft. high, whatever may be the length of the sticks. In some states the law specifies what shall be understood as a cord of wood.

To find the number of cords in a pile of wood, find the product of its three dimensions expressed in feet, and divide the product by 128.

233. *Written*

1. Find the number of cords in a pile of 4-foot wood:

a. 36 ft. long and 6 ft. high.
b. 20 ft. long and 5 ft. high.
c. 50 ft. long and 8 ft. high.
d. 100 ft. long and 6 ft. high.
e. 8 ft. long and $7\frac{1}{2}$ ft. high.

2. What must be the length of the sticks in a pile of wood 4 ft. high and 32 ft. long in order that it may contain:

a. 512 cu. ft.? *c.* 256 cu. ft.? *e.* 192 cu. ft.?
b. 384 cu. ft.? *d.* 128 cu. ft.? *f.* 160 cu. ft.?

3. What must be the length of a pile of 4-foot wood in order that a pile 4 ft. high may contain $5\frac{1}{2}$ cords?

4. What must be the height of a pile of 4-foot wood in order that a pile 20 ft. long may contain:

a. $2\frac{1}{2}$ cords? *b.* 4 cords? *c.* $1\frac{7}{8}$ cords?

BUILDING WALLS

234. There are no universal rules for the measurement of masonry. Some masons measure around the outside of a cellar wall to determine its dimensions, while others make allowance for the corners. The method of measurement should be specified in the contract in every case.

Quantities of uncut stone are bought by the cord, and usually 99 cu. ft. are taken for a cord.

From 21 to 23 bricks $8'' \times 4'' \times 2''$ are estimated to make a cubic foot of brick wall.

Some masons estimate the number of bricks required for a wall by multiplying the number of square feet in one side of the wall by 7, when the wall is one brick thick, by 14 when it is two bricks thick, and by 21, when it is three bricks thick, allowing for all openings.

A *perch* of stone or masonry is $24\frac{3}{4}$ cu. ft.

Concrete walls are estimated by the cubic yard, and the methods of measurement vary. Foundation walls are generally measured without regard to openings. When there are many openings some contractors allow one half for openings, and some make full allowance.

235. *Written*

1. A retaining wall is 220 ft. long and 8 ft. high. It has an average thickness of 3 ft. Find the cost of the stone used, at \$5.40 per cord, a cord of stone making 99 cu. ft. of wall.

2. Find the cost of the brick for a wall 120 ft. long, $12\frac{1}{2}$ in. thick, and 40 ft. high, at \$6.50 per M., estimating 21 bricks for a cubic foot, and making no allowance for openings.

3. Find the cost, at \$9.50 per M., of a brick veneer 4 in. thick on the outside of a house measuring 45 ft. by 30 ft. and 20 ft. high, making an allowance of 200 sq. ft. for doors and windows, and allowing 7 bricks for a square foot of surface.

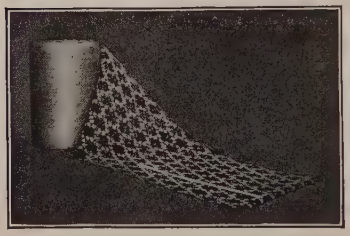
4. The walls of a rectangular cellar 87 ft. 6 in. by 45 ft. (outside measure), and 9 ft. deep, are 18″ thick. Find the cost of the stone at \$6.30 per cord, estimating a cord of stone to make 100 cu. ft. of wall, and deducting 18 cu. ft. for corners.

5. Find the cost of 24 concrete pier foundations each 21 in. square and 5 ft. deep, at $5.00 per cubic yard.

6. A garden wall 55 ft. long, 6 ft. high, and 18 in. thick cost how much at $4.20 a perch?

7. A contractor built a concrete cellar wall 54 ft. by 30 ft. (outside measure), 8 ft. high, and $1\frac{1}{2}$ ft. thick, receiving $5 per cubic yard. He used 75 barrels of cement costing $1.80 a barrel, 12 loads of sand costing $1.25 a load, and 50 cu. yd. of crushed stone at $1.30 per cubic yard. How much had he left for labor and profit?

FLOOR COVERING

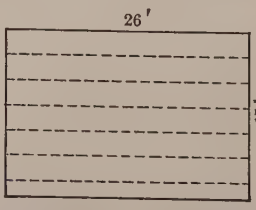


236. A **yard** of carpet or matting is a *yard of the length of the piece*, as it is unrolled, regardless of its width.

The exact number of yards of material to be purchased for the covering of any given floor is difficult to determine, because of the waste in fitting, and in matching figures.

237. *Written*

1. If this floor is covered with carpet $\frac{3}{4}$ yd. wide, how many strips, running lengthwise, must be purchased?



NOTE. — When a part of the width of a strip is needed, a whole strip must generally be purchased.

2. How many yards of carpet must be purchased for this floor, allowing 1 yd. for waste in matching?

3. A room is 20′ × 18′.

a. Find the cost of carpeting the floor with Brussels carpet, 27 in. wide, at \$1.25 per yard, adding 8 cents a yard for making and laying, and allowing $1\frac{1}{2}$ yd. for waste in matching figures.

b. Find the cost of covering the floor with matting one yard wide, at 60¢ a yard, adding 7¢ a yard for laying and allowing nothing for waste.

4. Find the cost of carpeting a floor $16' 6'' \times 14'$ with ingrain carpet, 1 yd. wide, at \$.75 a yard, allowing $1\frac{1}{2}$ yd. for waste in matching, and covering the floor first with carpet paper at 4¢ a square yard.

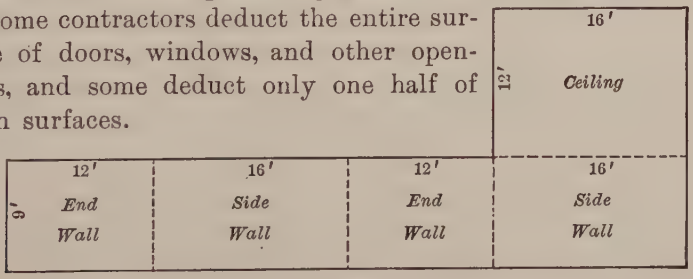
5. A room is 45 ft. by 25 ft. How many yards of carpet $\frac{3}{4}$ yd. wide are needed to cover the floor, running the strips so as not to divide a strip?

6. An office floor $18' \times 27'$ is covered with inlaid linoleum $1\frac{1}{2}$ yd. wide. Find its cost at \$1.40 per square yard, allowing one square yard for matching.

7. Measure your schoolroom and compute the cost of carpeting a room of the same size with velvet carpet $\frac{3}{4}$ yd. wide at \$1.30 per yard.

PLASTERING

238. The cost of plastering is estimated by the square yard. Some contractors deduct the entire surface of doors, windows, and other openings, and some deduct only one half of such surfaces.



239. *Written*

1. The cut on page 105 represents the four walls and ceiling of a room.

a. What is the entire length of the end and side walls? What is the height?

b. How many square feet are there in all the walls?

c. How many square feet are there in the ceiling?

d. How many square feet are there in the walls and ceiling together?

e. How many square yards are there in all?

f. What will it cost to lath and plaster this room at 35 cents a square yard, taking out $5\frac{5}{9}$ square yards for openings?

2. A schoolroom is 40 ft. square and 14 ft. high. The wainscoting is 3 ft. 8 in. high.

a. Find the cost of lathing and plastering the four walls at 38¢ per square yard, making full allowance for 10 windows 4 ft. $\times$ $7\frac{1}{2}$ ft., and no allowance for doors.

b. Find the cost of a steel ceiling for this room at 9¢ per square foot.

3. Find the cost of lathing and plastering the walls and ceiling of a room 19 ft. by 36 ft. and 12 ft. high at 36¢ per square yard, making one half allowance for 3 doors each 3 ft. 8 in. by 8 ft., and six windows each 4 ft. by $7\frac{1}{2}$ ft.

4. Measure the plastered parts of your schoolroom to the nearest half of a foot.

a. Find the cost of metal laths at 18¢ a square yard, sufficient for this room, making full allowance for doors and windows.

b. If a contractor received 60¢ per square yard for lathing and plastering the room, using the answer to question *a* for the

cost of the laths, find what the labor and the remaining materials cost.

c. Find the cost of wood laths sufficient for this room at $5.75 per M., estimating a bundle of 50 laths to cover $2\frac{1}{2}$ square yards.

WALL COVERINGS

240. A roll of figured wall paper is usually 8 yards long and $\frac{1}{2}$ yard wide. How many square yards of paper does it contain?

Ingrain paper is 30 inches wide.

Paper hangers generally estimate that a roll of paper will cover from 30 to 34 square feet of wall, after allowing for waste.

Woven wall coverings are sold by the square yard.

241. *Written*

1. A room $22' \times 16\frac{1}{2}'$ and 10 feet high was papered entirely with figured wall paper costing 30 cents a roll.

A molding costing 5 cents a lineal foot extended around the top of the wall. Two men did the work in one day and received $3.75 each.

a. Find the cost of decorating the room, allowing for one window $4\frac{1}{2}$ ft. by 6 ft., two windows 3 ft. 4 in. by 6 ft., and a baseboard 12 in. high, and estimating a roll of paper to cover 32 square feet of surface.

b. How much would the ceiling have cost if, instead of being papered, it had been covered with prepared muslin costing 20 cents a square yard and tinted with material costing 45 cents and requiring 1 day's labor for two men?

2. Find the cost of decorating a dining room $14' \times 18'$ and $9\frac{1}{2}$ ft. high, as follows: the side walls covered with plain burlap at 25 cents a square yard; the ceiling covered with paper at

10 cents a roll, a roll covering 30 square feet; picture molding and plate rail costing $12; water colors, glue, flour, etc., 65 cents; allowance made for 100 square feet of openings; labor, $2\frac{1}{2}$ days for two men at $3\frac{1}{2}$ per day for each man.

3. *a.* Select a room in your own home. Find the cost of decorating it as your mother would like to have it done. Ask her what she would like to have put on the walls; then you make the measurements, compute the amount of material and labor, and the cost.

b. Decorate in the same way a room 15 ft. long, 12 ft. wide, and 9 ft. high.

LUMBER MEASURE

242. A piece of wood 1 ft. long, 1 ft. wide, and 1 in. thick is a **board foot** (bd. ft.).

To the Teacher. — As material for this lesson, a real board foot — a piece of board exactly 1 ft. long, 1 ft. wide, and 1 in. thick — should be provided. Refer to it in obtaining answers to the oral questions below and whenever pupils seem to answer wide of the mark in this subject. This is very important.

243. *Oral*

1. A board foot contains how many cubic inches?

2. How many board feet piled one upon another would make a cubic foot of lumber?

3. A board foot is what part of a cubic foot?

4. A piece of lumber $10' \times 1' \times 1''$ contains how many board feet?

5. If the lumber in Question 4 were 2 in. thick, how many board feet would it contain? If it were 5 in. thick? 7 in. thick?

6. The floor of a room 10 feet square is 1 in. thick. How many feet of boards does it contain?

7. A bridge 15 ft. long and 10 ft. wide is floored with 3-inch plank. How many feet of plank are there in the floor?

8. A board 8 ft. long, 3 in. wide, and 1 in. thick contains how many board feet?

9. A timber is 30 ft. long and one foot square. Walk as far as this timber would reach. Show with your hands its width and thickness. How many cubic feet of lumber does it contain? How many board feet?

10. A board 16 ft. long and 1 in. thick must be how wide to contain 8 board feet?

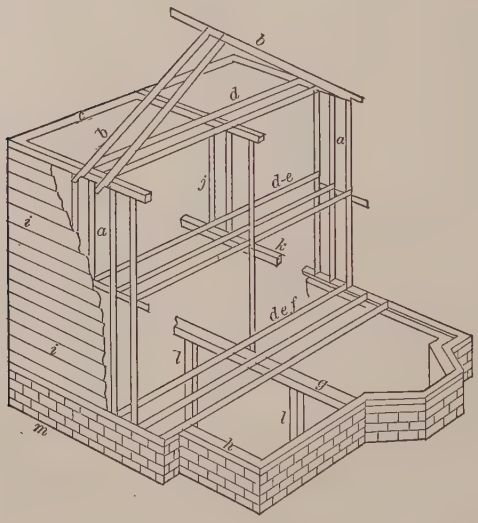
244. We may find the number of board feet in a piece of lumber by multiplying the number of cubic feet by 12. The rule commonly used by dealers and mechanics gives the same result, and is stated as follows:

To find the number of board feet in any piece of lumber, multiply together its three dimensions, **two** *of them expressed* **in feet** *and the* **other in inches.**

The cost of 25 planks each 16 ft. long, 11 in. wide, and 3 in. thick, at \$ 28 per thousand feet, may be found thus:

$$\frac{16}{1} \times \frac{11}{\cancel{12}_{4}} \times \frac{\cancel{3}^{1}}{\cancel{1}} \times \frac{25}{1} \times \frac{\cancel{\$28}^{7}}{\cancel{1000}} = \$30.80. \quad \textit{Ans.}$$

Lumber that is less than 1 in. thick is counted as 1 in. thick in measuring.



KEY TO ILLUSTRATION

a. Outside studding	*g.* Girder
b. Rafters	*h.* Sills
c. Plates	*i.* Sheathing
d. Ceiling joists	*j.* Partition studs
de. Second floor joists	*k.* Partition heads
def. First floor joists	*l.* Piers
	m. Foundation

245. *Written*

1. Find the cost of the following bill of lumber:

NOTE.—M. stands for thousand feet.

4 sills	$6'' \times 10'' \times 16'$, \$27 per M.
2 sills	$6'' \times 10'' \times 18'$, \$27 per M.
1 girder	$8'' \times 10'' \times 18'$, \$27 per M.
26 rafters	$2'' \times 6'' \times 14'$, \$27 per M.
60 pieces of studding	$2'' \times 4'' \times 16'$, \$27 per M.

Flooring for three floors $18' \times 30' \times \frac{7}{8}''$, \$38 per M.
2000 feet of sheathing, \$30 per M.
200 feet of casings, \$45 per M.

2. What is the cost of 10 joists, each 16 ft. long, 10 in. wide, and 3 in. thick, at \$26 per M.?

3. Find the cost of a stick of timber 8 in. square, and 30 ft. long, at \$18 per M.

4. What is the cost of 8 sticks of timber each 36 ft. long, 10 in. wide, 8 in. thick, at \$18 per M.?

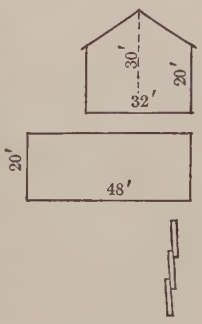
5. I need 213 planks 4 ft. 8 in. long, 1 ft. wide, and $1\frac{1}{2}$ in. thick, to build a sidewalk. How much will they cost at \$25 a thousand?

6. A builder bought 425 half-inch boards 16 ft. long and $2\frac{1}{2}$ in. wide. How many feet of lumber did he buy?

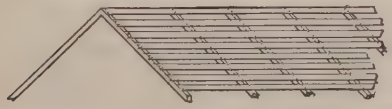
7. How many board feet are there in 24 joists $16' \times 14'' \times 3''$?

8. How many feet of 2-inch plank will cover a barn floor 20 ft. wide and 60 ft. long?

9. *a.* These figures represent one end and one side of a building covered with clapboards $\frac{5}{8}$ of an inch thick that cost \$36 per M. Allowing $\frac{1}{5}$ of all the lumber purchased, for waste in cutting and overlapping, how much did the clapboards for this building cost?



HINT. — If there were no waste, how much lumber would be needed? This is what part of the lumber purchased, when $\frac{1}{5}$ of the lumber purchased is wasted?



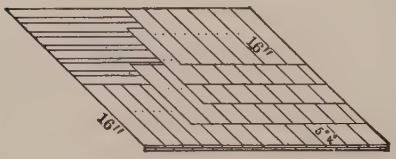
b. The rafters are $20' \times 2'' \times 6''$ and 25 rafters are used on each side of the roof. How much did they cost at \$27 per M.?

c. The roof-boards are nailed to the rafters, with spaces between the boards, so that only $\frac{3}{4}$ of the surface of the roof is covered with boards. What is the cost of the roof-boards for the roof of this building at $24 per M.?

ESTIMATING SHINGLES

246. *Oral*

1. In measuring shingles, the average width of the shingles is supposed to be 4 inches. The length varies, but they are always laid so that more than two thirds of the shingle is covered by the courses of shingles above. If they are laid so that 5 inches of the length are exposed to the weather, a shingle 4 inches wide will really make how many square inches of roof? 1000 shingles will make how many square inches of roof?



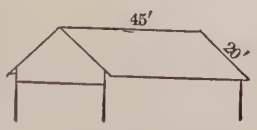
2. When shingles are laid 6 inches to the weather, each shingle will make how many square inches of roof? How many shingles will make one square foot of roof? How many shingles are required for one square (100 square feet) of roof?

3. When shingles are laid $4\frac{1}{2}$ inches to the weather, one shingle makes how many square inches of roof? How many shingles will make one square foot of roof? One square of roof?

247. *Written*

1. *a.* How many shingles laid 6 inches to the weather are required for one square foot of roof? *b.* For one square of roof? *c.* For a roof 20 ft. long, each slanting side of which measures 9 ft. in width?

2. Find the cost of the shingles for this roof at \$4.50 per M. (1000 shingles), estimating that 675 shingles will make one square of roof.



3. Find the cost of the shingles for a roof 36 ft. long, each slanting side 19 ft. wide; the shingles being laid so that seven shingles make one square foot of roof, and costing \$5.20 per M.

4. A shed roof $18' \times 40'$ slants only one way. Find the cost of the shingles required for it at \$4.80 per M., the shingles being laid so that $7\frac{1}{2}$ shingles make one square foot of roof.

5. Find the cost of the shingles at \$6 per M. to cover 15 squares of roof, the shingles being laid 6 inches to the weather.

6. The shingles for a roof cost \$68.25. Each side of the roof measured 25 ft. by 35 ft., and the shingles were laid so that $6\frac{1}{2}$ shingles made a square foot of roof. Find the price per M.

VOLUME AND CAPACITY

248. *Oral*

1. In what denominations are measures of volume expressed? Measures of capacity?

2. One gallon is equal to how many cubic inches?

3. One bushel is equal to how many cubic inches?

4. When the volume, in cubic inches, of a tank, cistern, or cask, is known, how may its capacity in gallons be found?

5. When the volume, in cubic inches, of a box, bin, or barrel, is known, how may its capacity in bushels be found?

6. What are the prime factors of 231?

7. How may we find the capacity of a bin in bushels, when we know its dimensions in inches? In feet?

8. When we know the capacity of a bin in bushels, how may we find its volume in cubic inches? In cubic feet?

9. A rectangular tin can 7 in. by 3 in. by 11 in. will hold how many liquid quarts?

10. The volume of a bin is 215,042 cubic inches. How many bushels will it hold?

11. The volume of a keg is 2310 cubic inches. How many gallons will it hold?

12. What is the volume of a cask that holds 100 gallons?

13. The volume of a rectangular solid is the product of how many dimensions?

14. The dimensions are what of the volume?

15. When three factors are known, how may their product be obtained?

16. When a product and two of its three factors are known, how may the other factor be found?

17. When the dimensions of a rectangular solid are known, how may its volume be found?

18. When the volume and two dimensions of a rectangular solid are known, how may the other dimension be found?

19. A box 6 in. by 8 in. must be how deep to contain 96 cu. in.?

20. A box $5'' \times 5'' \times ?$ contains 100 cu. in.

21. A box $? \times 11'' \times 3''$ holds 231 cu. in.

22. A box $7'' \times ? \times 3''$ holds 231 cu. in.

23. A rectangular tin box is 11 inches long and 7 inches wide and holds a gallon. How deep is it?

249. *Written*

1. Find in gallons the capacity of a cistern 11 ft. square and 6 ft. deep.

2. Find to the nearest hundredth (*a*) the number of gallons that are equivalent to one cubic foot; (*b*) the number of bushels that are equivalent to one cubic foot.

3. Find to the nearest hundredth the number of gallons that are equivalent to one bushel.

4. A box car is 33 ft. long, 8 ft. 8 in. wide, and 7 ft. 6 in. high, inside measure. It is strong enough to carry 30 tons. A bushel of corn weighs 60 pounds.

a. How many bushels of corn can the car carry?

b. How many cubic feet (to the nearest tenth of a cubic foot) will the load occupy?

c. How many cubic feet of space will be left unoccupied?

d. A bushel of oats weighs 32 pounds. How much space, to the nearest tenth of a cubic foot, will be left when the car contains 20 tons of oats? (Allow $1\frac{1}{4}$ cu. ft. for a bushel.)

5. A cellar 35 ft. long and 21 ft. wide was flooded, during a storm, to a depth of 3 ft. 8 in. What was the cost of pumping out the water at $.03 a barrel?

6. A watering trough in the form of a rectangular box is 11 ft. long, 18 in. wide, and 14 in. deep. How many barrels of water will it hold? (Result correct to hundredths.)

7. A farmer, having ten 44-gallon casks, used them for storing wheat. How many bushels of wheat could he store in them? (Result correct to hundredths.)

8. I have in my attic a rectangular copper water tank 14 ft. by 9 ft., into which the rain-water from the roof is carried. During a shower, the tank was filled to a depth of 11 inches. How many barrels of water ran into it?

9. A reservoir from which a city is supplied with water has a surface of 35 acres. If no water ran into it, the surface of the water would be lowered 5 inches a day by the pipes that supply the city. How many gallons are used daily?

10. A teamster wanted to know how many gallons of water he could carry in his watering-pail. He had no measure except a foot rule. He measured a feed box and found the inside dimensions to be: length 2 ft. 9 in., width 1 ft. 9 in., depth 1 ft. He filled the pail with oats and emptied them into the box, repeating the process till the box was full. The box held twelve pails of oats. Find; (*a*), the volume of the box in cubic inches, (*b*), the volume of the pail, (*c*), the capacity of the pail in gallons.

11. Some boys found a bowlder, and guessed the number of cubic inches of stone that it contained. To find which was the best guesser, they filled a large pail with water and set it in an empty washtub. Then they placed the bowlder in the pail of water so that the bowlder was entirely submerged, and found that 5 qt. 1 pt. of water had run over into the washtub. The nearest guess was 350 cu. in. Was it too large, or too small, and how much?

12. A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of a gallon of water? (Correct to 3 dec. places.)

13. The water displaced by a floating body weighs the same as the floating body. A log containing 20 cu. ft. of wood, floating in a stream, was three fourths under water.

a. How many gallons of water did it displace? (2 dec. places.)

b. What was its weight?

14. My house covers a surface equivalent to a rectangle $20' \times 40'$. During a rain storm, water fell to an average depth of .8 of an inch, according to the record at our weather station. How many barrels of water fell on my roof? (2 dec. places.)

15. A farmer's wagon box was 3 ft. 4 in. wide, 16 ft. 6 in. long, and 20 in. deep. Find, to the nearest tenth, the number of bushels that it holds.

16. A wagon box 12 ft. long and 3 ft. 6 in. wide holds 40 bushels. Find its depth to the nearest tenth of an inch.

17. A fruit grower made some bushel crates that were 2 ft. long and 1 ft. deep. Find their width to the nearest tenth of an inch.

18. An aquarium is 7 ft. long and 22 in. wide.

a. When it contains 40 gallons of water, how deep is the water?

b. How deep is the water when it contains one hogshead of water?

c. When the water is two feet deep, how many gallons does the aquarium contain?

d. When the water is 8.64 in. deep, how many pounds of water are there in the aquarium? (See Question **12.**)

19. How many barrels of water will a rectangular cistern $6' \times 5' \times 4\frac{1}{2}'$ hold?

REVIEW AND PRACTICE

250. *Oral*

1. Read CLI; MCMIX; CDLXXXVIII; CCXVI.

2. Read 10.0010; 100.00100; 101.00001; 101.100.

3. Give results rapidly:

$38 + 45$; $98 - 79$; $98 + 34$; 78×99; 60×80; 1.047×100; 96×25; $315 \times .33\frac{1}{3}$; 12×25; $1300 \div 25$; 48×125; $428.3 \div 1000$; 125×2000; $360{,}000 \div 400$.

4. What is the smallest number that exactly contains 2, 3, 4, 6, and 8?

5. What is the largest number that will exactly divide 45, 60, and 75?

6. 27 is a power of what number?

7. Name four powers of 10.

8. How is the value of a figure affected by moving it three places to the left?

9. Of what number are 5, 2, and 13 the prime factors?

10. The product of two or more numbers is found by what operation?

11. One of the two factors of a number is found by what operation, when the product and the other factor are known?

12. Describe two tests for examples in subtraction.

13. The product of three factors contains five decimal places. One of the factors has three decimal places and another two. How many decimal places has the third factor?

14. Name four signs of aggregation.

15. $3 + 18 \div 6 - 2 \times 3 = ?$

16. How can you tell whether a number is divisible by

(*a*) 2, (*b*) 3, (*c*) 4, (*d*) 5, (*e*) 6, (*f*) 8, (*g*) 10, (*h*) 9?

17. The sum of the digits in a number is 27. What numbers will divide it?

18. The sum of the digits in a number is 18 and the figure in units' place is 8. What numbers will divide it?

19. The figure in units' place in a given number is 7. What kind of numbers will not divide the given number?

20. Name a number that has no integral factor but itself and one. What kind of number is it?

21. Two of the three factors of a number being given, how can the remaining factor be found?

22. Reduce to simplest form:

$$\frac{48}{11}, \frac{63}{9}, \frac{12}{42}, \frac{85}{10}, \frac{40}{9}, \frac{84}{16}, \frac{75}{25}, \frac{600}{25}.$$

23. Change $\frac{5}{9}$ to a fraction whose denominator is 81.

24. Change $8\frac{5}{12}$ to an improper fraction.

25. Divide 375 by 25.

26. What is the cost of 48 horses at \$125 each?

27. The average price per dozen paid for eggs by an egg buyer during a season was $\$.16\frac{2}{3}$. At that rate, what did he pay for 1000 dozen? How many eggs could he buy for \$100?

28. A merchant bought 700 yards of damaged cloth at $\$.14\frac{2}{7}$ a yard. He sold 200 yards of it at \$.50 a yard, and the rest at \$.10 a yard. How much did he gain?

29. Name six parts that a bill should contain.

30. A gallon of spirits of camphor will fill how many 8-ounce bottles?

31. A stationer bought paper at \$1.00 a ream and sold it at \$.20 a quire. How much did he gain on 10 reams?

32. How many degrees are there in all the angles of a rectangle?

33. Eighteen straight lines are drawn from the center to the circumference of a circle, making equal angles at the center. What is the size of each angle? What is the size of each arc formed in the circumference?

34. What U. S. coin is most nearly like the English shilling?

35. What German coin is most nearly like the U. S. 25-cent piece?

36. What is the silver piece, coined in this country, whose value is most nearly like that of the franc?

37. What is the area of a triangle whose base and altitude are respectively 25 rods and 20 rods?

38. The area of a triangle is 5 acres. What is the area of a parallelogram having the same base and altitude?

39. A pile of stove-wood is 12 ft. long and 8 ft. high. What is it worth at $2.50 a cord?

40. How many strips of carpet 27 in. wide, running lengthwise of the room, are required to carpet a room 9 ft. wide?

41. A piece of timber 1 ft. square and 20 ft. long contains how many board feet?

42. The volume of a grain bin is 2,150,420 cubic inches. How many bushels of grain will it hold?

43. Make and solve a problem that requires multiplication of fractions.

44. Make and solve a problem that requires reduction of denominate numbers.

45. Make and solve a problem about capacity or volume.

46. $75 worth of steel wire will make $1000 worth of needles. How much is the value of the wire increased by being made into needles?

47. A man can drill 60,000 needle-eyes in a week. That is how many per day? How many per hour, if he works eight hours a day?

48. If 750,000 medium-sized needles weigh 1 cwt., how many would it take to make a pound?

49. 112 sheets of $14'' \times 20''$ IC tin roofing plates weigh 107 pounds. What is the weight of 560 such plates? Of 56 such plates?

50. (Ideas of Proportion.) *a.* 10 is how many times $2\frac{1}{2}$?

b. If $2\frac{1}{2}$ quarts of berries weigh $4\frac{1}{2}$ lb., what will 10 quarts weigh?

c. How many quarts will weigh 9 lb?

d. What will 15 quarts weigh?

51. If a boy can carry 150 apples weighing 3 ounces apiece, how many apples weighing 9 ounces apiece can he carry? $1\frac{1}{2}$ ounces?

251. *Written*

This table, compiled from the records of the United States Weather Bureau, shows in inches the average precipitation of moisture for each month of the year in different sections of the country.

	Boston	Binghamton	Buffalo	Philadelphia	Key West	New Orleans	Minneapolis	Des Moines	Topeka	Denver	Salt Lake Cy.	Spokane	Seattle	Los Angeles
Jan.	3.84	1.72	3.21	3.28	2.05	4.53	.65	1.19	.98	.48	1.33	2.34	4.33	2.64
Feb.	3.50	2 00	2.99	3.39	1.64	4.62	.80	1.08	1.46	.50	1.40	1.99	4.03	2.85
Mar.	4.27	2.93	2.70	3.40	1.27	5.14	1.77	1.53	2.09	.91	1.99	1.44	3.31	2.87
Apr.	3.46	2.11	2.40	2.89	1.21	4.98	2.46	3.00	2.74	1.98	2.13	1.29	2.97	1.15
May	3.45	2.69	3.14	3.16	2.77	4.01	3.34	4.78	5.28	2.58	1.97	1.40	2.26	.49
June	3.02	3.24	3.52	3.18	4.14	6.19	3.75	4.88	4.76	1.49	.73	1.48	1.60	.09
July	3.47	3.11	3.42	4.19	3.64	6.36	4.22	3.83	4.90	1.65	.52	.69	.80	.01
Aug.	4.06	3.35	3.07	4.50	4.72	5.68	3.80	3.57	4.46	1.36	.74	.50	.50	.03
Sept.	3.19	2.82	3.15	3.41	6.91	4.63	3.17	2.99	3.37	.86	.80	.99	2.12	.08
Oct.	3.96	2.98	3.33	3.01	5.31	2.96	2.75	2.75	1.99	.90	1.50	1.34	2.96	.81
Nov.	4.16	2.04	3.34	3.18	2.25	3.74	.98	1.45	1.08	.53	1.40	2.27	6.31	1.35
Dec.	3.26	2.27	3.37	2.95	1.66	4.25	1.00	1.36	.93	.64	1.43	2.40	5.96	2.99

1–14. Find, to the nearest hundredth, the average monthly precipitation in each of the cities named. Can you do it in 30 minutes, testing your work?

15. The population of the Japanese Empire is 42,352,620, and of the Russian Empire 128,932,173 according to a recent census. Find the difference between them.

16. The earth, in its revolution around the sun, passes through space at the rate of about 19 miles a second. How far does it travel during a 30-minute recitation in arithmetic?

17. *a.* How many years and days, taking no account of the extra day in leap year, would be required for a railroad train, traveling day and night at a uniform rate of 50 miles per hour, to travel 93,000,000 miles, the approximate distance from the earth to the sun?

b. The planet Neptune is about thirty times as far from the sun as the earth is. Using the answer to *a*, find the time in which such a train could travel a distance equal to that from the sun to Neptune.

18. A newspaper, folded into four leaves, each $17'' \times 24''$ in size, has seven columns on a page. The average number of copies of this paper printed per day during the twenty-seven week-days of January, 1908, was 48,400.

a. How many columns were printed?

b. If all these papers were spread out in single sheets, how many acres of land would they cover? (Indicate and cancel.)

19. Multiply 2496 by 329 and write each partial product in words.

20. $(94.7 + 8.456 + 37.92 \times 84 - 93.6 \div 1.8) \div \overline{14.4 \div .04}$.

21. The roof of my barn is sixty feet long. The slant height, from the eaves to the ridge, is 25 feet on each side. It is covered with redwood shingles costing \$4.50 per M., laid 4 inches to the weather. $7\frac{1}{2}$ pounds of nails were used with each thousand shingles and cost \$2.90 per hundredweight. The men

who laid the shingles averaged 1350 shingles per day for each man, and received $3.00 each, per day.

a. What did the shingles cost?
b. What did the nails cost?
c. What did the labor cost?
d. What did the roof cost?

22. A tile roof is 40 ft. long and 15 ft. 6 in. from eaves to ridge on each side.

a. What was its cost at $13.80 per square?

b. What is the weight of the tile in tons, if 975 lb. of tile will make a square of roof?

23. A factory roof is made of sheets of tin 20″ by 28″. To make the seams, $2\frac{3}{4}$ inches are taken from the width, and $\frac{3}{4}$ of an inch from the length of each sheet.

a. How many square inches of roof will one sheet make?

b. How many sheets will make a square of roof?

24. Find the prime factors of 4503.

25. Determine which of the following numbers are composite: 529, 403, 143, 397, 1943, 407.

26. Reduce $\frac{14630}{31570}$ to a fraction whose numerator and denominator are prime to each other.

27. $\frac{39}{2\,8\,0} \div 2\frac{1}{6} = ?$

28. Multiply in the shortest way:

a. 8697 by $.33\frac{1}{3}$.
b. 9456 by .25.
c. 793,051 by $.14\frac{2}{7}$.
d. 6050 by .125.
e. 39,764 by 99.
f. 4807 by 60,000.
g. 817 by 25.
h. 9796 by .125.
i. 8796 by $16\frac{2}{3}$.
j. 74,583 by $11\frac{1}{9}$.

29. Divide in the shortest way:

a. 39,474 by 25.
b. 9,726,250 by 125.
c. 9438 by $33\frac{1}{3}$.
d. 8753 by $.16\frac{2}{3}$.
e. 93,742 by $16\frac{2}{3}$.
f. 42,835 by $14\frac{2}{7}$.
g. 7648 by .25.
h. 93,042 by $.33\frac{1}{3}$.
i. 9843 by $.14\frac{2}{7}$.
j. 86,728 by 16,000.

30. Add: $17\frac{3}{7}$, $19\frac{11}{14}$, $\frac{19}{36}$, and $3\frac{13}{30}$.

31. Find the smallest number that will exactly contain 24, 42, 54, and 360.

32. Make out a bill containing 3 debit and 2 credit items, your teacher being the debtor and you the creditor. Receipt the bill in full after computing the balance.

33. The surveyor found a rectangular piece of land to be 2640 feet long and 880 feet wide. How many acres did it contain?

34. Find the number of seconds in a solar year.

35. How many fathoms deep is the ocean at a place where a sounding line one third of a mile long will just reach the bottom?

36. Find the cost of a flat tin roof $32' \times 24'$ at \$9.60 per square.

37. *a.* Find the exact number of days from the ninth day of last January to the present time.

b. How many days have passed since the last Fourth of July?

c. How many days will elapse between now and the next Memorial Day?

38. What is the value of a pile of uncut building stone, $33'$ by $6'$ by $3'$, at \$6 per cord (99 cu. ft.)?

39. It requires 4 cu. ft. of water to run the motor of our washing machine ten minutes.

a. How much water is used in 2 hr.?

b. If the motor runs two hours every week, what is the annual cost of the water used, at 14¢ per 100 cu. ft.?

c. How many gallons of water are used?

d. If all the water used for this purpose during a year were collected in a tank 12 ft. by 13 ft., how deep would the water be?

COMPUTATION IN HUNDREDTHS

252. Decimals in hundredths are used very generally in business calculations. The merchant calculates his gain or loss as a certain number of hundredths of the cost of the goods. Banks compute interest in hundredths. Agents who sell goods sometimes figure their earnings as a certain number of hundredths of the selling price of the goods. The relations of numbers are expressed generally in hundredths.

Problems involving computation in hundredths usually present one of the two questions of relation between product and factors, namely:

a. Two factors given, to find the product, or,

b. The product and one factor given, to find the other factor; *e.g.:*

1. A merchant bought pears at $1.60 a bushel and sold them so as to gain .25 of the cost. How much did he gain on one bushel?

Statement of Relation: .25 of $1.60 = gain on one bushel. Here 1.60 and .25 are factors, and the product is to be found. How shall we find it?

2. .40 of the pupils in a school are boys. If there are 600 boys, how many pupils are there in the school?

Statement of Relation: .40 of —— pupils = 600 pupils. Here 600 is a product and .40 one of its factors. How may the other factor be found?

3. A man's salary is $1500. He saves $250. How many hundredths of his salary does he save?

Statement of Relation: ——of $1500 = $250. Here 250 is a product and 1500 one of its factors. How may the other be found?

253. *Written*

In each of the following examples, give the statement of relation and find the answer:

1. A farm worth $4500 rents for .05 of its value. For how much does the farm rent?

2. .90 of the pupils in a class were promoted. If 36 pupils were promoted, how many were there in the class?

3. It cost $24 to decorate a room. The labor cost $18. How many hundredths of the entire expense were for labor?

4. A farmer's crop of apples amounted to 960 bushels, of which 864 bushels were fit for market. How many hundredths of the crop were fit for market?

5. A speculator sold some property for $78,000, and invested $.33\frac{1}{3}$ of the money in grain and $39,000 in real estate. He put the remainder in the bank.

a. How much did he invest in grain?

b. How many hundredths of his money did he invest in real estate?

c. How many hundredths of his money were left?

6. How many dollars' worth of goods must an agent sell to earn $513.40, if he receives .17 of the value of all the goods which he sells?

7. How many hundredths of $142.60 is $7.13?

8. 24 quarts are how many hundredths of six bushels?

9. A grocer bought 8 bushels of potatoes at 75 cents a bushel and sold them for $7.80. He gained how many hundredths of the cost?

10. Three clerks received wages as follows: A, $15 a week; B, $10 a week and .02 of the amount of his sales; C, .05 of the amount of his sales. What was each clerk's yearly income, if the sales of each amounted to $400 per week?

11. .85 of a certain number is 595. What is .14$\frac{2}{7}$ of the number?

12. A boy paid .24 of his money for books, .07 of his money for stationery, and .22 of his money for a football. If he then had $3.76 left, how much had he at first?

13. Mr. Markell bought a house for $4200 and sold it for $4830. How many hundredths of the cost did he gain?

14. By selling his automobile for $1860, Dr. Smith received .66$\frac{2}{3}$ of its cost. What did it cost?

15. The list price of suits for a baseball team was $4.75 apiece. The dealer sold 11 suits for .80 of the list price. How much did he receive for them?

NOTE. — The price at which goods are marked in the price list is called the list price.

16. By selling goods at a reduction of .15 of the list price,

a. What part of the list price is received?

b. What is the list price of goods that are sold for $155.55?

c. What reduction is made on goods that are sold for $170?

17. A contractor makes concrete by mixing 5 barrels of cement, 10 barrels of sand, and 25 barrels of crushed stone. How many hundredths of the mixture is: *a.* cement? *b.* sand? *c.* crushed stone?

PERCENTAGE

254. **Per cent** means *hundredths*.

Seven per cent of \$100 means .07 of \$100, or \$7.

Ten per cent of 300 pounds means .10 of 300 lb., or 30 lb.

Twenty-five per cent of 24 hours means .25 of 24 hours, or 6 hours.

Thirty-three and one third per cent of 276 means $.33\frac{1}{3}$ of 276, or 92.

The sign % indicates per cent; *e.g.*

19% of 200 = .19 of 200, or 38.
9% of \$80 = .09 of \$80, or \$7.20.
$\frac{1}{2}$% means $\frac{1}{2}$ of 1%, or $.00\frac{1}{2}$.
$\frac{3}{5}$% means $\frac{3}{5}$ of 1%, or $.00\frac{3}{5}$.

255. *Oral*

Read each of the following expressions, using the word **hundredths** *instead of the sign %, and find its value:*

1. 9% of \$300
2. 21% of 200
3. 75% of 1000
4. 13% of 30
5. 44% of 20
6. 89% of 1000 lb.
7. $4\frac{1}{2}$% of 100 bu.
8. 96% of 10,000
9. 3% of 120
10. $62\frac{1}{2}$% of 1000
11. 130% of 100
12. 99% of 2
13. $6\frac{3}{8}$% of 800
14. $2\frac{2}{3}$% of 2100
15. 125% of 200
16. $\frac{1}{2}$% of 200
17. $\frac{5}{9}$% of 900
18. $\frac{6}{7}$% of 1400
19. $\frac{1}{10}$% of 1000
20. $1\frac{1}{9}$% of 1800
21. $4\frac{7}{8}$% of 800
22. 15% of 40
23. $1\frac{5}{11}$% of 22
24. $6\frac{4}{7}$% of 70
25. $\frac{3}{4}$% of 140
26. $3\frac{1}{5}$% of 15

27. $4\frac{9}{11}\%$ of \$100

28. $66\frac{2}{3}\%$ of \$1000

29. 108% of \$1000

30. 33% of 2 cents

31. $2\frac{1}{2}\%$ of 400

32. 25% of 40

33. 250% of 4

34. $8\frac{1}{3}\%$ of 30 days

35. $6\frac{1}{6}\%$ of 12 feet

36. $4\frac{3}{4}\%$ of 1600 miles

37. $3\frac{4}{5}\%$ of 15

38. 100% of $\$.37\frac{1}{2}$

39. $\frac{5}{10}\%$ of \$100

40. $\frac{1}{2}\%$ of 18 quarts

41. $\frac{3}{4}\%$ of 40 sheep

42. $\frac{7}{13}\%$ of 52 weeks

256. *The number of hundredths indicated as per cent* is called the **rate per cent**; *the number of which a certain number of hundredths are indicated by the rate* is called the **base**; *the product of the base and rate* is called the **percentage**; *the sum of the base and percentage* is called the **amount**; *the difference between the base and percentage* is called the **difference**; *e.g.*

25% of \$300 is \$75. 25% is the rate; \$300 is the base; \$75 is the percentage; \$375 is the amount; \$225 is the difference.

257. The relations of product and factors usually determine the method to be employed in solving problems in percentage; *e.g.*

1. A man bought some land for \$4500, and sold it so as to gain 12% of the cost. How much did he gain?

Statement of Relation: 12% of \$4500 = gain

.12 of \$4500 = ?

.12 and \$4500 are factors, and the product is to be found. How may we find it?

2. A house rents for \$630, which is 9% of its value. Find its value.

Statement of Relation: 9% of (value of the house) = \$630

.09 of —— = 630

630 is a product and .09 one of its factors. How may we find the other factor?

3. A merchant gained \$19 on an article that cost \$95. The gain was what per cent of the cost?

Statement of Relation: —— of \$95 = \$19

19 is a product and 95 one of its factors. Find the other factor.

258. In finding the rate per cent, or number of hundredths, how many decimal places must there be in the quotient? Then how must the number of decimal places in the dividend compare with the number of decimal places in the divisor?

Summary

Before dividing, to find the rate per cent, arrange the dividend and divisor so that the dividend contains two more decimal places than the divisor. This may be done by annexing ciphers to one or the other of these terms, as may be necessary.

If the quotient is not exact when two decimal places have been reached, express the remainder as a common fraction, in the quotient, thus:

a. 7 bushels are what per cent of 8.5 bushels?

Statement of Relation: —— of 8.5 bu. = 7 bu.

$$8.5\overline{)7.0\,0\,0} \qquad .82\tfrac{30}{85} = .82\tfrac{6}{17} \text{ or } 82\tfrac{6}{17}\%. \quad \textit{Ans.}$$

680

200

170

30

b. A lake in Maine is 152.875 rods long and 92 rods wide. Its length is what per cent of its width?

Statement of Relation: —— of 92 rd. = 152.875 rd.

$1.66\frac{15}{92}\frac{5}{0} = 1.66\frac{31}{184}$ or $166\frac{31}{184}\%$. *Ans.*

$$
\begin{array}{r|l}
92.0 & 152.875 \\
 & 920 \\
 & \overline{6087} \\
 & 5520 \\
 & \overline{5675} \\
 & 5520 \\
 & \overline{155}
\end{array}
$$

NOTE. — Care should be taken to express the decimal rate per cent properly, as hundredths. Every fractional part of 1% must be written at the right of the hundredths' place.

$1\% = .01$	$12\frac{1}{2}\% = .12\frac{1}{2}$ or .125
$9\% = .09$	$\frac{1}{2}\% = .00\frac{1}{2}$ or .005
$10\% = .10$	$10\frac{7}{10}\% = .10\frac{7}{10}$ or .107
$90\% = .90$	$33\frac{1}{3}\% = .33\frac{1}{3}$
$100\% = 1.00$	$8\frac{1}{4}\% = .08\frac{1}{4}$ or .0825
$900\% = 9.00$	$\frac{1}{4}\% = .00\frac{1}{4}$ or .0025
$125\% = 1.25$	$\frac{1}{8}\% = .00\frac{1}{8}$ or .00125

259. *Written*

1. *Express decimally:*

a. 7%	*f.* $6\frac{1}{4}\%$	*k.* 101%	*p.* $\frac{1}{2}\%$
b. 6%	*g.* $12\frac{1}{2}\%$	*l.* 110%	*q.* $\frac{3}{4}\%$
c. 2%	*h.* $15\frac{3}{4}\%$	*m.* 250%	*r.* $\frac{2}{5}\%$
d. 12%	*i.* $37\frac{1}{2}\%$	*n.* 200%	*s.* $\frac{5}{8}\%$
e. 78%	*j.* $4\frac{5}{8}\%$	*o.* $127\frac{1}{2}\%$	*t.* $\frac{18}{20}\%$

2. \$291 is $16\frac{2}{3}\%$ of what?

3. 35% of a number is 700. Find the number.

4. 84.20 is what per cent of 421?

5. Find $\frac{3}{8}\%$ of \$5600.

6. Find $66\frac{2}{3}\%$ of 927 tons.

7. 39.744 is what per cent of 900?

8. A short ton is what per cent of a long ton?

9. 386% of $244 =$ what?

10. $23\frac{1}{3}\%$ of a number is 7000. Find the number.

11. A nautical mile is 6086.07 feet. A statute mile is what per cent of a nautical mile?

12. Find a number, $\frac{7}{8}$ per cent of which is 287.

13. 48 rods are what per cent of a mile?

14. $57\frac{3}{4}$ cubic inches are what per cent of one gallon?

15. Find $3\frac{7}{11}\%$ of \$235.

16. 17% of 2475 is what per cent of 720?

17. .043 is what per cent of 17.2?

18. A man's salary is \$1850 and his expenses \$1757.50. His expenses are what per cent of his salary?

19. A man's expenses are \$2140 a year. His salary is 125% of this sum. Find his salary.

20. A man bequeathed 18% of his estate to a hospital, 7% to a missionary society, and 30% to his wife. The remainder was divided equally among his three brothers. If the estate amounted to \$72,600, how much did each of the brothers receive?

21. For how much a month must a house worth \$6000 be rented in order that the rent may amount to $7\frac{1}{2}\%$ of the value of the house?

22. A grocer bought 12 cases of coffee, each containing 50 one-pound packages, and sold 240 packages. What per cent of the coffee did he sell?

23. Which is greater, and how much, 50% of 75, or 75% of 50?

24. A field is 375 feet long and 150 feet wide.

a. Its breadth is what per cent of its length?

b. Its length is what per cent of its breadth?

25. Find in acres the area of a rectangular field whose width is 45 rods and whose length is $142\frac{2}{9}\%$ of its width.

26. A manufacturing company employing 272 persons, whose weekly wages average \$12 apiece, raises the wages of its employees $8\frac{1}{3}\%$. How much per year is then paid to all of them?

27. 18% of the men in an army died of disease. If the loss from this cause was 1260 men, how many men were there in the army at first?

28. A man having \$20,000 in the bank drew out 30% of it and then 25% of what was left. How many dollars still remained in the bank?

29. 83% of the boys in a military school attended a game of football. If 166 boys attended the game, how many boys were there in the school?

PER CENTS EQUIVALENT TO COMMON FRACTIONS

260. All percentage problems involving the relation of product and factors may be solved in decimals. But in many cases the work may be shortened by changing the per cents to common fractions.

261. *Oral*

1. The whole of anything is how many hundredths of it? What per cent of it?

2. $\frac{1}{2}$ of anything is how many hundredths of it? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{1}{8}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{6}$? $\frac{5}{6}$? $\frac{1}{12}$? $\frac{1}{20}$?

3. What common fraction is the same as .10? .20? .30? .40? .50? .60? .70? .80? .90? .25? .33$\frac{1}{3}$? .14$\frac{2}{7}$? 62$\frac{1}{2}$? .37$\frac{1}{2}$? .66$\frac{2}{3}$? .12$\frac{1}{2}$? .87$\frac{1}{2}$? .75? .16$\frac{2}{3}$? .83$\frac{1}{3}$? .08$\frac{1}{3}$. .05?

4. What per cent is the same as $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{1}{20}$? $\frac{1}{16}$? $\frac{1}{25}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$?

5. *Learn this table:*

$\frac{1}{2} = 50\,\%$	$\frac{1}{3} = 33\frac{1}{3}\,\%$	$\frac{5}{8} = 62\frac{1}{2}\,\%$
$\frac{1}{4} = 25\,\%$	$\frac{2}{3} = 66\frac{2}{3}\,\%$	$\frac{7}{8} = 87\frac{1}{2}\,\%$
$\frac{3}{4} = 75\,\%$	$\frac{1}{6} = 16\frac{2}{3}\,\%$	$\frac{1}{10} = 10\,\%$
$\frac{1}{5} = 20\,\%$	$\frac{5}{6} = 83\frac{1}{3}\,\%$	$\frac{1}{12} = 8\frac{1}{3}\,\%$
$\frac{2}{5} = 40\,\%$	$\frac{1}{7} = 14\frac{2}{7}\,\%$	$\frac{1}{20} = 5\,\%$
$\frac{3}{5} = 60\,\%$	$\frac{1}{8} = 12\frac{1}{2}\,\%$	$\frac{1}{16} = 6\frac{1}{4}\,\%$
$\frac{4}{5} = 80\,\%$	$\frac{3}{8} = 37\frac{1}{2}\,\%$	$\frac{1}{25} = 4\,\%$

6. What per cent of anything is left after 50 % of it has been taken away? After 75 % of it has been taken away? 30 %? 40 %? 35 %? 33$\frac{1}{3}$ %? 62$\frac{1}{2}$ %? 37$\frac{1}{2}$ %? 83$\frac{1}{3}$ %? 16$\frac{2}{3}$ %? 49 %? 1 %? 2$\frac{1}{2}$ %? 98$\frac{1}{2}$ %?

7. What per cent of anything is left after 15 %, 10 %, and 5 % of it have been taken away?

8. A girl used 12 % of her Christmas money on one day, 18 % the next day, and 15 % the next day. What per cent of her money remained?

9. 97$\frac{1}{8}$ per cent of the pupils belonging to a certain school were present. What per cent of the pupils were absent?

10. 2$\frac{3}{7}$ % of the pupils in a school were absent. What per cent of the pupils were present?

11. 37 % of a shipment of peaches were spoiled. What per cent of the peaches were good?

12. *Using common fractions instead of decimals, find:*

a. 50 % of \$124
b. 25 % of 36
c. 75 % of 24
d. $33\frac{1}{3}$ % of 999
e. $66\frac{2}{3}$ % of 42
f. $83\frac{1}{3}$ % of 30
g. $16\frac{2}{3}$ % of 48 bushels
h. $14\frac{2}{7}$ % of 49 feet
i. $12\frac{1}{2}$ % of 30 inches
j. $37\frac{1}{2}$ % of 64 days
k. $87\frac{1}{2}$ % of 72 pounds
l. $8\frac{1}{3}$ % of 132 square miles
m. 5 % of \$200,000
n. $6\frac{1}{4}$ % of 32 quarts
o. 4 % of 75 cents
p. $37\frac{1}{2}$ % of 56 minutes
q. $16\frac{2}{3}$ % of 180 grains
r. $12\frac{1}{2}$ % of 64 miles

13. 5 cents are $12\frac{1}{2}$ % of what sum?

14. Frank paid $16\frac{2}{3}$ % of his money for a book that cost \$.20. How much money had he?

15. Arthur earned \$1.60 and paid 75 % of it for a football. How much had he left?

16. Wallace earned a sum of money, paid $87\frac{1}{2}$ % of it for a suit of clothes, and had \$2 left.

a. \$2 is what per cent of the money which he earned?

b. How much did he earn?

17. A farmer raised a crop of potatoes, sold $66\frac{2}{3}$ % of them, and had 200 bushels left.

a. 200 bushels were what per cent of the crop?

b. How many bushels were raised?

c. How many bushels were sold?

18. One morning Lucy cut a basket of roses; 25 % of them were pink, $33\frac{1}{3}$ % yellow, $16\frac{2}{3}$ % red, and the rest white.

a. What per cent of the roses were not white?

b. What per cent were white?

c. If there were 15 white roses, how many roses did Lucy cut?

d. How many were yellow?

e. How many were pink?

f. How many were red?

19. A wholesale grocer bought a carload of flour. He sold $12\frac{1}{2}\%$ of it to A, 30% to B, $37\frac{1}{2}\%$ to C, and the remainder, which was 48 barrels, to D.

a. What per cent of the flour did he sell to D?

b. How many barrels of flour did the carload contain?

20. A and B hired a horse. B paid $66\frac{2}{3}\%$ of the expense and A the remainder, which was \$1.50. What was the entire expense?

21. Three coal dealers furnished the coal for the schools of a city. The first furnished 50% of it, the second 1400 tons, and the third $16\frac{2}{3}\%$ of it.

a. What per cent of the coal did the second dealer furnish?

b. How many tons did all furnish?

c. How many tons did the first furnish?

22. A man bought a lot and built a house on it. The lot cost \$800, which was 50% of the cost of the house. How much did both cost?

23. $\frac{5}{8}$ of a farm is cultivated. What per cent of it is uncultivated?

24. A pole stands in a pond of water so that $\frac{1}{4}$ of the length of the pole is in the mud, and $\frac{3}{8}$ in the water. What per cent of the length of the pole is above the water?

25. 25% of 60 added to $\frac{3}{4}$ of 60 equals what?

26. Edward solved 16 problems and $93\frac{3}{4}\%$ of them were correct. How many were incorrect?

27. 50 % of 80 % of anything is what per cent of it?

28. 80 % of 50 % of anything is what per cent of it?

29. 100 % of anything added to 25 % of it equals what per cent of it?

30. A man had a sum of money and gained a sum equal to 40 % of what he had at first. He then had what per cent of the first sum?

31. 20 % more than \$5 is what per cent of \$5?

32. 20 % more than \$5 is how many dollars?

33. 50 % more than \$8 is how many dollars?

34. \$250 is 125 % of what sum?

35. \$250 is 25 % more than what sum?

36. A colt worth \$100 increased 25 % in value in eight months. How much was it then worth?

37. A liveryman bought a horse for \$200. Its value decreased 50 % in one year. What was it then worth?

38. What number, increased by 50 % of itself, equals 300?

262. *Written*

1. *Find:*

a. $12\frac{1}{2}$ % of 896 bu.

b. $66\frac{2}{3}$ % of 927 T.

c. $87\frac{1}{2}$ % of 240 gal.

d. $16\frac{2}{3}$ % of 636 qt.

e. $\frac{1}{2}$ % of \$15,000

f. 5 % of 15,000

g. 500 % of 15,000

h. 130 % of 480

2. 39.40 is $16\frac{2}{3}$ % of what?

3. 3.27 is what per cent of 8.72?

4. 8.72 is what per cent of 3.27?

5. $312\frac{1}{2}$ is $62\frac{1}{2}$ % of a certain number. What is $87\frac{1}{2}$ % of the same number?

6. Express decimally $\frac{1}{2}$, $\frac{1}{2}\%$, $\frac{3}{5}$, $\frac{3}{5}\%$, $\frac{1}{8}$, $\frac{1}{8}\%$, $\frac{3}{8}$, $\frac{3}{8}\%$, $\frac{3}{4}$, $\frac{3}{4}\%$, $\frac{7}{8}\%$, $\frac{1}{10}\%$, $\frac{5}{8}\%$.

7. A floor is 16 ft. square. What per cent of it may be covered by a rug that is 3 yd. long and 2 yd. wide?

8. A man paid \$175 for a horse, \$125 for a wagon, and \$25 for a harness. What per cent of the entire cost was the cost of each?

9. From a field containing 40 A., a rectangular piece 40 rd. long and 20 rd. wide was sold. What per cent of the field remained?

10. A man withdrew 35% of his deposits from the bank, leaving \$3250 in the bank. How many dollars did he withdraw?

11. Robert attended school 86 days during a term and was marked $95\frac{5}{9}\%$ in attendance. How many days were there in the term?

12. A piece of cloth shrank 4% in sponging, after which it contained 48 yd. How many yards did the piece contain before it was sponged?

13. A telephone was placed in my house on the 20th day of October, 1907. Beginning with that day, I had the use of the telephone what per cent of the year 1907?

14. A telephone company increased its charge from \$30 a year to \$36 a year. What was the rate per cent of increase?

15. A depositor withdrew 40% of his balance at the bank, and bought a piece of furniture for \$72, which was 12% of the sum withdrawn.

a. What was his balance before withdrawing?

b. What per cent of it did he pay for furniture?

c. How many dollars of his withdrawal remained unused?

16. My city tax bill amounts to $82.60 this year. If I do not pay it before Nov. 15, two per cent will be added to the amount of the bill. What must I pay if I wait till Nov. 16?

17. I obtained a 5% reduction by paying my semi-annual water bill before the 15th of July. The bill, after being reduced, was $2.375. What was it at first? How much did I have to pay?

18. The population of a city has increased 12% in the last three years. It is now 112,000. What was it three years ago?

19. The cost of living was estimated to be 40% greater in 1907 than in 1897.

a. Assuming this estimate to be correct, how much money was necessary to support in 1907 such a family as was supported for $1250 in 1897?

b. A workingman's wages of $15 a week in 1897 were equivalent to what sum per week in 1907?

c. A salary of $1820 in 1907 was equal to what in 1897?

20. The daily sales of a department store last year averaged $8262, which was 2% greater than the daily average for the year before. What was the daily average for the year before?

PROFIT AND LOSS

263. When property is sold for more or less than it cost, *the gain or loss is always computed as a certain per cent of the cost.*

Each of the following expressions, when used in a problem, means that the profit or gain is 10% of the cost:

At a profit of 10%; at 10% gain; at 10% above cost; at an advance of 10%.

264. *Oral*

1. A book that cost $5 was sold at a gain of 25%. What was the gain?

Statement of Relation: 25% of $5 = gain.

Which term of relation (factor or product) is to be found?

2. A grocer paid 80 cents a bushel for potatoes and sold them at a profit of 20 cents a bushel. What per cent did he gain?

Statement of Relation: ——% of \$.80 = \$.20.

Which term of relation is to be found?

3. A furniture dealer sold a desk at a gain of 25%. He gained \$5. What did the desk cost?

Statement of Relation: 25% of cost = \$5.

Which term of relation is to be found?

4. The whole of anything is what per cent of it?

If the cost of an article is 100% of the cost, and the gain is 10% of the cost, the selling price, which is the sum of the cost and the gain, is what per cent of the cost?

5. An article that cost \$8 was sold at a gain of 10%. Find the selling price.

Statement of Relation: 110% of \$8 = selling price.

Which term of relation is to be found?

6. A fruit dealer lost 40% on a shipment of peaches that cost him \$200. How much did he lose? What did he receive?

7. A produce dealer sold potatoes at \$2.20 a barrel, thereby gaining 10%. What did they cost per barrel?

Statement of Relation: 110% of cost = \$2.20.

Which term of relation is to be found?

8. A man sold his farm at \$32 per acre, thereby losing 20%. What price per acre did he pay for the farm?

100% of the cost less 20% of the cost = what per cent of the cost?
Statement of Relation: 80% of the cost = \$32.

Which term of relation is to be found?

9. An article that cost \$200 was sold at a gain of 50%.

a. What was the selling price?

b. What was the gain?

10. On an article that sold for \$180 the dealer lost 10%.

a. What was the cost?

b. How much was lost?

11. On an article that sold for \$2.40 the dealer gained 20%.

a. What was the cost?

b. What was the gain?

12. Cloth that cost \$2 a yard was sold for \$3 a yard.

a. What was gained on a yard?

b. What per cent was gained?

13. A dealer bought hops at 40¢ a pound and sold them at 30¢ a pound.

a. What was the loss on a pound?

b. What per cent was lost?

14. What per cent was gained on a city lot bought for \$400 and sold for \$500?

15. What per cent was lost on a city lot bought for \$500 and sold for \$400?

16. At what price must goods costing \$7.20 be sold to yield a profit of $16\frac{2}{3}\%$?

17. \$1 profit on a pair of shoes costing \$4 is what per cent profit?

18. A profit of \$1 on a pair of shoes sold for \$4.00 is what per cent profit?

265. *Written*

1. *a.* What is the profit on 1 ton of pork bought at \$7.50 per hundredweight and sold at \$.10 per pound?

b. What is the rate of profit?

2. A contractor gained $12\frac{1}{2}\%$ on a job of grading that cost him \$2448. How many dollars did he gain?

3. A carriage dealer gained 18% by selling a carriage for \$36 more than he paid for it. Find its cost.

4. *a.* What must a grocer receive per barrel for flour, in order that he may make a profit of $22\frac{2}{9}\%$ on flour that costs \$4.50 per barrel?

b. What is his gain on 75 barrels?

5. A stock of paper costing \$2345 was damaged by water so that it had to be sold at a loss of 15%. What was the selling price?

6. *a.* A grocer selling sugar at \$5.50 per hundredweight makes a profit of 10%. How much per ton does the sugar cost him?

b. How much does he gain on 7 T. of sugar?

c. How many pounds must he sell in order to gain \$25?

7. A hardware merchant bought 75 hundred-pound kegs of nails for \$206.25.

a. When he sells them at $3\frac{1}{2}¢$ a pound, what per cent profit does he make?

b. When he sells them at \$2.90 per keg, what per cent profit does he make?

c. At what price per keg must he sell them to make a profit of 16%?

8. The proprietor of a market received a shipment of 600 lb. of hams, costing \$15 per hundredweight. He allowed for a shrinkage of 10 lb. while they were being sold, and marked them so as to gain $31\frac{1}{9}\%$. At what price per pound did he mark them?

9. Mr. Jennings sold his automobile for $2142, thereby losing 16%. What did it cost? Make and solve another problem based on the facts given in this problem.

10. What per cent is gained on carpets bought at 90 cents a yard and sold at $1.25 a yard? Make and solve another problem based on the facts given in this problem.

11. A grocer makes a profit of 10% by selling sugar at 50 cents per hundredweight above cost. At what price per pound does he sell it? Make and solve another problem based on the facts given in this problem.

12. Hats that cost $27 a dozen were sold for $3.50 apiece. What was the rate per cent of profit? Make and solve another problem based on the facts given in this problem.

13. A man bought a city lot for $2400 and sold it so as to gain 20%. How much did he receive for the lot?

14. A man sold a house and lot for $2400, thereby gaining 20%. How much did the house and lot cost?

15. By selling a horse for $189 the owner lost 10%. At what price must he have sold the horse to gain 10%?

16. A horse dealer bought a span of horses for $240 apiece. He sold them so as to gain 20% on one and lose 20% on the other. What was his gain or loss by the transaction?

17. A jeweler sold two watches for $60 apiece. He gained 20% on one and lost 20% on the other.

a. How much did he gain or lose by the transaction?

b. What per cent did he gain or lose by the transaction?

18. A merchant sells goods at an average profit of 30%. 60% of his goods are sold for cash and the remainder are sold on credit. He loses 5% of his credit sales in bad debts.

a. How much cash does he receive for a stock of goods that cost $36,000?

b. How many dollars does he charge on his books from the sale of this stock?

c. How much does he lose in bad debts?

d. What is his net gain?

e. What per cent does he gain, making allowance for bad debts?

19. A huckster buys sweet corn at $1.25 per hundred ears and sells it at 20¢ a dozen.

a. What per cent profit does he make?

b. At what price per dozen must he sell it in order to make a profit of 40 %?

c. How many ears must he sell at an advance of 10 % in order to gain $3.00?

20. A farmer bought a piece of land, and, after keeping it a number of years, desired to sell it. He asked 40 % more than he paid for it, and then sold it for $3780, which was 90 % of his asking price.

a. What was his asking price?

b. What did the farmer pay for the land?

c. How much did he gain?

d. What per cent did he gain?

e. For how much should he have sold the land to gain $16\frac{2}{3}$ %?

21. A manufacturer sold his goods at 60 % above the actual cost of manufacture, and was able to collect only 96 % of his sales. He collected $18,000 from one month's sales.

Make and solve four problems, using these facts.

22. Make and solve a problem that requires the gain to be found.

23. Make and solve a problem that requires the rate per cent of loss to be found.

24. Make and solve a problem that requires the cost to be found.

25. Make and solve a problem that requires the rate per cent of gain to be found.

26. Make and solve a problem that requires the selling price to be found.

COMMISSION

266. *One who transacts business for another* is an **agent.**

Agents are known by various names according to the kind of business transacted by them. Those who buy and sell merchandise on commission are called commission merchants or commission brokers; those who buy and sell stocks and bonds are called stock brokers; those who collect money are called collectors. Can you mention other kinds of agents?

267. *The percentage allowed an agent as compensation for transacting business* is called **commission.**

268. *The commission of a broker* is called **brokerage.**

269. *Commission for buying goods* is computed as a certain per cent of the *cost of the goods; commission for selling goods* is computed as a certain per cent of the *selling price of the goods;* commission generally is computed as a certain per cent of the money handled, or the value of the property with which the agent deals. The principal exception to this rule is brokerage for buying and selling stocks and bonds, which will be treated later.

270. *A quantity of goods delivered to a commission merchant to be sold* is called a **consignment.**

271. *The party sending a consignment of goods to be sold by a commission merchant* is the **consignor.**

272. *The party to whom a consignment of goods is delivered for sale* is the **consignee.**

273. *The sum received from the sale of goods, after all expenses, such as commission, freight, and cartage, have been deducted,* is called the **net proceeds** *of the sale.*

274. *The party who employs an agent* is called the **principal.**

275. *Oral*

1. A college student sold 200 books at \$3 apiece during a summer vacation. What was his commission, at 40 %?

2. A real estate agent received \$80 for selling a house. His commission was 2 %. What was the selling price of the house?

Statement of Relation: 2 % of —— = \$80.

Which term of relation is to be found?

3. A lawyer received \$30 for collecting \$200. What was the rate of his commission?

Statement of Relation: —— % of \$200 = \$30.

Which term of relation is to be found?

4. A commission merchant sold 1000 pounds of butter at 25 cents a pound, retained his commission of 10 %, and sent the remainder to his principal.

a. What did his commission amount to?

b. How much did the principal receive?

5. An auctioneer sold, on 10 % commission, household goods to the amount of \$700. What were the net proceeds of the sale?

6. When an agent sells goods on 20 % commission, what per cent of the selling price of the goods does the principal receive?

7. A manufacturing company sold its entire product through a commission merchant who received 10 %. What was the selling price of a consignment for which the company received $900?

8. The net proceeds of a sale were $85. The commission was $15. What was the rate of commission?

9. What rate of commission is received when a sale amounting to $100 yields $80 net proceeds?

10. A commission merchant receives 2 cents a dozen as his compensation for selling eggs.

a. That is equivalent to what per cent commission when eggs sell at 20 cents a dozen?

b. When they sell at 16 cents a dozen?

c. When they sell at 24 cents a dozen?

11. A collector for a daily newspaper received 5 % commission. How much must he collect daily in order to earn $4 a day?

12. A collector working on 10 % commission must collect how many dollars in order that his principal may receive $180?

13. An agent collected a sum of money, took out his commission of 20 %, and paid the remainder, which was $40, to his employer. What was his commission?

276. *Written*

1. What is an agent's commission at $4\frac{1}{2}$ % for selling 850 barrels of flour at $5.25 a barrel?

2. A commission merchant sold a consignment of goods for $2470, took out his commission of 8 %, paid $28 freight and $5 storage, and sent the remainder to the consignor. How much did the consignor receive?

3. An agent receives 6 % commission for buying wool at 21 cents a pound.

a. What is his commission for buying 50 tons of wool?

b. How many pounds must he buy in order to earn $1690.50 in commissions?

4. An agent's commission for selling 479 books at $3.50 apiece was $670.60. What was the rate of his commission?

5. A lawyer procured a loan for an improvement company, charging $1\frac{1}{2}$ % commission. His commission was $4500. What was the amount of the loan?

6. A dealer in typewriters in a Western city sold typewriters manufactured in New York State. His commission was 35 %, out of which he paid freight charges at the rate of $4.50 per hundredweight.

a. If the weight of the typewriters averaged 50 pounds apiece when packed for shipment, and they were sold at an average price of $103 each, how much did the dealer clear on a shipment of 100 typewriters?

b. This dealer employed an agent, paying him $10 a week, and 20 % commission. The agent sold two typewriters in one week. What did he receive for his week's work?

c. How much did the dealer gain from this agent's work?

7. An agent took grocery orders on a commission of $12\frac{1}{2}$ %. He sold goods amounting to $1352, took out his commission, paid freight charges amounting to $30.75, and sent the remainder of his collections to his principal.

a. What were the net proceeds of the sale?

b. How many dollars' worth of goods must the agent sell to earn $568 in commissions?

8. *a.* An agent who receives \$15 per week and $5\frac{1}{2}\%$ commission, must sell how many dollars' worth of goods in a year to obtain an income of \$1566.50?

b. If he receives no compensation but his commission, what must be the amount of sales to yield him the same income as in Question *a*?

9. An agent who had charge of a business block received as his commission 2% of the first year's rent and 1% of all rents for succeeding years.

a. What was the amount of his commission on five-year leases of two stores, one at \$250 per month and the other at \$300 per month?

b. What was his commission on a three-year lease of an office 16 ft. by 20 ft., the annual rent being at the rate of \$.80 per square foot of floor?

c. His commission on a ten-year lease of a suite of banking rooms was \$253. What was the annual rent?

10. A real estate agent sold my property in Boston, took out his commission of 2%, and remitted to me the remainder, which was \$5880. What was the amount of his commission?

11. A commission merchant received a consignment of goods on which he paid \$82.50 freight charges, \$15.60 for cartage, and \$6 for storage. He sold the goods, deducted his commission of 8%, and his disbursements for freight, cartage, and storage, and then had \$7255.90 net proceeds of the sale, which he remitted to his principal. For how much did he sell the goods?

12. A commission merchant sold a consignment of goods, paid freight charges and drayage to the amount of \$39.85, retained his commission of 8%, and sent the remainder, which was \$1685.15, to his principal.

a. What was the amount of the sales?

b. What was the agent's commission?

13. A real estate agent sold a tract of land, and bought a business block with the money received for the land. His commission at 2% for selling and $\frac{1}{2}$ of 1% for buying amounted in all to $1325. For how much did he sell the tract of land?

14. A manufacturer in Pittsburg sells his products through a commission house in Philadelphia, paying 8% commission. What is the selling price of goods for which the manufacturer receives $6440 net proceeds?

15. A collector receives $8\frac{1}{3}$% commission on all the money he collects. How much does his principal receive out of collections for which the collector receives $317.65 in commissions?

16. A farmer sells his produce through a commission merchant in the city. If the merchant's commissions average $9\frac{1}{2}$%, how many dollars' worth of produce must the farmer sell in order to receive $1810 net proceeds?

COMMERCIAL DISCOUNT

277. It is customary for manufacturers, wholesale merchants, and others transacting a large amount of business to distribute among their customers printed lists of the articles which they offer for sale, with the price of each article. These lists are called price lists. The goods are often sold at a lower price than that given in the price list. A reduction in price is made sometimes because the customer buys a large quantity of goods; sometimes because other dealers are selling the same kind of goods at a lower price; sometimes because the dealer desires to close out his entire stock to make room for other goods; sometimes as an inducement to the customer to pay cash instead of paying at a certain time after the purchase of the goods. Can you mention other reasons for a reduction in price?

Two or more reductions are often made in the price of the same bill of goods, as, for instance, one reduction because the market price of that kind of goods has fallen, another on account of the quantity sold, and still another for cash payment.

When no price list is published, goods are often marked at a certain price, but sold at a reduction from that price.

278. *The marked price, or the price given in a price list,* is called the **list price.**

279. *A reduction from the list or marked price of goods* is a **commercial discount** or **trade discount.**

A discount for cash payment is sometimes called a *cash discount.* A discount because of the quantity of goods sold is sometimes called a *quantity discount.*

280. *The sum received for an article, after all discounts have been made,* is the **net price.**

281. *When two or more discounts are made from the price of an article, they* are called **successive discounts.** The first discount is a certain per cent of the list price, the second a certain per cent of the remainder, the third a certain per cent of the second remainder, and so on.

282. *Oral*

1. The whole of anything is what per cent of it?

2. When an article is sold at a discount of 10 % from the list price, it is sold for what per cent of the list price? When sold at a discount of 20 %?

3. I bought a copy of Longfellow's poems listed at $1.50, the bookseller allowing me 20 % discount. How much did I pay?

4. I can buy a bicycle for \$40 and pay for it in 30 days, or obtain a discount of 2 % by paying cash. How much will I save by paying cash? What is the cash price?

5. A man bought a bill of goods at 10 % discount. He paid \$180 for them.

a. What per cent of the list price did he pay?

b. What was the list price?

6. By paying cash for a bill of goods I obtained a discount of 2 %, thereby saving \$2. What was the amount of the bill?

7. A merchant bought from a jobber goods listed at \$2000, receiving a discount of 40 %. What was the entire discount? What did he pay for the goods?

8. A merchant bought a bill of goods at a discount of $33\frac{1}{3}$ %. What was the discount on goods listed at \$90? What was the net price?

9. What is the net price of goods listed at \$200 and bought at a discount of 30 %? What is the discount?

10. The net price of a bill of goods is \$12. The rate of discount is 40 %. What is the list price? What is the discount?

11. The net price of a bill of goods is \$30. The rate of discount is 40 %. What is the discount?

12. A fruit dealer sold me ten barrels of apples at \$2.50 a barrel. They arrived in poor condition and he discounted the bill 20 %. How much did I pay?

13. A discount of $\frac{1}{3}$ is equivalent to what per cent discount? $\frac{1}{6}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{8}$?

283. There are two ways of treating successive discounts. For example, let it be required to find the net price of a bill of

goods listed at $400, on which successive discounts of 15%, 10%, and 5% are allowed.

15% of $400 = $60. *First discount.*
$400 − $60 = $340. *First remainder.*
10% of $340 = $34. *Second discount.*
$340 − $34 = $306. *Second remainder.*
5% of $306 = $15.30. *Third discount.*
$306 − 15.30 = $290.70. Net price.

Or

The net price is 95% of 90% of 85% of $400. Find the net price in this way and compare results. The latter method is the more direct and in most cases the shorter.

Written

1. Goods listed at $3241 are sold at a discount of 30%. What is the net price?

Statement of Relation: 70% of $3241 = net price.

2. A man bought goods at 15% discount. What was the list price of goods that cost him $59.50?

Statement of Relation: 85% of the list price = $59.50.

Which term of relation is to be found?

3. A merchant saved $4.50 by paying cash, thus obtaining a discount of $1\frac{1}{2}$% on a bill of goods. What was the amount of the bill?

Statement of Relation: $1\frac{1}{2}$% of the amount = $4.50.

Which term of the relation is to be found?

4. Find the net prices of the following bills of goods:

	List Price	Discounts		List Price	Discounts
a.	$240	2%, 10%, 8%	*d.*	$312.50	10%, 10%, 10%
b.	$300	10%, 5%, 2%	*e.*	$214	2%, 10%, 20%
c.	$870	30%, 5%, 2%	*f.*	$300	15%, 10%, 5%

5. A bookseller bought books at an average discount of 38 % from the list price and sold them to a library association at an average discount of $\frac{1}{3}$ from the list price. How much did he gain on a bill of books listed at $735?

6. A druggist sold headache powders at 23 cents a box. They were listed at 25 cents a box.

a. What per cent discount did he allow?

b. If he bought them at 40 % discount, what did he pay for seven dozen boxes?

c. What per cent profit did he make?

7. A merchant bought carpet at 60 cents a yard. He marked it so that he might give a discount of 10 % and still make 20 %.

a. At what price did he sell the carpet?

b. At what price did he mark it?

8. At what price must goods costing $285 be marked so that the dealer may give a discount of 5 % and still make a profit of 18 %?

9. A merchant sold his stock of goods at a discount of 10 % from the marked price and still made a profit of 14 %.

a. If he received $4560, what was the marked price?

b. What was the cost?

10. A bill of goods was marked at 45 % above cost, and sold at a discount of $8\frac{1}{3}$ % from the marked price. The marked price was $725.

a. Find the cost.

b. Find the selling price.

11. Steel screws are listed at $8 a great gross, and successive discounts of 30 %, 40 %, 15 %, and 8 % are allowed. What must be paid for 40 great gross?

12. Find the net price of goods listed at $720, and discounted at 5 %, 10 %, and 20 %.

13. A speculator bought a quantity of peaches for \$280, and marked them 40 % above cost. They began to spoil and he was obliged to sell them at a discount of 40 % from the marked price. Did he gain or lose, and how much?

14. A man sold two vacant lots for \$960 apiece. By so doing he sold one at a discount of 4 % from his asking price and the other at a discount of 20 % from his asking price. Both were marked 20 % above cost.

a. What did each cost?

b. What was his entire gain?

15. Two merchants have the same kind of goods marked at the same price. One offers discounts of 25 %, 20 %, and 5 %. The other offers discounts of 5 %, 20 %, and 25 %. Which is the better offer?

16. Two merchants have goods exactly alike, listed at \$200. One offers discounts of 20 %, 10 %, and 10 %. The other offers a single discount of 37 %. Which is the better offer, and how much better?

17. A carload of corn containing 700 bushels was bought on 60 days' time at 48 cents a bushel. The purchaser obtained a discount of $2\frac{1}{2}$ % by paying cash. What did the corn cost him?

18. *What single discount is equal to successive discounts of:*

a. 10 and 5 per cent?		*j.* 25 and 10 per cent?	
b. $12\frac{1}{2}$ and 5 per cent?		*k.* 25, 10, and 5 per cent?	
c. 15 and 5 per cent?		*l.* 30 and 5 per cent?	
d. 15 and 10 per cent?		*m.* 30 and 10 per cent?	
e. $16\frac{2}{3}$ and 10 per cent?		*n.* 30, 10, and 5 per cent?	
f. 20 and 5 per cent?		*o.* $33\frac{1}{3}$ and 5 per cent?	
g. 20 and 10 per cent?		*p.* $33\frac{1}{3}$ and 10 per cent?	
h. 20, 10, and 5 per cent?		*q.* $33\frac{1}{3}$, 10, and 5 per cent?	
i. 25 and 5 per cent?		*r.* 40 and 5 per cent?	

s. 40 and 10 per cent? *w.* 45 and 10 per cent?
t. 40, 10, and 5 per cent? *x.* 50 and 5 per cent?
u. 40 and 20 per cent? *y.* 50 and 10 per cent?
v. 40, 20, and 5 per cent? *z.* 50, 10, and 5 per cent?

19. \$144 was sufficient to pay a bill on which discounts of 20% and 10% were given. What was the amount of the bill before the discounts were made?

Statement of Relation: 90% of 80% of the amount = \$144.

When the product of three factors and two of the factors are given, how may the remaining factor be found?

20. What is the list price of a bill on which discounts of 10%, 10%, and 5% make the net price \$153.90?

21. Two successive discounts reduced to \$108 the price of an article listed at \$160. One of the discounts was 25%. What was the other?

Statement of Relation: ——% of 75% of \$160 = \$108.

When the product of three factors and two of the factors are given, how may the remaining factor be found? That factor subtracted from 100% is the required discount.

22. What discount, in addition to one of 20%, will reduce a price from \$50 to \$39.20?

23. What list price will give a net price of \$113.40 when discounts of 30%, 10%, and 10% are made?

24. A merchant bought goods at a discount of 35% from the list price and sold them at a discount of 25% from the list price.

HINT.—Goods listed at \$100 cost him \$65 and he sold them for \$75.

a. What was his profit on goods listed at \$350?

b. What was his rate per cent of profit?

c. What was his profit on goods which cost him \$195?

d. What was the list price of goods that cost the merchant \$1300?

25. A man bought goods at successive discounts of 25%, 10%, and 10%, and sold them at successive discounts of 10% and 5% from the list price.

a. What was his gain on goods listed at $80?

b. His gain was what per cent of the cost?

CONTRACTS

284. *A* **contract** *is an agreement between two or more parties for doing or not doing a particular thing.*

In making a contract it is necessary that all the parties agree to the same thing. For instance, in bargains for the purchase of property, if the seller has in mind one piece of property, while the buyer thinks he is buying a different piece of property, there is no contract.

It is generally held, also, that there must be a consideration. That is, when one party makes a contract with another, he must pay, or agree to pay, a sum of money, or render some service, or give something of value, in return for what he receives from the other.

There are many kinds of contracts. Among the commonest ones are the following:

Contracts for the purchase of property.

Contracts for the rental of property.

Contracts for the payment of money — such as notes, bonds, and mortgages.

Contracts of insurance.

Contracts of employment — as when one person agrees to work for another for a certain time at a specified salary.

INSURANCE

285. **Insurance** *is a contract whereby one party (usually an insurance company) agrees to pay to another party a specified sum*

of money in case a certain event shall happen, such as the death of some person, injury to the person by accident, destruction of property by fire or water, or loss of property by theft or accident.

The different forms of insurance are known as **life** insurance, **accident** insurance, **fire** insurance, **marine** insurance, etc., according to the kind of risk that is assumed by the insurer.

286. *The written or printed document that contains the terms of an insurance contract* is called an **insurance policy.**

287. *The sum which the insurer agrees to pay* is called the **face of the policy.**

288. *The sum paid by the insured to the insurer* is called the **premium.**

Life insurance policies are in force for a term of years or during the life of the insured ; but the premium is usually paid in annual, semi-annual, or quarterly installments. *Installments after the first* are called **renewals.**

Most other kinds of insurance policies are for a shorter time, and the premium is paid in one sum when the policy is issued.

Accident policies are usually made out for one year, though some special kinds, like railroad accident policies, are sold for shorter periods.

Fire insurance policies are usually for three years.

The premium on a fire insurance policy is computed at a certain sum for each $100 of insurance, or a certain per cent of the face of the policy, this single rate covering the entire time for which the policy is given.

The premiums on life insurance policies are generally computed at a certain sum for each $1000 of the face of the policy, the sum varying according to the age of the insured when the policy was issued, and according to the conditions of the contract.

289. The following forms illustrate some kinds of insurance policies. Only the essential parts of each contract are given.

FIRE INSURANCE POLICY

No. 258683 $2000

THE

MECHANICS INSURANCE COMPANY

Incorporated A.D. 1834. OF BOSTON

In Consideration of the Stipulations herein named and of ---------- *Twenty Four and 80/100* ---------- Dollars' Premium

Does Insure *Jacob P. Goettel* for the term of *one year*

from the *11th* day of *October* 19*04*, at noon,

to the *11th* day of *October* 19*05*, at noon,

against all direct loss or damage by fire, except as hereinafter provided,

To an amount not exceeding *Two Thousand* Dollars,

to the following described property, while located and contained as described herein, and not elsewhere, to wit:

Jacob P. Goettel

$2000 **On the three- and four-story brick building, including elevators and all attachments, gas and water pipes, and fixtures, heating apparatus and fixtures, and plate glass in doors and windows, occupied for storage purposes, situate on the east side of and known as No. 240 North Salina Street, Syracuse, N. Y. Mechanic's permit attached.**

Permission given for the use of gas, kerosene oil, or electric lights on said building.

Other insurance permitted without notice until required. Lightning clause attached.

* * * * * * * * * * * *

In Witness Whereof, this Company has executed and attested these presents this *6th* day of *October* 19*04*. This Policy shall not be valid until countersigned by the duly authorized Agent of the Company at *Syracuse, N.Y.*

Attest: *Jno. A. Snyder,* Secretary. *Samuel Martin,* President

Countersigned by *Phillips & White* Agent.

ORDINARY LIFE INSURANCE POLICY

THE NORTH STAR MUTUAL LIFE INSURANCE COMPANY

In Consideration of the application for this Policy, a copy of which is attached hereto and made a part hereof, and in further consideration of the payment of

Two Hundred and Seventy Nine $\frac{80}{100}$ Dollars,

the receipt whereof is hereby acknowledged, and of the Annual payment of a like sum to the said Company, on or before the First day of January in every year during the continuance of this Policy, promises to pay at its office in Milwaukee, Wisconsin, unto Mary Doe, Beneficiar y, Wife of John Doe the Insured, of Des Moines in the State of Iowa subject to the right of the Insured, hereby reserved, to change the Beneficiary or Beneficiaries the sum of Ten Thousand Dollars,

upon receipt and approval of proof of the death of said Insured while this Policy is in full force, the balance of the year's premium, if any, and any other indebtedness on account of this Policy being first deducted therefrom; provided, however, that if no Beneficiary shall survive the said Insured, then such payment shall be made to the executors, administrators or assigns of the said Insured.

In Witness Whereof, THE NORTH STAR MUTUAL LIFE INSURANCE COMPANY, at its office in Milwaukee, Wisconsin, has by its President and Secretary, executed this contract, this *First* day of *January* one thousand nine hundred and *eight.*

S. A. Hawkins, Secretary. *L. H. Perkins*, President.

ACCIDENT INSURANCE POLICY

United Casualty Company

In Consideration of ___*Twenty-five*___ $\frac{}{100}$ Dollars' premium and the warranties and agreements in the application for this policy, which application is hereby made a part hereof, the United Casualty Company, herein called the Company, insures, subject to the provisions, conditions, definitions and limits herein, ___*John Doe*___ of ___*Harrisburg, Pa.*___ by occupation ___*Printer*___ herein called the Insured, for ___*twelve*___ months, beginning at noon, standard time, on the ___*first*___ day of ___*January*___, 190*8*, against loss as herein provided caused by bodily injury effected exclusively and directly by external, violent and accidental means which, independently of any and all other causes, immediately, wholly and continuously disables him, to wit:

I A LOSS OF LIFE, ___*Ten Thousand*___ Dollars ($ *10,000*);
B LOSS OF BOTH EYES, the amount stipulated for loss of life;
C LOSS OF BOTH HANDS, the amount stipulated for loss of life;
D LOSS OF BOTH FEET, the amount stipulated for loss of life;
E LOSS OF ONE HAND and ONE FOOT, the amount stipulated for loss of life;
F LOSS OF ONE ARM, three-fifths the amount stipulated for loss of life;
G LOSS OF ONE LEG, three-fifths the amount stipulated for loss of life;
H LOSS OF ONE HAND, one-half the amount stipulated for loss of life;
I LOSS OF ONE FOOT, one-half the amount stipulated for loss of life;
J LOSS OF ONE EYE, one-quarter the amount stipulated for loss of life;
K TOTAL LOSS OF TIME, ___*Twenty-five*___ Dollars per week, not to exceed 104 consecutive weeks;
L PARTIAL LOSS OF TIME, one-half the amount stipulated for total loss of time per week, not to exceed 30 consecutive weeks.

* * * * * * * * * * *

In witness whereof the United Casualty Company has caused this policy to be signed by its President and Secretary, but it shall not be in force until countersigned by a duly authorized representative of the Company.

Richard Johnson, President. *Dan J. Seward*, Secretary.

Countersigned
Herbert W. Greenland, Agt.

290. *Written*

1. *a.* A wooden dwelling house in a city was insured for three years for $3500, the rate of premium being $.65 on $100 of insurance for three years. Find the premium.

b. How much did the owner pay in premiums in twelve years, at this rate?

c. The rate of premium for brick dwellings in the same city is 55¢ on $100, for three-year policies. Find the premium for $4200 of insurance on a brick dwelling in that city.

d. The insurance agent who wrote the policy in question *c* received as his commission 25% of the premium. Find the agent's commission.

2. A schoolhouse in a Western city is insured for three years for $28,000, at $\frac{3}{4}$%. The agent's commission is 20% of the premium. Find the agent's commission.

3. A farmer in Pennsylvania has his house insured for $900 and his barns for $1350, the premium for the former being $\frac{3}{4}$% and for the latter $1\frac{1}{5}$%, for three-year policies. What is the farmer's annual expense for insurance?

4. *a.* The premium for insuring a mill, in a small village, for $2000, amounted to $75 a year. What was the annual rate of premium?

Statement of Relation: ——% of $2000 = $75.

Which term of relation is to be found?

b. What was received by the agent who wrote three annual policies on this mill, his commission being 15% of the premiums?

5. A merchant's stock of goods is insured for $\frac{4}{5}$ of its value, for three years, at $\frac{4}{5}$%. If the stock is worth $7500, what is the annual expense for insurance?

6. I pay $28.50 for three years' insurance, the rate of premium being $\frac{3}{5}$%. How much insurance have I?

Statement of Relation: $\frac{3}{5}$% of $—— = $28.50.

7. The premium for insuring my house, at 70¢ per $100, is $38.50. What is the face of the policy?

8. A machine shop is insured for three years at a cost of $114. If the rate is $1\frac{1}{2}\%$, what is the face of the policy?

9. An agent received $5.25 as his commission for insuring a house for $\frac{4}{5}$ of its value. The rate of premium was $\frac{3}{4}\%$ and the agent received 25% of the premium.

a. What was the premium?

b. What was the face of the policy?

c. What was the value of the house?

10. How many dollars of insurance must an agent secure in order that he may obtain $46.35, if his commission is 15% of the premiums and the premiums are $1\frac{1}{5}\%$ of the insurance?

11. A man had an accident insurance policy which cost him $25 a year. After he had paid three years' premiums, he was injured by an accident and received $20 a week for six weeks.

a. The man received how much more than he paid?

b. If the agent received 30% of the premiums, how much did the insurance company lose by insuring this man?

c. If the company insured ten other men for the same time at the same rate, and none of them made any claim for injuries, how much more did the company receive from the eleven men than it paid out on account of the one man's injuries?

12. A house worth $3600 was insured for $\frac{2}{3}$ of its value, and the contents, worth $2800, were insured for $\frac{1}{2}$ of their value. The rate of insurance was 65¢ on $100. The house and contents were entirely destroyed within a year.

a. What did the company lose by insuring the property?

b. What did the owner lose by the fire?

c. What did the owner gain by having the property insured?

13. A mill owner had his mill insured every year by one company, for $23,000, at $\frac{1}{2}\%$. After he had paid five annual

premiums, the mill was damaged by fire to the amount of $3150, which was paid in full by the insurance company.

a. How much did the owner gain by having the mill insured?

b. How much did the company lose by insuring the mill?

14. Property worth $48,600 is insured for $\frac{5}{6}$ of its value at a cost of $364.50. What is the rate?

15. *a.* Turn to the life insurance policy on page 160. If John Doe lives to be seventy years old, and pays 35 renewals, besides the first premium, how much money will he have paid to the insurance company?

b. How much would the first premium amount to in 35 years, if put on interest at the rate of 5% per year, simple interest?

c. If the agent through whom John Doe obtained this policy received as his commission 40% of the first premium and 5% of all renewals for the first ten years, how much did he receive in all?

16. The premium on Mr. Wilson's accident policy was at the rate of $5.00 per $1000 of the face of the policy. The agent's commission was 25% of the premium. If the agent received $6.25, what was the face of the policy?

17. A man took out a $5000 ten-year endowment life insurance policy, on which the semi-annual premium was $55.95 per $1000 of the face of the policy. If he lived ten years after taking the policy, and paid all the premiums when due, how much did he pay to the company?

18. A merchant has his stock of goods insured for $18,000 at $2\frac{1}{4}$% for three years, his building for $12,500 at 2% for three years, one boiler at $15 a year, an elevator at $35 a year, and plate glass at $12 per year, with 30% off, on the plate glass. How much does his insurance cost in three years?

19. A man starting on a journey bought a 12-day accident policy at $2.50. When that expired he bought a 10-day policy at $2.00, and after that a 7-day policy at 25 cents a day. How much would he have saved by investing at first in a 30-day policy that cost $4.50?

INTEREST

291. *Money paid for the use of money* is **interest.**

292. *Money for the use of which interest is paid* is the **principal.**

293. *The sum of the principal and interest* is the **amount.**

294. *The sum to be paid for the use of money is always determined by taking a certain per cent of the principal.*

295. *The number of hundredths of the principal taken as the interest for one year* is the **rate of interest.** For instance, if a sum of money is borrowed and 6% of that sum is the interest for one year, the rate of interest is 6%.

296. *The rate of interest which is fixed by law* is called the **legal rate.** In a majority of the states the legal rate is 6%. In some states it is greater than 6%, and in some states less.

A lower rate than the legal rate is always allowed by law if the debtor and creditor so agree. In some states a higher rate than the legal rate is allowed, if the debtor and creditor so agree; in others a higher rate than the legal rate is forbidden by law. What is the legal rate where you live?

297. *Interest at a higher rate than that permitted by law* is **usury.**

298. *Oral*

1. Mr. Smith borrowed $100 from Mr. Arnold for 1 yr. At the end of the year Mr. Smith repaid the money which he had

borrowed and also paid Mr. Arnold 6 % interest. How much was the interest? What was the principal? How much did Mr. Arnold receive in all? What is this sum called? Who was the debtor? Who was the creditor?

2. What is the interest on $500 at 5 % for 1 yr.? On $800? On $900? On $300? On $1000? On $250?

3. What is the interest on $500 for 1 yr. at 5 %? At 10 %? At 7 %? At 4 %? At 3 %? At 8 %?

4. What is the interest on $1000 at 5 % for 1 yr.? For 2 yr.? For 3 yr.? For 8 yr.? For 10 yr.?

5. What is the interest on $100 for 2 yr. at 6 %? At 4 %? At 3 %? At 9 %? At 8 %?

6. Six months are what part of a year? 3 mo.? 4 mo.? 8 mo.? 9 mo.? 10 mo.? 1 mo.?

7. What is the interest on $600 for 1 yr. at 6 %? For 6 mo.? For 3 mo.? For 4 mo.? For 8 mo.? For 9 mo.?

8. Make and solve many problems similar to the above, multiplying the interest for one year by the number of years, treating months as fractions of a year.

299. *Written*

1. *a.* What is the interest on $800 for 2 yr. 6 mo. at 5 %?

$$\frac{\$\cancel{800}^{4}}{1} \times \frac{5}{\cancel{100}} \times \frac{5}{\cancel{2}} = \$100 \quad \textit{Ans.}$$

How do we find the interest for one year? For $2\frac{1}{2}$ years?

b. What is the interest on $840 for 1 yr. 9 mo. at $4\frac{1}{2}$ %?

$$\frac{\$\cancel{840}^{\cancel{210}^{105}}}{1} \times \frac{9}{\cancel{200}_{100}} \times \frac{7}{\cancel{4}} = \$66.15 \quad \textit{Ans.}$$

How does $\frac{9}{200}$ compare in value with $\frac{4\frac{1}{2}}{100}$?

We multiply $4\frac{1}{2}$ and 100 by 2 to obtain $\frac{9}{200}$.

1 yr. 9 mo. = how many months?

2. *Find the interest on*

a. \$750 for 2 yr. at 6%.

b. \$375 for 1 yr. 6 mo. at 6%.

c. \$500 for 2 yr. at $3\frac{1}{2}$%.

d. \$625 for 6 mo. at 4%. (6 mo. = what part of a year?)

e. \$342.40 for 1 yr. 3 mo. at $4\frac{1}{2}$%.

f. \$279.75 for 1 yr. 2 mo. at 6%.

g. \$364.50 for 2 yr. 8 mo. at $4\frac{1}{2}$%.

h. \$640 for 1 yr. 9 mo. at $5\frac{1}{2}$%.

300. *Oral*

1. In computing interest, one year is assumed to be 360 days, and one month 30 days. On that assumption, 1 day is what fraction of a year? 2 days? 3 days? 7 days? 245 days? 430 days? 83 days? 792 days? 879 days? 90 days?

2. The interest on a sum of money for 1 day is what part of the interest for a year?

3. The interest on a sum of money for 9 days is what part of the interest for a year? The interest for 231 days is what part of the interest for a year?

4. 1 yr. and 15 da. are how many days, counting 360 days as a year?

5. The interest on a sum of money for 1 yr. 15 da. is how many 360ths of the interest for a year?

6. The interest on a sum of money for 3 mo. 20 da. is how many 360ths of the interest for one year?

7. The interest for 5 mo. 10 da. is how many 360ths of the interest for one year?

301. *Written*

1. Find the interest on \$240 for 1 yr. 3 mo. 18 da. at $3\frac{1}{2}\%$.

$$\begin{array}{r} 1 \text{ yr.} = 360 \text{ da.} \\ 3 \text{ mo.} = 90 \text{ da.} \\ \underline{18 \text{ da.}} \end{array} \qquad \frac{\overset{3}{\cancel{6}}\,\cancel{\$240}}{1} \times \frac{7}{\cancel{200}\,100} \times \frac{52}{\cancel{468}\,\cancel{360}\,\cancel{4}} = \$10.92 \quad \textit{Ans.}$$

1 yr. 3 mo. 18 da. $= \overline{468}$ da., or $\frac{468}{360}$ yr.

2. *Find the interest on*

a. \$700 for 30 da. at 6%.
b. \$450 for 45 da. at 5%.
c. \$1380 for 82 da. at $4\frac{1}{2}\%$.
d. \$3000 for 2 mo. 20 da. at 7%.
e. \$6540 for 1 yr. 15 da. at 5%.
f. \$2700 for 1 yr. 2 mo. 12 da. at 4%.
g. \$450 for 1 yr. 6 mo. 6 da. at $5\frac{1}{2}\%$.
h. \$280 for 2 yr. 2 mo. 17 da. at 3%.
i. \$519.16 for 173 da. at 5%.
j. \$249.83 for 1 yr. 5 mo. 14 da. at 6%.
k. \$931 for 1 yr. 11 mo. 19 da. at $2\frac{1}{2}\%$.
l. \$67,000 for 2 yr. 17 da. at 3%.
m. \$864.13 for 9 mo. 16 da. at $4\frac{1}{2}\%$.
n. \$4182 for 1 yr. 4 mo. 11 da. at 8%.
o. \$180.55 for 10 mo. 23 da. at $6\frac{1}{2}\%$.

302. *Oral*

1. *Find the interest on*

a. \$300 for 1 yr. at $3\frac{1}{2}\%$.
b. \$200 for 2 yr. at $5\frac{1}{2}\%$.
c. \$300 for 30 da. at 4%.
d. \$700 for 6 mo. at 3%.

2. From Jan. 1, 1908, to June 1, 1908, is what part of a year? Find the interest on \$2500 at 6% for that time.

3. Find the interest on $200 from July 1, 1908, to Jan. 1, 1909, at the legal rate where you live.

4. What is the interest on $400 from Nov. 1, 1907, to Nov. 1, 1909, at the legal rate in your state?

5. If I borrow $250 on the first of January and pay the debt on the first of the following January, with interest at the legal rate in your state, how much do I pay?

303. *Written*

1. What is the amount of $700 when put at interest at 5% from Nov. 21, 1907, to June 3, 1909?

1909 yr.	6 mo.	3 da.	
1907	11	21	
1 yr.	6 mo.	12 da.	*Diff. in Time*
7	46		$700.00
$\frac{\cancel{700}}{1} \times \frac{5}{\cancel{100}} \times \frac{\cancel{552}}{\cancel{360}} = \$53.67$ *Interest*			53.67
			$753.67 *Amount, Ans.*

2. *Find the amount of*

a. $250 from April 7, 1905, to Oct. 19, 1906, at 6%.
b. $5000 from Sept. 15, 1905, to May 21, 1907, at 6%.
c. $348 from July 25, 1902, to March 11, 1904, at 5%.
d. $1000 from Jan. 28, 1907, to Jan. 21, 1909, at $5\frac{1}{2}$%.
e. $875 from Sept. 30, 1908, to Feb. 24, 1909, at $4\frac{1}{2}$%.
f. $3980 from March 2, 1901, to July 2, 1903, at $4\frac{1}{2}$%.
g. $600 from Oct. 12, 1899, to April 12, 1901, at 7%.
h. $1350 from Aug. 25, 1907, to Dec. 5, 1908, at $5\frac{1}{2}$%.
i. $163.50 from Dec. 16, 1907, to Jan. 1, 1909, at 8%.

3. Mr. Anderson borrowed $700 May 15, 1907, and agreed to pay it June 3, 1908, with 6% interest. How much did he have to pay?

4. What is the amount of $600 when put at interest from June 24, 1904, to May 19, 1906, at the legal rate where you live?

5. Compute the interest on $428.70 from Oct. 18, 1908, to July 13, 1910, at the legal rate where you live.

INTEREST FOR SHORT PERIODS

304. When money is on interest for less than a year, it is customary to compute the time in days.

1. What is the interest on $1575.25 from Jan. 9, 1904, to March 15, 1904, at 3%?

The money is on interest for
22 da. left in Jan.
29 da. in Feb.
15 da. in March
66 da. *Term of Interest*

2. *Compute the interest on*

a. $600 from April 21 to Aug. 3, at 7%.
b. $845.60 from Sept. 1 to Dec. 24, at 6%.
c. $570 from April 25 to Aug. 13, at $5\frac{1}{2}$%.
d. $473.70 from June 1 to July 31, at 8%.
e. $1857 from Nov. 30 to Dec. 31, at 7%.

3. *Compute the interest on*

a. $900 from Dec. 18, 1903, to Feb. 21, 1904, at $6\frac{1}{2}$%.
b. $388.20 from Dec. 18, 1906, to Feb. 21, 1907, at 6%.
c. $1880 from Dec. 19, 1905, to March 1, 1906, at $3\frac{1}{2}$%.
d. $1230 from Dec. 19, 1907, to March 1, 1908, at 6%.
e. $870 from Nov. 1, 1908, to April 1, 1909, at 6%.

4. *Compute the amount of*

a. $496 from June 15 to Oct. 15, 1901, at 5%.
b. $4000 from Dec. 1, 1903, to Feb. 1, 1904, at 6%.
c. $460.80 from May 8 to July 7, 1908, at 5%.
d. $500 from Sept. 30, 1905, to Feb. 10, 1906, at 6%.

5. On the first day of May, 1907, Mr. Blank borrowed $1800 with which to buy an automobile, agreeing to pay the money with interest at 6% on the 10th day of September. He sold the automobile for $1350 on the 10th of September. How much money must he put with what he received, in order to pay his debt?

EXACT INTEREST

305. When a day is called $\frac{1}{360}$ of a year, in computing interest, the interest obtained is a trifle greater than it would be if each day were taken as $\frac{1}{365}$ of a year—its exact value. Interest computed by the usual method is therefore slightly inexact; yet business men seem to consider that its greater convenience compensates for its lack of accuracy.

306. Exact interest is *interest computed by taking as many 365ths of the interest on the given principal for one year as there are days in the interest period.*

The exact method of computing interest is employed by the United States government and, to a limited extent, elsewhere.

The process is the same as that given in the preceding pages, except that the last factor has 365, instead of 360, for its denominator.

307. *Written*

1. What is the exact interest on $731.46, at 8%, from Jan. 29 to July 22, 1908?

Int. Period
- Jan. 2 da.
- Feb. 29 da.
- Mar. 31 da.
- Apr. 30 da.
- May 31 da.
- June 30 da.
- July 22 da.

Total, 175 da.

$$\frac{\$731.46}{1} \times \frac{8}{100} \times \frac{175}{365} = \$28.056 \text{ or } \$28.06 \textit{ Ans.}$$

2. *Find the exact interest on*

a. \$5000 at 5% from Oct. 5, 1905, to April 3, 1906.

b. \$584 at 4% from Jan. 7 to May 5, 1908.

c. \$109.50 at 3% from May 5 to Sept. 6, 1905.

d. \$2190 at 7% from Nov. 15, 1908, to April 1, 1909.

e. \$75.50 at $3\frac{1}{2}$% for 90 da.

3. A man borrowed \$500 on the 5th of May. How much is due on the debt July first, computing exact interest at 5%?

4. What is the difference between the common and the exact interest at 5% on \$525,600 for 15 da.?

5. What is the amount of \$328.50, computing exact interest at 7%, from June 12 to Aug. 28?

6. Find the amount of \$1095 for 146 da., computing exact interest at $5\frac{1}{2}$%.

7. Find the exact interest on \$8760, at $4\frac{1}{2}$%, from Oct. 15, 1908, to Feb. 15, 1909.

PROBLEMS IN INTEREST

308. *Oral*

1. In the preceding examples in interest we have found in every case that the interest is the product of what factors?

2. When we have given any number of factors, what must we do to find the product?

3. When we have given the product of two factors, and one of the factors, how may we find the other factor?

4. Which term in division is always a product? Which terms are factors?

5. Which term in multiplication is the product? Which terms are factors?

6. $8 \times 7 = ?$ Which terms are given? Which is to be found?

7. $8 \times ? = 56$. Which terms are given? Which is to be found?

8. $? \times 7 = 56$. Which terms are given? Which is to be found?

9. When we have given the product of three factors, and two of the factors, how may we find the remaining factor?

10. $5 \times 3 \times 2 = ?$ Which terms are given? Which is to be found?

11. $5 \times 3 \times ? = 30$. Which terms are given? Which is to be found?

12. $5 \times ? \times 2 = 30$. Which terms are given? Which is to be found?

13. $? \times 3 \times 2 = 30$. Which terms are given? Which is to be found?

14. In each of the following examples, tell which terms are given, and which is to be found, and find the term which is wanting.

a. $3 \times 7 \times 2 = ?$
b. $? \times 7 \times 2 = 42$
c. $3 \times ? \times 2 = 42$
d. $3 \times 7 \times ? = 42$
e. $7 \times 5 \times 2 = ?$
f. $8 \times ? \times 3 = 48$
g. $9 \times 6 \times ? = 108$
h. $4 \times 7 \times ? = 112$
i. $10 \times ? \times 10 = 10{,}000$
j. $3 \times 11 \times ? = 99$
k. $6 \times ? \times 5 = 120$
l. $7 \times 2 \times ? = 700$
m. $? \times 12 \times 5 = 600$
n. $? \times 13 \times 4 = 104$
o. $5 \times 5 \times ? = 125$
p. $? \times 6 \times 7 = 210$

15. When we have given the principal, rate, and time expressed in years, how is the interest found?

16. The principal, rate, and time expressed in years are what of the interest?

17. When the principal, rate, and interest are given, how may the time be found?

18. When the principal, time, and interest are given, how may the rate be found?

19. When the rate, time, and interest are given, how may the principal be found?

309. *Written*

1. The interest on \$720 for 1 yr. 8 mo. 11 da. is \$61.10. Find the rate.

Statement of Relation: $\frac{\$720}{1} \times \text{Rate} \times \frac{611}{360} = \$61.10.$

Which terms of relation are given? Which is to be found? How shall we find it?

Solution

$$\frac{\overset{2}{\cancel{720}}}{1} \times \frac{611}{\underset{}{\cancel{360}}} = 1222.$$

$$\text{Rate} = 61.10 \div \left(\frac{720}{1} \times \frac{611}{360}\right) = 61.10 \div 1222 = .05, \text{ or } 5\%. \quad \textit{Ans.}$$

2. At what rate of interest will \$2350 gain \$94 in 8 mo.?

3. When the interest on \$240 for 1 yr. 7 mo. is \$30.40, what is the rate?

4. At what rate will \$1600 amount to \$1718.60 in 1 yr. 7 mo. 23 da.?

Statement of Relation: $\frac{\$1600}{1} \times \text{Rate} \times \frac{593}{360} = \$118.60.$

What is \$118.60? How is it obtained?

5. At what rate will \$52.50 double itself in 16 yr. 8 mo.?

6. At what rate will any sum double itself in 14 yr.?

7. At what rate will any sum double itself in 16 yr. 8 mo.?

8. At what rate must $960 be put at interest to gain $99.20 in 1 yr. 3 mo. 15 da.?

9. Interest $110.72, principal $3460, time 8 mo. 16 da. Find the rate.

310. *Written*

1. In what time will $5000 gain $375 if put at interest at $4\frac{1}{2}\%$?

Statement of Relation: $\frac{\$5000}{1} \times \frac{4\frac{1}{2}}{100} \times \left\{\begin{matrix}\text{Time} \\ \text{in years}\end{matrix}\right\} = \$375.$

Which terms of relation are given?
Which term is to be found?

Solution

$$\frac{5000}{1} \times \frac{4\frac{1}{2}}{100} = 225$$

$$\text{Time} = \frac{375}{1} \div \left(\frac{5000}{1} \times \frac{4\frac{1}{2}}{100}\right) = 375 \div 225 = 1\tfrac{2}{3} \text{ yr.}$$

or, 1 yr. 8 mo. *Ans.*

2. For what time will $101.50 pay the interest on $725 at 7%?

3. A young man borrowed $3000 from his father, paying him $4\frac{1}{2}\%$ interest every year. How long must the father permit the debt to run in order to receive $945 in interest?

4. In what time will $4816 on interest at $3\frac{1}{2}\%$ earn $421.40?

5. In what time will $1200 earn $306 if put on interest at 6%?

6. In what time will \$210 bear \$25.62 interest, at 9% per annum?

Statement of Relation: $\frac{\$210}{1} \times \frac{9}{100} \times \left\{ \begin{matrix} \text{Time} \\ \text{in years} \end{matrix} \right\} = \$25.62.$

Solving as indicated above, the time is $1\frac{16}{45}$ yr.

$$\tfrac{16}{45} \text{ yr.} = \frac{16}{\cancel{45}_{15}} \times \frac{\overset{4}{\cancel{12}}}{1} \text{ mo.} = \tfrac{64}{15} \text{ mo.} = 4\tfrac{4}{15} \text{ mo.}$$

$$\tfrac{4}{15} \text{ mo.} = \frac{4}{\cancel{15}} \times \frac{\overset{2}{\cancel{30}}}{1} \text{ da.} = 8 \text{ da.} \qquad 1 \text{ yr. } 4 \text{ mo. } 8 \text{ da. } \textit{Ans.}$$

7. \$217 will pay the interest on \$2000 for how long at 6%?

8. For what time will \$25.62 pay the interest on \$210 at 9%?

9. In what time will \$231 put at interest at 5% amount to $\$243.70\frac{1}{2}$?

10. \$630 will pay the interest on \$3500 at 5% for what time?

11. In what time will \$810 amount to \$823.23 if put at interest at 7%?

12. In what time will \$1896 amount to \$2006.60 at 5%?

13. A note for \$1800 with interest at 6% amounted to \$1828.50 when it was paid. How long had the note run?

14. A man borrowed \$1280 at $4\frac{1}{2}$% interest and paid the debt when it amounted to \$1341.60. How long did he have the use of the money?

15. A debt of \$10,000 on interest at $5\frac{1}{2}$% amounted to \$10,618.75 when it was paid. How long had it run?

311. *Written*

1. What principal on interest at 6 % will gain \$90 in 1 yr. 1 mo. 10 da.?

Statement of Relation: Principal $\times \frac{6}{100} \times \frac{400}{360} = \$90$.
Which terms of relation are given? Which is to be found?

Solution

$$\frac{\cancel{6}}{\cancel{100}} \times \frac{\cancel{400}}{\cancel{360}} = \frac{1}{15}$$

$$\cancel{90}$$

$$15$$

Principal $= 90 \div (\frac{6}{100} \times \frac{400}{360}) = 90 \div \frac{1}{15} = \$1350$ *Ans.*

2. What principal will earn \$80 in two years at 5 %?

3. A farmer owed a debt on which he paid \$495 interest in three years, the rate being $5\frac{1}{2}$ %. How much did he owe?

4. A certain city borrowed money at $3\frac{1}{2}$ % interest, with which to build a city hall. In 7 yr. 6 mo. the city paid \$78,750 interest on this debt. How much money was borrowed?

5. What principal will yield \$26.40 interest in 1 yr. 4 mo. at $8\frac{1}{4}$ %?

6. What principal, at 7 %, will bring \$153.93 interest in 2 yr. 6 mo.?

7. A man paid \$46.41 for the use of a sum of money for 7 mo. 11 da. The rate was 7 %. What was the principal?

8. A man paid \$209 interest on a sum of money for 9 mo. 15 da. If the rate was $5\frac{1}{2}$ %, what was the principal?

312. *Written*

1. What principal will amount to $584.65 in 1 yr. 18 da. at 6%?

$$\$1 \times \frac{\cancel{6}}{100} \times \frac{\cancel{378}}{\cancel{360}} = \$.063 \text{ interest on } \$1 \text{ for 1 yr. 18 da.}$$

63
$\cancel{60}$
10 $1.063 amount of $1 for 1 yr. 18 da.

Statement of Relation: $1.063 × Principal = $584.65.
Which term of relation is to be found? Find it.

2. What principal will amount to $431.20 in 2 yr. at 6%?

3. What principal on interest at 5% will amount to $430 in 1 yr. 6 mo.?

4. Mr. Smith borrowed a sum of money at $4\frac{1}{2}$% interest for eight months. When the debt became due, he had to pay $2060. What was the sum borrowed?

5. A farmer bought a hay press, agreeing to pay for it in six months, with 5% interest on the purchase price. When the money became due, it took $491.20 to settle the bill. What was the purchase price?

6. Mr. Jacobs bought a house Nov. 23, 1905, paying three fifths of the price in cash and the remainder with 5% interest on the 5th of February, 1907, when it required $1696 to cancel the debt.

a. How much was left unpaid at the time of purchase?

b. What was the purchase price of the house?

7. A dealer in real estate offered me a lot for $1317.50, to be paid 15 mo. after date of purchase, without interest. This was equal to what cash price, money being worth 6%?

313. *Written*

In examples 1–20 *find the terms indicated by interrogation points:*

	Principal	Rate	Time	Interest	Amount
1.	$364.24	6%	1 yr. 4 mo.	?	
2.	$2700	5%	1 yr. 1 mo.		?
3.	$2350	5%	1 yr. 3 mo. 6 da.	?	?
4.	$292	$5\frac{1}{2}$%	90 da.	Exact	?
5.	$1730	4%	?	$318.32	
6.	?	$4\frac{1}{2}$%	2 yr. 9 mo.	$1556.775	?
7.	$387.50	?	7 mo. 24 da.	$20.15	
8.	$3500	5%	?	$630	
9.	$1000	6%	?	?	$2000
10.	$250	?	90 da.	?	$252.50
11.	$3500	?	July 18 to Nov. 9	$70	
12.	?	5%	Jan. 1 to May 25, 1908	?	$3580
13.	?	$5\frac{1}{2}$%	?	$132	$4132
14.	$1800	?	Feb. 20 to Sept. 21, 1907	$86.75	
15.	?	6%	6 mo. 6 da.		$494.88
16.	$620.50	$5\frac{1}{2}$%	30 da.	Exact	?
17.	?	5%	146 da.	Exact	$765
18.	$800	?	73 da.	Exact	$811.20
19.	$2500	5%	?	Exact	$2550
20.	$350	$5\frac{1}{2}$%	Apr. 1 to Nov. 6	Exact	?

21. What is the difference between the exact interest and the common interest on \$657 for 90 da. at 5%?

22. How long will it take \$1440 to earn \$244.80 interest at $4\frac{1}{2}$%?

23. What sum must a lady have invested at 5% per annum to yield her an income of \$125 a month?

COMPOUND INTEREST

314. Compound interest *is interest computed by adding the unpaid interest to the principal at regular interest periods, and taking the sum for a new principal for each succeeding interest period.*

315. Simple interest *is interest computed on the original principal for the entire time.*

In ordinary business transactions, "with interest" is understood to mean simple interest, although the debt may run for several years.

It is customary to insert in contracts for the payment of interest, where the debt runs for a longer period than one year, a provision that the interest shall be paid at regular periods, usually of three months, six months, or one year. This is especially true in the case of insurance companies, loan associations, and other institutions doing a large loan business; so that they are enabled to compute their income on a compound interest basis by loaning the interest as fast as it is paid in.

Savings banks and trust companies generally allow compound interest on all deposits remaining for a full interest period, which is usually three or six months.

316. *Written*

1. Find the compound interest of \$350 for 2 yr. and 6 mo. at 6%.

Solution

$350.00	Principal
21.00	Interest for 1st year
$371.00	Amount taken as new principal
22.26	Interest for 2d year
$393.26	Amount used as new principal
11.80	Interest for 6 mo.
$405.06	Amount for 2 yr. 6 mo.
350.00	1st principal
$55.06	Compound interest for 2 yr. 6 mo.

NOTE 1. — When the interest is compounded semi-annually, the rate for each period is one half the annual rate; when quarterly, one fourth.

When no interest period is mentioned, interest is compounded annually.

NOTE 2. — In actual practice, compound interest is computed by means of compound interest tables similar to that on page 410. The table gives the amounts of one dollar for from one to twenty periods, at various rates for each period. The required amount is obtained by multiplying the amount of one dollar, for the required number of interest periods, at the given rate, by the given principal. If the compound interest is desired, omit the 1 at the left of the decimal point in the multiplicand.

2. What is the compound interest of $830 for 3 years at 5 %?

3. What is the amount of $650 for 4 years at 4 % interest, compounded semiannually?

4. What is the compound interest of $365 for 2 yr. 7 mo. 18 da. at 6 %, compounded semiannually?

5. What is the compound interest on $640 for 4 years at 5 %?

6. What is the interest, compounded quarterly, on $538.25 for 2 yr. 6 mo., rate 4 %?

7. What is the interest, compounded annually, on $683.48 for 4 years at 6 %?

8. What is the compound interest on $437.50, for 3 yr. 6 mo., at 5 %, compounded semiannually?

PROMISSORY NOTES

317. *A* **promissory note** *is a written promise made by one party to pay absolutely a specified sum of money to another party at a certain time.*

Since the term "note" in business transactions always refers to a promissory note, we shall henceforth omit the word "promissory" in speaking of a note.

FORMS OF NOTES

318. The following forms illustrate various kinds of notes:

Note 1

$500 — Springfield, Mass., Aug. 3, 1907

Ninety days after date, I promise to pay

to the order of John H. Blodgett

Five Hundred dollars, with interest.

Value received.

Lucius Thomas.

Note 1. — Back

Pay to the order of
C. F. Harper
John H. Blodgett

Note 2

$250 — Los Angeles, Cal., April 1, 1908

One year after date, I promise to pay W. J. Barr

or bearer Two hundred fifty dollars,

with interest at seven per cent.

Value received.

William R. Hamilton

Note 2.—Back

Received on the within note:
Apr. 1, 1909, $15.00
May 8, 1909, $100.00

Note 3

$1000 — Scranton, Pa., May 12, 1908

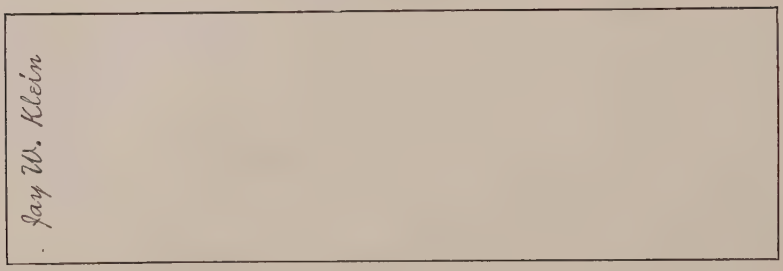
On demand I promise to pay Jay W. Klein

or order One thousand dollars,

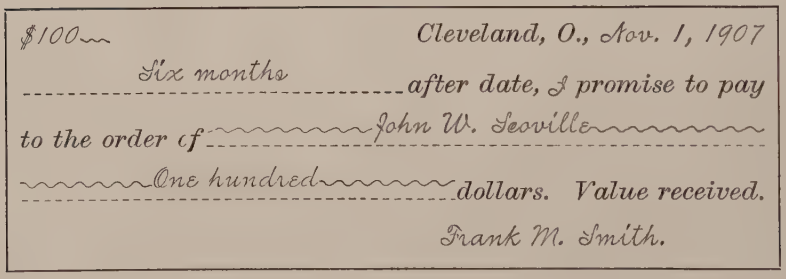
for value received, with interest.

William H. Waldorf.

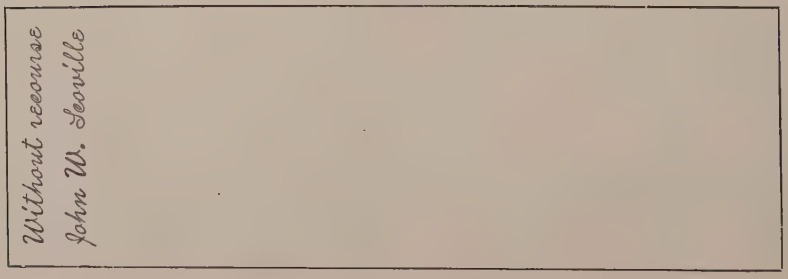
Note 3. — Back



Note 4


$100
Cleveland, O., Nov. 1, 1907
Six months after date, I promise to pay
to the order of John W. Seville
One hundred dollars. Value received.
Frank M. Smith.

Note 4. — Back



Note 5

$425 — Rochester, N.Y., May 1, 1908

Thirty days after date, I promise to pay George K. Walden

Four hundred twenty-five dollars.

Value received.

Henry G. Roberts

KINDS OF NOTES

319. *The party who makes the promise* is the **maker** *of a note.*

320. *The party to whom the money is promised to be paid* is the **payee** *of a note.*

321. *The party who owns a note* is the **holder.**

322. *The sum promised to be paid, not including interest,* is the **face** *of a note.*

323. *A note in which the maker promises to pay interest* is an **interest-bearing note.**

324. *A note in which the maker does not promise to pay interest* is a **non-interest-bearing note.**

325. *A* **time note** *is a note payable at a specified time after date.*

326. *A* **demand note** *is a note payable on demand of the holder.*

A note payable one day from date becomes a demand note, for the holder may require payment at any time after date.

327. *A note is* **negotiable** (i.e. *transferable*) *when it is drawn payable —*

a. To the bearer, b. To the payee or bearer, or *c. To the order of the payee.*

NOTE. — Besides the previous conditions, a note to be negotiable—
In Alabama, must be payable at a fixed place.
In Indiana, must be payable at a bank.
In West Virginia, must be payable at a banking office.

328. *A note is* **non-negotiable** *when it is drawn payable only to the payee.*

329. A note should contain the following things, in addition to the words of the promise:

a. The time and place at which the note is made.
b. The face, expressed both in figures and in words.
c. The name of the payee.
d. The time of payment.
e. The name of the maker.
f. The words "with interest," and the rate, if the note is intended to be interest bearing.
g. "Value received."

A note is valid without the words "Value received," but there is a legal advantage in using them.

NOTE. — There are many kinds of notes, such as "joint" notes, "joint and several" notes, "judgment" notes, "collateral" notes, and others, which are not in general use and involve legal distinctions that do not come within the scope of elementary arithmetic. Hence no treatment of them is here given.

INDORSEMENT

330. *An* **indorsement** *is a name or other writing on the back of a note.* Usually an indorsement contains either

a. The name of the payee, or of some other person or persons, or,
b. A record of payments made on the note.

331. A person indorses a note **in blank** by merely writing his name across the back of it.

332. A person indorses a note **in full** by writing "Pay to the order of _____" (the name of the person to whom the note is transferred) and signing his name below.

333. *One who indorses a note* is called an **indorser.**

334. *One to whose order a note is made payable by the indorsement* is called the **indorsee.**

335. An indorser, by the act of indorsement, agrees to pay the note when due if the maker does not; but an indorser may avoid this liability for the payment of a note by writing "Without recourse" above his signature.

336. When the payee of a note drawn payable to the payee's order transfers the note, he must indorse it in order to make it payable to the new holder.

If he indorses it *in blank*, it becomes payable to the holder, whoever he may be, and can be transferred again without further indorsement. If he indorses it *in full*, it becomes payable only to the person designated in the indorsement, until it is in turn indorsed by that person.

He may make a **restrictive indorsement**, by writing over his signature "Pay to _____" (naming some person). With such an indorsement, the note cannot again be transferred, for it is payable only to the person designated. A restrictive indorsement is sometimes written, "Pay to _____ only."

MATURITY

337. *The day on which a note becomes due, or payable*, is the **day of maturity.**

In most states, a note becomes due, or payable, on the day specified for payment in the note; in a few states, the note does not become due until three days after the time specified in the note. These three days are called **days of grace.** The

debtor's legal right to days of grace has been recognized by the courts because of the prevalent custom, in early times, of allowing this extra time for payment.

The present tendency is toward a restriction of the custom, and the states are, one by one, enacting laws abolishing days of grace.

If a note falls due on Sunday, or a legal holiday, it is generally not collectible until the next business day. In a few states it becomes due on the last preceding business day. In New York State, a note falling due on Saturday is not collectible until the following Monday.

If no time of payment is mentioned in a note, it is payable *on demand.*

DEFAULT OF PAYMENT

338. When the maker of a note fails to pay it on the day of maturity, it is the duty of the holder to notify the indorsers of that fact. If they are not so notified within a reasonable time, they are freed from liability for its payment. Can you think of some reason for this rule?

When the maker does not pay a note on the day of maturity, the indorser may pay it and then collect it from the maker. When there are several indorsers, and the maker fails to pay the note when due, the first indorser may pay it and sue the maker; or any other indorser may pay it and sue the maker and all the previous indorsers.

EXERCISES

339. *Oral*

1. From the forms on pages 182–185 select, giving reasons for the selections, —

a. A time note.

b. A demand note.

c. A negotiable note.

d. A non-negotiable note.

e. A note that may be transferred without indorsement.

f. A note that cannot be transferred without indorsement.

g. A note that cannot be transferred.

h. An indorsement in blank.

i. An indorsement in full.

j. An indorsement that does not make the indorser liable for payment of the note.

k. An indorsement that makes the note transferable again without further indorsement.

l. A note that is partly paid.

m. An interest-bearing note.

n. A non-interest-bearing note.

2. Name the maker of each note.

3. Name the payee of each note.

4. Name the indorser of each note.

5. Who can collect note 1?

6. If C. F. Harper sells note 1, what must he do to make it payable to the one who buys it?

7. Who can collect note 3?

8. Who can collect note 5?

9. Who is liable for the payment of note 1?

10. Who is liable for the payment of note 4?

340. *Written*

Pupils number around the class, "one, two, three; one, two, three," etc, until each pupil has a number.

1. *a.* Each of the number 1's write a note that can be transferred only by being indorsed, making himself the maker, and number 2 the payee.

b. Deliver the note to number 2.

c. Number 2 transfer the note to number 3, indorsing it in full.

d. Number 3 transfer the note to the teacher, indorsing it so that the teacher may transfer it again without indorsing it.

e. To whom may the teacher look for payment of the note?

f. Number 1 is which party? Number 2? Number 3? The teacher?

2. *a.* Each of the number 3's write a note payable to number 2 or bearer.

b. Deliver the note to number 2.

c. Number 2 transfer the note to number 1.

d. Number 1 transfer the note to the teacher.

e. How many indorsements are necessary in making these transfers?

f. To whom may the teacher look for payment?

g. Both number 1 and number 2 might have indorsed the note. Would their indorsement in blank have affected the value of the note? If so, how and why?

3. *a.* Number 2 write a note payable to number 1 or order.

b. Deliver it to number 1.

c. Number 1 transfer it to number 3, indorsing it in full.

d. Number 3 transfer it to the teacher, indorsing it without recourse.

e. To whom can the teacher look for payment?

4. *a.* Every pupil write a non-negotiable demand note bearing interest at the legal rate where made, making the teacher the payee.

b. Deliver the note.

c. Who can collect the note?

d. Who must pay the note?

e. How could a third party become liable for the payment of the note?

COMPUTING INTEREST ON NOTES

341. An interest-bearing note bears interest from the *day of date* to the *day of payment.*

A non-interest-bearing note, *if not paid at maturity*, bears interest *from the day of maturity until paid*, at the legal rate where made.

If no rate of interest is mentioned in an interest-bearing note, interest must be computed *at the legal rate in the state in which the note is made.*

342. *The face of a note* is the **principal.**

343. *The sum of the principal and interest* is the **amount** *of the note.*

344. When the time mentioned in a note is expressed in months, *calendar months* are always understood. Thus, a note for three months given July 15 is due Oct. 15, or, where grace is allowed, Oct. 18. A 90-day note given July 15 is due 90 days after July 15, or Oct. 13.

345. *Written*

1. Find the amount of note 2, page 183.

2. Find the amount of note 1, page 182, the legal rate of interest in Massachusetts being 6%.

3. Find the amount of note 3, page 183, if paid on the third day of January, 1909, the legal rate of interest in Pennsylvania being 6%.

4. Find the amount of note 4, page 184, if not paid until Aug. 11, 1908, the legal rate of interest in Ohio being 6%.

5. How much can Mr. Walden collect on note 5, page 185, if it is paid Aug. 20, 1908, the legal rate of interest in New York being 6%?

6. A demand note for $711 with interest was dated at Ogden, Utah, July 7, 1905, and paid Sept. 30, 1905. How much was paid, the legal rate of interest in Utah being 8%?

7. A 90-day note for $960, with interest at 7%, was made July 1, 1906, at Lincoln, Neb., where grace is allowed.

a. On what day did the note mature?

b. How much was due at maturity?

8. A 60-day note for $1200 without interest, dated at Cairo, Ill., Jan. 1, 1904, was not paid until May 15, 1904. What sum was then due, the legal rate of interest in Illinois being 5%?

9. Find the amount at maturity of a 30-day interest-bearing note for $700 in the state where you live.

10. What must be the face of a 90-day note that will amount to $263.90, computing interest at 6%, without grace?

11. Find the amount at maturity of the following note, the rate of interest in Louisiana being 5% and grace being allowed:

600\frac{00}{100}$. NEW ORLEANS, Sept. 1, 1908.

On the 15th day of December, 1908, I promise to pay to the order of Henry P. Emerson, six hundred dollars, with interest. Value received. JOHN H. GARDNER.

12. Write a note for $1000 that will give James Thorne the right to collect $1020 from you 90 days from the date of the note.

13. Find the amount due June 15 on an unpaid non-interest-bearing 30-day note for $250, dated March 3, in a state where the legal rate of interest is 6%.

14. Write a negotiable note dated at your city or town, Jan. 15, due May 7 of the present year, and find the amount due at maturity.

PARTIAL PAYMENTS

346. When payments are made in sums less than the entire amount of a note, the holder indorses them on the back of the note, and they are known as **indorsements**, or **partial payments.**

The rule given below is the one adopted by the Supreme Court of the United States for determining the amount due on a debt on which partial payments have been made. It is the legal rule in most of the states of the Union. Classes in any state having a different rule should follow the legal rule of their own state, in solving the partial payment problems given in this book.

United States Rule for Partial Payments

347. *Find the amount of the debt to the time when a payment, or the sum of the payments, equals or exceeds the interest due, and from that amount subtract such payment or sum of payments. With this remainder for a new principal, proceed as before to the time of settlement.*

This rule means that neither the whole interest nor any part of it shall be used to increase the principal on which interest is paid; but whenever more than enough to cover the interest has been paid, the excess shall be used to diminish the principal.

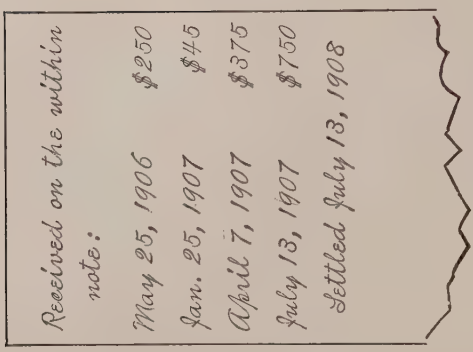
348.

$1820 Watertown, N.Y., Jan. 1, 1906

On demand, for value received, I promise to pay to the order of A. D. Parsons

One thousand eight hundred twenty dollars with interest.

Robert S. White



The diagram at the left shows a part of the back of the note, on page 193, on which indorsements were made as given. The amount due at date of settlement is computed below by the United States rule.

NOTE. — The legal rate of interest in New York State is 6 per cent.

Subtracting each date from the one above to find interest periods:

	Yr.	Mo.	Da.		Remainders
a.	1908	7	13		
b.	1907	7	13	1 yr.	*5th Int. per.*
c.	1907	4	7		3 mo. 6 da., or 96 da., *4th Int. per.*
d.	1907	1	25		2 mo. 12 da., or 72 da., *3d Int. per.*
e.	1906	5	25		8 mo. 0 da., or 240 da., *2d Int. per.*
f.	1906	1	1		4 mo. 24 da., or 144 da., *1st Int. per.*
	2	6	12	= 2 yr.	6 mo. 12 da., *Proof of int. periods*

Subtracting *f* from *a*, we obtain 2 yr. 6 mo. 12 da., which is the same as the sum of the remainders. This proves that the interest periods are correct.

$$\frac{\$1820}{1} \times \frac{6}{100} \times \frac{\overset{24}{\cancel{144}}}{\underset{6}{\cancel{360}}} =$$

$43.68	*Interest for 1st period*
1820.00	*First principal*
$1863.68	*Amount*
250.00	*First payment*
$1613.68	*New principal*

$$\frac{\$1613.68}{1} \times \frac{6}{100} \times \frac{\overset{4}{\cancel{240}}}{\cancel{360}} = \$64.55$$ *Interest for 2d period exceeds payment*

$$\frac{\$1613.68}{1} \times \frac{\overset{6}{\cancel{6}}}{100} \times \frac{\overset{12}{72}}{\cancel{360}} = 19.36$$ *Interest for 3d period*

$\$1697.59$ *Amount*

$\underline{420.00}$ *Sum of 2d and 3d payments*

$\$1277.59$ *New principal*

$$\frac{\$1277.59}{1} \times \frac{\overset{6}{\cancel{6}}}{100} \times \frac{\overset{16}{96}}{\cancel{360}} = 20.44$$ *Interest for 4th period*

$\$1298.03$ *Amount*

$\underline{750.00}$ *4th payment*

$\$548.03$ *New principal*

$$\frac{\$548.03}{1} \times \frac{6}{100} = 32.88$$ *Interest for 5th period*

$\$580.91$ *Due at date of settlement*

349. *Written* *Ans.*

1. Write a demand note for $792 with interest, dated Jan. 15, 1902, at Springfield, Ill. Indorse payments as follows: Dec. 15, 1902, $50; Aug. 30, 1903, $12.50; Oct. 25, 1903, $155. Find the amount due Dec. 1, 1903, computing interest at 5%.

2. A note without interest, dated Lexington, Ky., Aug. 15, 1903, promising to pay $1200 thirty days from date, has $200 indorsed Nov. 16, 1903, and $350, March 4, 1904. How much was due April 1, 1904, the legal rate in Kentucky being 6%?

3. What was due March 1, 1901, on a note for $1000 with interest at 9%, dated March 1, 1900, with indorsements as follows: Aug. 10, 1900, $300; Sept. 1, 1900, $100; Jan. 1, 1901, $50?

4. What amount was necessary to settle, Oct. 20, 1905, a note for $2000, with interest at 6%, dated July 20, 1903, bearing indorsements of $700, Sept. 10, 1903, and $75, Oct. 20, 1904?

5. A note for $700 with interest at 7% was given Dec. 12, 1906. Payments of $200, Dec. 12, 1907, and $159, April 5, 1908, were made. What was due Oct. 30, 1908?

6. How much was due Aug. 1, 1906, on a note for $380, with interest at 5%, dated Aug. 1, 1904, on which were indorsed payments of $15, May 30, 1905, and $90, Jan. 1, 1906?

7.

$300 Troy, N.Y., *Oct. 12*, 1899

On demand, for value received, *I* promise to pay ~~~~~~~~~~*S. D. Cleveland*~~~~~~~~~~or order, *Three hundred* dollars, with interest.

J. H. Van Alstyne.

The following payments were made on this note: June 27, 1901, $150; Dec. 9, 1902, $150. What was due Oct. 9, 1905?

8. On a note for $573.25, with interest at 6%, dated June 10, 1900, were the following indorsements: April 5, 1901, $14.30; July 14, 1902, $250. How much was due Sept. 20, 1903?

9. A note of $850 was dated June 21, 1902, bearing interest at 6%. On this note were the following indorsements: Sept. 15, 1902, $150.90; Nov. 21, 1903, $45; Jan. 15, 1904, $256.88. What remained due June 21, 1904?

10. A man bought a farm, Jan. 1, 1901, giving in part payment a bond and mortgage for $1900, due on demand, with interest at $4\frac{1}{2}$%. He paid $40, July 1, 1901; $300, Feb. 15, 1902; and $240, July 20, 1902. How much was due at time of settlement, Jan. 1, 1903?

11. On a note for $832.26 dated Aug. 3, 1899, the following payments were indorsed: $350, Oct. 5, 1900; $468.37, May 15, 1902. How much was due Dec. 12, 1903, interest at 7%?

12. Face, $2950. Date, July 1, 1905. Interest, 7%. Indorsements: Oct. 1, 1905, $750; Jan. 15, 1906, $600; Dec. 1, 1906, $300; March 1, 1907, $450. What was due July 1, 1907?

350. When notes and accounts, upon which partial payments have been made, are settled within a year after interest begins, business men sometimes make use of the following

Merchants' Rule

Find the amount of the entire debt at date of settlement.
Find the amount of each payment at date of settlement.
Subtract the amount of the payments from the amount of the debt.

351. *Written*

Find the balance due at time of settlement on each of the debts in examples 1–5, *using the Merchants' Rule.*

1. A note for $700, dated Jan. 1, 1904. Indorsements: $215, April 15; $124.68, April 30; $21.04, July 7; $130, Oct. 20. Settled Jan. 1, 1905. Rate 5%.

2. A note for $250, dated March 31, 1906. Indorsements: $10.45, July 1, 1906; $130, Dec. 4, 1906; $50, Jan. 1, 1907. Settled Feb. 28, 1907. Rate 6%.

3. A debt of $1240 contracted July 1, 1907, with payments of $289, Jan. 1, 1908, and $135, April 15, 1908. Settled June 1, 1908. Rate $4\frac{1}{2}$%.

4. $700 borrowed Oct. 22, 1905; payments made Jan. 1, 1906, and March 14, 1906, of $280.50 and $35.90 respectively. Settled April 1, 1906. Rate 7%.

5. A man bought a house for $5500, May 1, 1904, paying $4000 at that time and $200 the 15th of each month, besides interest at 5%. Settled Dec. 1, 1904.

REVIEW AND PRACTICE

352. *Oral*

1. Read MCMXII; 305.0070100; $\frac{\frac{3}{4}\times\frac{7}{9}}{12\frac{1}{8}-7\frac{2}{3}}$.

2. Name three powers of 10; two powers of 5.

3. Count by 12's to 132.

4. What term in division corresponds to the product in multiplication?

5. State two ways of testing subtraction.

6. State two ways of testing division.

7. How many decimal places does the quotient contain?

8. Give results rapidly, adding tens first:

$$28+35;\ 46+43;\ 53+17;\ 82+49.$$

9. $2\times15-14\div2+3\times10=?$

10. What problems can be solved by cancellation?

11. How can we tell, without actually dividing, whether a number is divisible by 3? By 9? By 2? By 4? By 8? By 5? By 25? By 6?

12. Give results at sight: $247\div100$; $.83546\times1000$; 36×25; $497.60\div100$; $9\div25$.

13. Without actually dividing, tell whether 247,658 will exactly divide 42,130,071,359, and why.

14. Name two composite numbers that are prime to each other.

15. How may we tell whether a number is prime or not?

16. How may we find a single divisor that will reduce a fraction to lowest terms?

17. When is a number in its simplest form?

18. What fractions cannot be reduced to exact decimals?

19. Using aliquot parts, answer the following questions:

a. At 25 cents a pound, what will 56 pounds of coffee cost?

b. How many packages of cereal, at $12\frac{1}{2}$ cents per package, will \$10 buy?

c. If the average price of the melons in a load is $16\frac{2}{3}$ cents apiece, what are 48 of them worth? How many can be bought for \$5?

d. At $14\frac{2}{7}$ cents a dozen, how many dozen pencils will \$2 buy? What will 28 dozen cost?

20. How many pence are there in 20 shillings?

21. Alice bought half a ream of note paper. How many sheets of paper did she buy? How many quires?

22. Name four kinds of figures that are quadrilaterals.

23. How many cords are there in a pile of 2-foot wood, 12 ft. long and 8 ft. high?

24. Describe a board foot and with your hands show its size.

25. How many feet of lumber are there in a piece of scantling 8′ by 4″ by 3″?

26. How many cubic inches of oil are there in 3 gal.?

27. What per cent is equal to $\frac{1}{3}$? $\frac{2}{5}$? $\frac{7}{8}$? $\frac{9}{10}$? $\frac{3}{8}$? $\frac{4}{5}$? $\frac{2}{3}$? $\frac{5}{8}$?

28. 18% of \$200 = ? 80% of 20? $66\frac{2}{3}$% of 12 oz.?

29. \$17 is 25% of what?

30. What is 170% of 10 quarts?

31. What per cent of \$65 is \$13?

32. What is $\frac{1}{2}$% of 800 miles?

33. $\frac{6}{7}$% of 49 gallons are how many gallons?

34. $\frac{6}{7}$ of 49 gallons are how many gallons?

35. 1 sq. ft. is what per cent of 1 sq. yd.?

36. Three quarts are what per cent of one gallon?

37. 80 % of £1 = how many shillings?

38. A man earns $40 a week and spends 60 % of it. How much does he save?

39. Frank missed three problems in a lesson of 15 problems. What per cent of the lesson did he have correct?

40. Mr. Peck sold a piano bench for $13, gaining $3. What per cent did he gain?

41. A furniture dealer bought a chair for $20 and gained 40 % on it. What was the selling price?

42. Upon what base are gain and loss always computed?

43. A merchant sold a bill of goods for $40 more than they cost, thereby gaining 20 %. What did the goods cost?

44. A merchant sold a bill of goods for $20 more than they cost, thereby gaining 10 %. What was the selling price?

45. A merchant sold a bill of goods for $24, thereby gaining 20 %. What did the goods cost?

46. A man sold a horse for $180, thereby losing 10 %. What did he pay for the horse?

47. What per cent is gained on chestnuts bought at $1.20 per peck and sold for 20 cents a quart?

48. What per cent is lost on chestnuts bought at 20 cents a quart and sold at $1.20 a peck?

49. A man sold $1500 worth of goods on a commission of 10 %. How much should he pay over to his principal?

50. An agent's commission at 12 % for selling a consignment of goods amounted to $48. What was the value of the goods sold? How much did the principal receive from the sales?

51. A real estate agent's commission at 2 % for selling a business block was $800. What was the selling price of the block?

52. The net amount of a bill which had been discounted 5% was \$9.50. What was the face of the bill? What was the discount?

53. What is the net amount of a bill of \$200 on which commercial discounts of 20% and 10% have been made?

54. What single discount is the same as two successive discounts of 10% each?

55. What is the premium for insuring a house for \$2000 for three years at the rate of 70 cents per \$100 of insurance?

56. How much is saved on \$1000 of insurance for three years by taking a three-year policy at 1% instead of three one-year policies at $\frac{1}{2}$% each?

57. When a burglar insurance policy for \$1000 costs \$12.50 per year, the premium is what per cent of the face of the policy?

58. Of what three factors is interest the product?

59. Define a promissory note.

60. Describe a negotiable note.

61. Describe a non-negotiable note.

62. How may a note be indorsed in blank?

63. By indorsing a note in blank, what contract does the indorser make?

64. How may a person indorse a note so as to avoid liability for its payment?

65. Give the United States rule for partial payments.

66. Give the Merchants' Rule for partial payments.

353. *Written*

The following table, compiled from the records of the United States Weather Bureau, shows in degrees the average temperature for each month in twenty different places:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Albany, N.Y.	31	32	40	56	69	78	82	80	72	60	46	36
Atlanta, Ga.	50	54	61	70	79	85	87	85	81	71	60	53
Baltimore, Md.	41	43	49	61	73	82	86	84	77	66	53	44
Binghamton, N.Y.	33	30	41	55	67	77	83	80	73	63	45	35
Bismarck, N. Dak.	17	20	32	54	67	75	82	81	70	56	37	25
Boston, Mass.	35	36	42	54	66	76	81	78	71	60	49	39
Carson City, Nev.	44	48	53	61	67	76	84	84	75	65	56	46
Cincinnati, O.	40	43	51	63	74	83	87	84	78	66	52	43
Galveston, Tex.	59	62	68	74	81	86	89	88	84	78	68	62
Harrisburg, Pa.	36	36	45	60	71	80	84	82	75	62	49	40
Indianapolis, Ind.	36	39	48	62	72	82	86	84	77	64	49	40
Jacksonville, Fla.	64	67	72	78	84	89	91	90	86	78	71	65
Minneapolis, Minn.	24	24	36	58	69	78	83	81	74	60	39	28
New Orleans, La.	61	65	70	76	82	87	89	88	85	78	69	63
Portland, Me.	30	32	39	51	62	72	77	75	68	57	45	35
Portland, Ore.	44	48	55	61	67	71	78	77	71	62	52	47
St. Louis, Mo.	40	43	52	66	75	84	88	86	79	68	53	44
Santa Fe, N.M.	39	43	52	60	69	78	81	79	73	62	50	43
Spokane, Wash.	33	38	48	59	68	74	83	83	71	59	44	37
Yuma, Ariz.	66	72	78	85	93	101	106	104	100	87	76	68

1–12. Find the average temperature of all the places for each month.

13–32. Find to the nearest hundredth of a degree the average annual temperature of each place.

33. Express in Roman numerals the number of the present year.

34. Multiply in the shortest way:

a. 39,742,568 by 25. *b.* 34,067 by 125. *c.* 394,708 by 99.

35. Divide in the shortest way:

a. 39,763 by 25. *b.* 9834 by 125. *c.* 796,453 by $.16\frac{2}{3}$.

36. Resolve 7511 into its prime factors.

37. Reduce $\frac{123}{287}$ to lowest terms.

38. Find the smallest number that will exactly contain 39, 36, and 84.

39. Find the greatest number that will exactly divide 2205 and 3024.

40. Reduce $\frac{123}{250}$ to a decimal.

41. Write a bill containing two debit items and one credit item. Foot the bill and receipt it as clerk for the creditor.

42. A field $\frac{1}{8}$ mile long and 30 rods wide contains how many acres of land?

43. 15*s.* 9*d.* are what part of one pound?

44. Divide 18° 17′ 30″ by 15.

45. Multiply 7 hr. 40 min. 8 sec. by 15.

46. What is the cost, at 28 cents a square yard, of painting the walls and ceiling of a room 33 ft. by 24 ft. and 11 ft. high, allowing for five windows, each 4 ft. by 8 ft., and three doors, each 4 ft. by 8 ft. 6 in.?

47. Find the cost of the following bill of lumber:

30 scantlings,	$18' \times 2'' \times 4''$ at \$26 per M.
40 joists,	$16' \times 3'' \times 12''$ at \$26 per M.
25 joists,	$16' \times 2'' \times 10''$ at \$26 per M.
120 boards,	$14' \times \frac{7}{8}'' \times 4''$ at \$35 per M.
300 pieces siding,	$10' \times \frac{1}{2}'' \times 5''$ at \$55 per M.

48. A box car 36 ft. long and $8\frac{1}{2}$ ft. high contains 102 cu. yd. of space. How wide is it?

49. \$2.80 is $\frac{7}{10}$ per cent of what sum?

50. $\frac{3}{8}$ of 50 bu. are how many quarts?

51. Three days are what per cent of two weeks?

52. A speculator bought 150 crates of eggs in April and May, paying 15¢ a dozen for one third of them and 17¢ a dozen for the remainder. Each crate contained 30 dozen eggs. In December, he sold them at a uniform price of 25¢ a dozen, and out of the profits paid a bill of 45¢ per crate for cold storage, and $13.80 for cartage and other expenses. What per cent net profit did he make?

53. During the month of December, at a certain place, there were 8 stormy days and 22 cloudy days, the remaining days being fair.

a. What per cent of the days were stormy?

b. What per cent were cloudy?

c. What per cent were fair?

54. A bill of goods listed at $700 was sold at a discount of 15%, 12%, and 5%. Find the net price.

55. *a.* Find, by the United States rule, the balance due at settlement on a debt of $630, contracted April 1, 1907, and settled Sept. 1, 1908, on which payments of $15.50, Dec. 11, 1907, and $125.00, Feb. 16, 1908, had been made. Interest allowed at 5%.

b. If this balance were computed by the Merchants' Rule, would it favor the debtor or the creditor, and how much?

56. A bill of hardware was discounted 80%, 10%, and 5%, and then amounted to $3.42. What was the list price?

57. Which is the better offer, successive discounts of 30%, 10%, and 5%, or successive discounts of 5%, 10%, and 30%?

58. Which is the better offer, successive discounts of 15%, 5%, and 2%, or successive discounts of 20% and 2%?

59. The premium for one kind of accident insurance policy is at the rate of \$5 per \$1000. The agent's commission is 30 % of the premium.

a. What is the face of the policy for which the company receives \$17.50 after paying the agent's commission?

b. What is the face of a policy that yields the agent \$3.75?

c. What is the agent's commission on a \$6000 policy?

60. A bill of \$20 was reduced by three successive discounts. If the first two discounts were 20 % and 10 %, and the net price was \$12.96, what per cent was the third discount?

BANKS AND BANKING

354. There are many kinds of banking institutions, but most of them may be included in three general divisions; viz. savings banks, banks of deposit, and trust companies.

355. Savings banks are designed to be safe places of deposit for small sums of money. These sums are usually the savings of people who have not the inclination or opportunity to engage in large business enterprises. Savings banks pay a low rate of interest on all balances of one dollar or more, and the interest is compounded quarterly, semi-annually, or annually. The interest is computed by means of tables, and each bank has its own method of calculation.

In order that the money of depositors may be safeguarded, savings banks are generally forbidden by law to make loans unless secured by mortgages on real estate, and from making investments, except in special kinds of property, such as government bonds and bonds of certain states and cities.

356. Banks of deposit, otherwise known as commercial banks, or banks of discount, transact a much wider range of business

than do savings banks. They may loan money on notes, collect accounts and notes for customers, issue bills of exchange and letters of credit, and make many kinds of investments which savings banks are not permitted to make. As a rule they pay no interest on deposits, but the services that they render to their customers are considered sufficient compensation for their use of the money on deposit.

Banks of deposit which are organized under Federal laws and are under the supervision of the United States government are known as **national banks**; those that are organized according to state laws and are under the supervision of state authorities are generally known as **state banks**, though each individual bank adopts a name of its own.

State and national banks transact in general the same kinds of business; but national banks also perform a special function in connection with the issuance of paper money, which will be considered later.

357. Trust companies are similar in some respects to savings banks, and in other respects to banks of deposit.

They resemble savings banks in that they pay interest on deposits. They are generally not allowed to loan money on notes, except when secured by collateral, *i.e.* some specific piece of property, put into the hands of the trust company to be sold by the company if the note is not paid when due.

Otherwise they are much like banks of deposit, having in some respects even greater latitude in the kinds of business which they may transact.

DEPOSITING AND WITHDRAWING MONEY

358. *One who has money on deposit in a bank* is called a **depositor.**

When a person deposits money for the first time in any particular bank, he receives from the bank a book in which he is credited with the sum deposited.

A depositor in a savings bank takes his book with him whenever he deposits or withdraws money. To deposit money he merely hands it to the receiving teller, who credits in the bank book the amount of the deposit. To withdraw money, he hands his book to the paying teller, and signs a receipt for the money to be withdrawn. The teller charges in the bank book the amount withdrawn and pays it to the depositor.

DEPOSIT SLIP

MARINE NATIONAL BANK
OF BUFFALO
Deposited to Credit of
Gerald W. Porter
Buffalo, N.Y. Apr. 23 1908

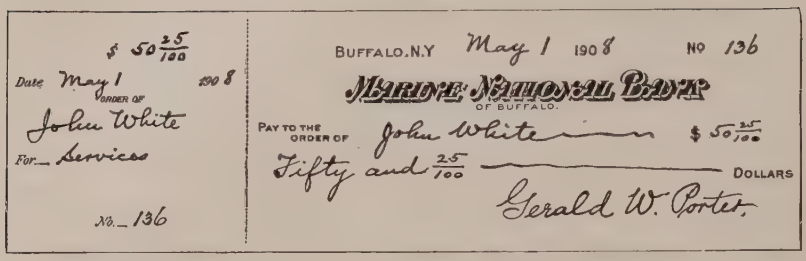
	DOLLARS	CTS.
CURRENCY,	125	
GOLD,		
SILVER,	18	75
CHECKS,		
Bank of Buffalo	38	70
Cleveland	183	40
AMOUNT,	365	85

In depositing money in any other bank than a savings bank, the depositor fills out a **deposit slip** stating in separate items the amount of paper money, of gold, of silver, and of checks which he deposits. This slip is handed in with the money and checks deposited, and is used by the teller in making up his balance at the close of the day's business.

Withdrawals from a bank of deposit are made by means of checks.

A **check** *is a written order, signed by a depositor, directing the bank to pay to a certain person, or to his order, or to the bearer, a specified sum of money.*

When the bank pays the sum directed to be paid, it charges the depositor's account with the amount paid.



Stub Check

359. *The amount named in a check* is called the **face.**

360. *The depositor who signs a check* is called the **drawer** *of the check.*

361. *The person to whom, or to whose order, a check is made payable* is called the **payee.**

362. *The bank on which a check is drawn* is called the **drawee.**

In the above check which party is John White? Gerald W. Porter?

Every depositor in a bank of deposit receives from the bank a **check book,** which consists of blank checks bound together, each check attached to a stub as shown above. When a check is filled out, the stub is filled out to agree with it, and the check is then torn off, through the perforated line. When all the checks have been used, there remains a book of stubs containing a record of all the checks, the number of each check, its date, its face, the name of the payee, and the purpose for which it was used. Some check books are so arranged that the stub may also show the balance remaining in the bank after each check is drawn.

Checks are convenient in paying bills; for by means of them the depositor may avoid carrying or sending money. To illustrate, let us suppose that Mr. A, a merchant in Cleveland,

buys a bill of goods from Mr. B, in Chicago. A fills out a check payable to B's order and mails it to B. B indorses the check, deposits it in his own bank at Chicago, and it is credited on his account. The banks attend to the rest of the business. The check is finally returned to A's bank in Cleveland, and the amount is charged to A's account, and credited to the account of B's bank in Chicago.

Most banks make a practice of returning all checks to depositors. These checks, being indorsed in each case by the payee, serve as receipts for the amounts paid.

363. If the drawer of a check is a stranger to the payee, the payee may be unwilling to accept the check in lieu of money, fearing that the maker may not have money on deposit sufficient to pay the check when presented at the bank for payment. Then the maker may be required to have the check **certified**. To do this, he takes the check to the bank, and the bookkeeper, teller, or other proper person stamps on its face the word "certified" with the name of the bank, and writes in his own name. He then makes a memorandum of the amount on the drawer's account. The bank is then obliged to cash the check when presented. The certification of the check is equivalent to the bank's promise to pay.

A COMPARISON OF CHECKS AND NOTES

364. 1. A note is a *promise* to pay money, while a check is an *order* to pay money.

2. A check always has *three* parties, while a note may have only *two*.

3. A check, like a note, may be *negotiable* or *non-negotiable*, according to the manner in which it is drawn.

4. A negotiable check may be *transferred by indorsement* in the same manner as a note, and the indorser is liable for its payment if it is not paid by the maker or drawee.

5. The different forms of indorsement have the same force when made on a check as when made on a note.

6. A note may draw interest, but a check does not.

EXERCISES

365. *Oral*

1. Name some similarities or differences between a check and a note, other than those given above.

2. Who is the drawer of the check on page 208?

3. Who is the payee? The drawee?

4. Tell whether the check is negotiable or non-negotiable.

5. How must a check be worded in order to be negotiable?

366. *Written*

Let the pupils of the class take numbers one, two, and three, as on page 189. Let the teacher be the First National Bank.

1. Number *two* make out a bill against number *one*, and receipt it when paid.

2. Number *one* write a negotiable check and give it to number *two* in payment of the bill.

3. Number *two* indorse the check to number *three* and take a receipt for the amount on account.

4. Number *three* indorse the check in blank and deposit it to his own credit in the bank.

5. Teacher mark "paid" and return to number *one*.

NOTE. — Repeat this and similar exercises until pupils are familiar with the use of checks.

BANK DISCOUNT

367. *A note that is payable to or at a bank* is a **bank note.**

$ 500 00/100 SYRACUSE, N.Y. Oct 10 1907

Ninety days AFTER DATE I PROMISE

TO PAY TO THE ORDER OF John Sherman

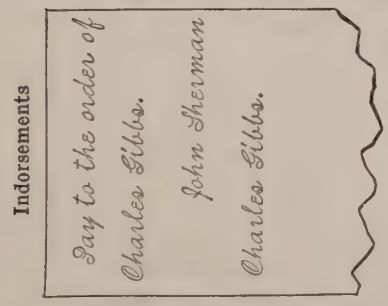
Five hundred no/100 DOLLARS

AT NATIONAL BANK OF SYRACUSE, SYRACUSE, N.Y.

VALUE RECEIVED Timothy S. Hughes

368. Banks come into possession of notes in two ways:

a. They may lend money directly to the maker and take his note, or,



b. The note may be drawn payable to another party and be bought by the bank, or deposited in the bank for collection.

Either of these ways is equivalent to a purchase of the note by the bank. When a bank thus buys a note, it pays less than the maturity value; hence the transaction is called **discounting** the note.

369. *The sum deducted from the maturity value of a note in determining the price to be paid for the note by a bank* is called the **bank discount.**

370. *The sum paid for a note by a bank, or the difference between the maturity value and the bank discount,* is called the **proceeds** *of the note.*

371. *The day on which a note is discounted* is called the **day of discount.**

372. *The time from the day of discount to the day of maturity* is the **term of discount.**

373. If the bank should buy the note in § **367** on the day of date, the proceeds would be determined as follows:

Day of maturity, Jan. 8, 1908.
Day of discount, Oct. 10, 1907.
Term of discount, 90 days.
Interest on $500 for 90 days at 6%, $7.50.
$500 − $7.50 = $492.50. *Proceeds.*

If the bank should buy the note Nov. 19, the proceeds would be determined as follows:

Day of maturity, Jan. 8, 1908.
Day of discount, Nov. 19, 1907.
Term of discount (Nov. 19, 1907, to Jan. 8, 1908), 50 days.
Interest on $500 for 50 days at 6%, $4.17.
$500 − $4.17 = $495.83. *Proceeds.*

In determining the proceeds of an interest-bearing note, the general practice of banks is to find the amount of the note at maturity and compute the interest on that amount for the term of discount. That interest is the bank discount. The bank discount subtracted from the maturity value (which is the amount in this case) gives the proceeds.

374. In all cases we may apply the following:

Rule for finding the bank discount and proceeds of a bank note.

1. *Find the amount due at maturity. This is the maturity value.*

2. *Find the time from the day of discount to the day of maturity. This is the term of discount.*

3. *Find the interest on the maturity value for the term of discount. This is the bank discount.*

4. *Subtract the bank discount from the maturity value to find the proceeds.*

NOTE 1.—When the time mentioned in a note is given in months, calendar months are understood. For example, a note dated July 12, payable three months after date, is due Oct. 12, or 92 days after date.

NOTE 2.—In most states, notes falling due on Sunday or a legal holiday are payable on the next business day, and interest and discount are reckoned to that day.

NOTE 3.—In states allowing days of grace, the date of maturity is three days later than the time mentioned in the note, and the term of discount three days longer than when grace is not allowed.

The local practice in regard to holidays, days of grace, etc., should be followed in solving problems.

375. *Oral*

1. How is the maturity value of an interest-bearing note found?

2. How does the maturity value of an interest-bearing note compare with the face of the note?

3. How does the maturity value of a non-interest-bearing note compare with the face of the note, if paid when due?

4. A 30-day note is dated Jan. 15. What is the day of maturity?

5. A 60-day note was dated Feb. 20, 1908. When did it mature?

6. Mr. Field, wishing to borrow from a bank, made out a 60-day bank note for $100 without interest, dated Sept. 11, 1907. What was the date of maturity? How much was due

at maturity? If Mr. Field had his note discounted on the day of date, what was the term of discount? What was the discount, the legal rate being 6 %?

7. Mr. Brown bought a horse from Mr. Martin, giving in payment a bank note for $200 without interest, dated July 9, 1906, payable 90 days from date. On the 8th day of August, Mr. Martin indorsed the note and deposited it in the bank, receiving credit for the proceeds. What was the day of maturity? The day of discount? The term of discount? The bank discount, the legal rate being 6 %? How much was credited to Mr. Martin's account?

8. A bank note for $500, without interest, due in 90 days, dated May 7, 1905, was discounted June 6, 1905. What were the proceeds, money being worth 6 %?

9. A note for $400, bearing interest at 7 %, dated Jan. 1, 1907, and due in 90 days, was discounted on the day of date. What was the maturity value? On what sum was the discount computed?

376. *Written*

1. A man gave his note for $720 for 90 days without interest. What was it worth at a bank where the discount rate was 6 %?

2. How much can I borrow from a bank by giving my 60-day note for $650 without interest, if the bank gives me a discount rate of 5 %?

3. A merchant bought a piano for $400 cash and sold it the same day, taking in payment a 90-day bank note for $500, which he immediately indorsed and deposited in his bank, receiving credit for the proceeds at a discount rate of 7 % per annum. What was his profit on the piano?

4. What were the proceeds of a note for $300 without interest, due Jan. 7, 1907, and discounted Nov. 15, 1906, the discount rate being 5%?

5. The following note was discounted at the rate of $4\frac{1}{2}$% per annum on the 21st day of January, 1905. What were the proceeds?

$9600 NEW YORK, December 7, 1904.

Ninety days after date I promise to pay to the order of the New York National Exchange Bank nine thousand six hundred dollars.

Value received. CHARLES H. REDMOND.

6. What are the proceeds of a six-months note for $800, without interest, dated May 7, 1903, and discounted Oct. 15, 1903, at the rate of 6% per annum?

7. A man in Seattle accepted a 30-day note for $975, without interest, in payment for furniture. Nine days later he had the note discounted at the rate of 8% per annum. What did he receive for it?

8. Silas Brown sold a vacant lot on the 15th day of April, 1906, to James Otis, taking in part payment a six-months note for $900 without interest, signed by Francis Fernald, dated Dec. 1, 1905, and payable to Mr. Otis at the Marine Bank. Mr. Otis indorsed the note to Mr. Brown's order and Mr. Brown immediately indorsed it in blank and had it discounted. The discount rate was 7%.

a. Write the note and make all the indorsements.

b. How much did Mr. Brown receive for the note?

9. A 90-day note for $1000 with interest at 6% was discounted at 6% on the day of date. What were the proceeds?

10. On the first day of March, 1907, Edward F. Jones borrowed $800 from John Ethridge, giving his note for one

year with interest at 8%, payable at the Corn Exchange Bank. On the first day of January, 1908, Mr. Ethridge had the note discounted at 6% per annum. How much did he receive for it?

11. A 90-day note for $690, bearing interest at 6%, was discounted at the same rate 60 days after date. What were the proceeds?

12. A merchant sold at 25% profit a bill of goods that cost him $150 cash, taking in payment a 60-day note without interest, which he had discounted immediately at 7% per annum. What was his net profit on the bill of goods?

13. A farmer received $297 as the proceeds of a note, without interest, due in 60 days, discounted at 6% per annum. What was the face of the note?

Solution

Discount for 60 da. = 1% of face. ($\frac{6}{100} \times \frac{60}{360} = .01$, or 1%.)

Proceeds for 60 da. = 99% of face.

Statement of Relation: 99% of face = $297.

14. I borrowed $591 from a bank, giving my note for 90 da. without interest, the rate of discount being 6%. What was the face of the note?

15. Edward H. Flint gave William G. Barrows his note, without interest, payable 30 days after date at the Third National Bank. Mr. Barrows indorsed the note and deposited it on his account on the day of date, receiving credit for $477.20, the rate of discount being 7%. Write the note and indorse it properly.

16. Robert M. Smith borrowed $715.26 from the Security National Bank, giving his note for 100 days, without interest,

which was discounted at 7%, and indorsed by Fred Howard. Write the note and indorse it.

17. A farmer gave in payment for farm machinery a bank note for $600, due six months from date, without interest, money being worth 8%. That was equivalent to how much in cash?

18. Mr. Walsh owed $700 at the bank. When it became due, he obtained 30 days' extension of time by paying the bank discount for that time at the rate of 7%. How much did he pay to secure the extension?

19. By paying $3.50, a debtor obtained a 15 days' extension of time on a debt at a bank, which made a discount rate of 6%. How much did he owe?

Statement of Relation: Face $\times \frac{6}{100} \times \frac{15}{360} = \$3.50$.

20. What are the proceeds of a six-months note for $400, bearing interest at 5%, discounted four months after date at 6%?

21. What is the face of a non-interest-bearing note payable 90 days after date which will bring $550 if discounted 70 days after date at 6%?

22. A non-interest-bearing note, dated May 7, 1904, due in three months, was discounted at 6%, June 8, 1904, yielding $574.20. What was its face?

23. Given the amount $896.50, term of discount 45 days, rate of discount $5\frac{1}{2}$%. Find the proceeds.

24. Given the proceeds $1541.99, rate of discount 7%, time 33 days. Find the face.

25. Write a 60-day bank note without interest, which will yield enough, if discounted at 6% on the day of date, to buy 25 acres of land at $29.70 per acre.

377. PROTESTING NOTES, CHECKS, AND DRAFTS

SYRACUSE, N. Y. *Jan. 8,* 1908

SIR:

PLEASE TO TAKE NOTICE THAT A *note* MADE BY *Timothy L. Hughes* DATED *Oct. 10,* 1907, FOR *$500* AND INDORSED BY YOU, WAS THIS DAY PROTESTED FOR NON-PAYMENT, AND THAT THE HOLDERS LOOK TO YOU FOR THE PAYMENT THEREOF, PAYMENT HAVING BEEN DEMANDED AND REFUSED.

YOURS RESPECTFULLY,

F. L. BARNES,
NOTARY PUBLIC.

TO *Charles Gibbs*

If a bank note, check, or draft (see page 231) is not paid at the time specified, a notice similar to the above is sent to each of the indorsers. This is called a notice of protest, and sending it is called protesting the note, check, or draft.

If notice of protest is not sent within a reasonable time after default in payment has been made, the indorsers are released from liability for payment. Banks usually protest a note after banking hours on the day of maturity. This notice enables an indorser to protect himself and avoid needless expense. It is customary to send a notice of protest to the maker, also, though he cannot avoid liability for payment if the notice is not sent.

The notice of protest is always signed by a notary public, who is generally an officer or employee of the bank, also.

Consult your dictionary to find the meaning of *notary public.* Most notaries public are not connected with banks.

378. *Oral*

1. Can you define a notice of protest?

2. Why is a note protested, when unpaid at the time of maturity?

3. The notice given above is the one that would have been sent to Charles Gibbs, if the note on page 211 had not been paid when due. To what other persons would the notice have been sent?

4. What is a notary public?

5. Name the men who are responsible for the payment of the note mentioned above, if it is properly protested when due and unpaid?

6. Who is responsible for its payment, if not protested when due and unpaid?

7. Who is always liable for the payment of a note?

379. *Written*

1. A bank note for $450, dated April 1, 1903, payable 60 days after date, without interest, was properly protested when due, and was finally paid by one of the indorsers on the 29th of August, 1903. The indorser was obliged to pay a fee of $1.25 for protesting the note, together with interest at 7% on the note from the day of maturity. How much did he pay?

2. If the note on page 211 was paid by the maker Jan. 18, 1908, including $1.25 for protesting, how much did he pay, the legal rate of interest in New York State being 6%?

3. A bank note for $1000, without interest, became due and was protested. Six days later, the maker took up the note by giving a new note for the same sum for 30 days, with a new indorser, and paying the bank discount on the new note at 6%, interest on the old note from the day of maturity at 6%, and the charge for protesting, which was $1.75. How much did he pay?

4. The maker of a bank note, without interest, paid the note 30 days after maturity, with interest at 6% from the day of

maturity, and a charge of $1.50 for protesting. If he paid $604.50, what was the face of the note?

TAXES

380. The support of a town, village, city, county, state, or national government requires a large sum of money. This money is used for many purposes, such as carrying on the schools, keeping roads and streets in good condition, paying the salaries of public officers, constructing bridges and public buildings, and taking care of the poor and unfortunate who are unable to care for themselves. This money is used for the benefit of all the people and the protection of their lives and property. Hence all the people are required to contribute toward paying the expense, according to the value of their property.

In some places each male citizen over twenty-one years of age is required to pay a certain sum toward the expenses of his town, regardless of the value of his property.

Can you think of some expenses, other than those given above, that occur in your city, village, or town for which all the people must pay? Can you tell how the valuation of the property belonging to any person is determined? Name as many different kinds of property as you can.

381. *A* **tax** *is a sum of money levied upon persons or property for public use.*

382. *A tax levied on persons* is a **poll tax.**

383. *A tax levied on property* is a **property tax.**

384. **Personal property** *is property that is movable*, as *money, notes, furniture, books, and tools.*

385. **Real property** *is immovable property*, as *houses and lands.*

386. **Assessors** *are officers chosen to make a list of the taxable property of a city, village, or town, estimate its value, and apportion the tax.*

387. *A* **tax budget** *is a list of all the items of expense in carrying on a state, county, city, or other government for a certain time, usually one year, or in carrying on a department of such government.* From this is deducted the income (from licenses, fines, sale of privileges, etc.) and the poll tax, if any, to find the **net amount of the budget.**

388. *An* **assessment roll** *is a list of all the taxable property in a town, village, or city, with the assessed value of each piece of property.*

389. *The* **tax rate** *is the decimal which shows what part of the assessed valuation is required for taxes.* It is determined by dividing the net amount of the tax budget by the entire assessed valuation of all the property upon which the tax is levied.

The rate is generally expressed in a decimal of four, five, or six places, showing the part of a dollar taken as the tax on one dollar. Sometimes this decimal is multiplied by 1000, the product showing the number of dollars taken as the tax on $1000.

390. The following examples illustrate the different forms in which the relation of tax rate, assessed valuation, and amount of taxes appears:

1. The money to be raised by tax in a certain town is $9000. The property of the town is valued at $600,000. What is the tax rate?

Statement of Relation: —— of $600,000 = $9000. What terms of relation are given? How is the other found?

2. The tax rate of a certain county is .003 and the property is valued at $24,567,800. What is the amount of the tax budget?

Statement of Relation: .003 of $24,567,800 = ——. How is the required term of relation found?

3. When it requires a tax rate of .0132 to raise $264,000 in taxes, what is the valuation of the property taxed?

Statement of Relation: .0132 of —— = $264,000. How may the required term of relation be found?

391. *Oral*

1. The tax budget of a township is $12,000. The assessed valuation of the property in the township is $1,200,000.

a. What is the tax rate?

b. Mr. A has property in this township assessed at $25,000. What is his tax?

c. Mr. B pays $15 taxes. What is the valuation of his property?

2. A man's city taxes were $40 on property valued at $2000. What was the tax rate?

3. The school tax in a village having property to the amount of $3,000,000 was $9000.

a. What was the school tax rate?

b. What amount did a man pay whose property was assessed at $15,000?

c. Mr. Jones's school tax was $12. What was the valuation of his property?

4. The tax rate of a certain county is .0025. The tax budget is $75,000. What is the value of the property?

5. A town has 352 citizens who pay a poll tax of $1 apiece. The entire tax budget of the town is $5252.

a. How much money must be raised by tax on the property?

b. The tax rate is .007. What is the valuation of the property?

c. How much are the taxes of a man in this town, who owns property assessed at $4000, and who pays one poll tax?

6. The poll tax in a certain town is $1.50, and there are 400 citizens who pay poll tax. The property of the town is assessed at $1,000,000, and the rate is .01. What is the entire amount raised by tax?

7. A man's property is assessed at $4000. The city tax rate is .014, the county rate is .004, and the state rate, .002. The poll tax is $1.50. What is this man's entire tax?

8. If the rate for county and state taxes together is .005, what is my bill for state and county taxes on an assessment of $9000?

9. The assessed valuation of the property in a certain county is $70,000,000. The tax rate is 3 mills on a dollar. The county has an income from various sources amounting to $40,000. After collecting all the taxes and other income and paying all the expenses, $5000 remains. What are the expenses of the county?

10. What is the rate when $24 will pay the tax on property assessed at $1200?

11. What is the tax on $10,000 of property when the rate is .009345?

12. When the entire budget of a town is $35,000 and 500 men pay $1 apiece poll tax, how much must be assessed on the property?

13. When the tax on $1000 is $18.57, what is the rate per dollar of assessed valuation?

14. $30 will pay the tax on how many dollars' worth of property, when the tax rate is .015?

392. *Written*

1. **City Tax Budget for One Year**

Interest	$ 49,755.44
Comptroller	11,620
City Treasurer	18,450
Department Public Instruction (School Funds)	463,780
Library Fund	35,000
Art Museum	5,000
Department Charities and Correction	85,129
Municipal Lodging House	4,071
Veteran Relief	8,000
City Engineer	35,959
Public Buildings and Grounds	15,000
Department Public Works (General Office)	14,462
Parks and Cemeteries	47,000
Walks and Sidewalk Repair	5,000
Street Cleaning	91,542
Collecting Garbage and Ashes	86,455
Street Repairs, Sewers, and Bridges	64,120
Municipal Baths	4,000
Public Markets	3,382
Lighting Fund	114,000
Boiler Inspector	900
Department of Law	13,726
Municipal Court	11,978
Police Court	6,000
Department of Public Safety (General Office)	7,520
Police Department	162,730
Fire Department	205,080
Health Department	55,925
Department of Taxes and Assessments	19,200
Executive Department	8,400
City Clerk	9,000
Civil Service Board	2,600
Election and Primary Fund	16,000
Printing and Publishing Fund	7,500
Sealer of Weights and Measures	1,200
Common Council	16,450
Smoke Inspector	1,200
Plum Street Bridge	6,000
Other Expenses	139,879
Total	$
Less Income from Licenses, etc.	246,228
Net Total	$

From the above city tax budget,

a. Find the total expenses of the city for the year.

b. Find the net total of the tax budget.

c. Find the tax rate, correct to four places of decimals, the assessed valuation of the real property in the city being $89,000,000 and of the personal property $9,000,000.

d. Find the amount of A's city tax on $15,000 of personal property and $ 5000 of real property.

e. In this city the county and state taxes are paid together, and the rate is .00363682. What is A's county and state tax?

f. Mr. B's county and state taxes, computed by the above rate, amount to $ 65.46276. He pays $65.47. What is the valuation of his property?

g. Mr. C owns two pieces of property in this city, one valued at $600 and the other at $ 3200. What is the entire amount of his city, county, and state taxes?

2. The valuation of property in a certain town is $ 1,500,000, and the rate is $\frac{1}{5}$ %. What is the tax?

3. The tax to be raised in a certain village is $ 37,500. The valuation of the taxable property is $2,500,000.

a. What is the rate?

b. What will be A's tax on $15,000 real estate, and $ 3000 personal property?

c. What is the valuation of property on which the tax is $37.50?

4. The property of a town is assessed at $ 1,250,000. The tax to be raised is $ 15,975. There are 650 polls, assessed at $ 1.50 each. What is B's entire tax, if his property is assessed at $ 2500, and he pays the poll-tax?

5. The officers of a town find that all the town expenses for a year will amount to $ 46,000. The tax-roll shows real estate valued at $ 2,000,000, and personal property at $ 300,000. What is the tax rate?

6. The tax rate in a certain city for the year 1906 was \$16.84 per \$1000 of assessment. The city treasurer began to receive taxes October 1, and taxpayers who failed to pay before the 1st of November had a one-per-cent fee added to their tax bills. What was the tax bill of Mr. K, whose property was assessed at \$7500 and who paid his taxes on the 5th of November?

7. If the assessed valuation of a village is \$2,384,564, and there are 750 polls taxed \$1.50 each, what must be the rate of taxation to meet an expense of \$29,807.05?

8. A sewer was built in a street 980 feet long, at a cost of \$1999.20, the expense being assessed to the owners of property on each side of the street, according to the number of feet of frontage they owned; that is, the number of feet their land extended along the street.

a. What was the total frontage on both sides of the street?

b. What was the rate per front foot?

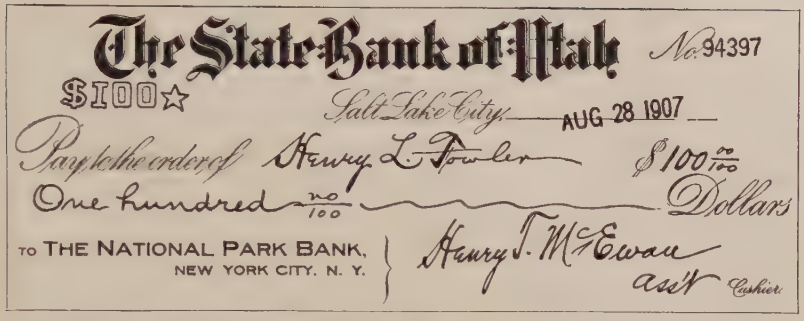
c. What was the sewer tax of Mr. M, who owned one lot 4 rods wide and another 50 feet wide?

EXCHANGE

393. *A* **draft** *is a written order for the payment of money, made in one place and payable in another.*

394. *A* **bank draft** *is an order made by a bank in one place, directing a bank in a different place, with which the drawer has funds on deposit, to pay a specified sum of money to some person, or to his order, or to the bearer.*

395. *The party who draws a draft* is the **drawer**; *the party to whom the order is addressed* is the **drawee**; *the party to whom a draft is payable* is the **payee**; *the* **face** *of a draft is the sum ordered to be paid.*


The State Bank of Utah No. 94397

$100☆ Salt Lake City, AUG 28 1907

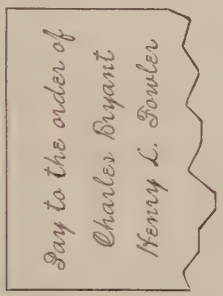
Pay to the order of Henry L. Fowler $100 00/100

One hundred no/100 Dollars

TO THE NATIONAL PARK BANK, NEW YORK CITY, N. Y.

Henry T. McEwan
Asst Cashier

A Bank Draft


Indorsement

Pay to the order of
Charles Bryant
Henry L. Fowler

In the draft given above, the drawer is the State Bank of Utah, of which Henry T. McEwan is assistant cashier; the drawee is the National Park Bank of New York, and the payee is Henry L. Fowler. The face of the draft is $100.

Observe that a bank draft is like an ordinary check, except that both the drawer and the drawee are banks, and that their places of business are in different cities or villages. A bank draft is sometimes called a *bank check*, because, like an ordinary check, it is an order drawn by one party upon another party, with whom the first party has funds deposited.

396. By means of drafts, payments may be made between different places without actually sending the money. The method of making such payments is as follows:

Let us suppose that Henry L. Fowler, in Salt Lake City, desires to send to Charles Bryant, at Portland, Me., $100. He goes to the State Bank of Utah, in Salt Lake City, and says to the teller or other person who waits upon him, "I wish to buy a New York draft for $100, payable to the order of Henry

L. Fowler." (Some banks require the purchaser of a draft to fill out a slip with the name of the payee and the amount of the draft.) The teller then fills out and hands to Mr. Fowler the draft (page 227), for which Mr. Fowler pays $100 plus a small fee to pay the bank for its services. This fee is called the **exchange.** The exchange is sometimes computed at a certain per cent of the face of the draft. It seldom exceeds $\frac{1}{4}$ %.

Banks often sell drafts to their *depositors* and *customers* with no charge for exchange.

Mr. Fowler indorses the draft as indicated above, incloses it with a letter, and mails it to Mr. Bryant, who takes it to a bank in Portland, indorses it in blank, and receives $100 for it. The transaction is complete so far as Mr. Fowler and Mr. Bryant are concerned.

Let us now study the transaction between the banks. Every bank of importance has money on deposit in some bank, called its **correspondent**, in one or more of the great money centers of the country.

The National Park Bank is the correspondent of the State Bank of Utah. The bank which cashes the check for Mr. Bryant in Portland, charges $100 to its correspondent in New York and sends the draft to its correspondent. The correspondent presents the draft to the National Park Bank (through the clearing-house), which pays $100 and charges the amount to the State Bank of Utah.

Each of the banks has now received and paid out $100 in cash or credit; Mr. Fowler, in Salt Lake City, has paid out $100, and Mr. Bryant, in Portland, has received $100; and yet no money has actually been transferred from one city to the other.

Whenever the State Bank of Utah cashes a New York draft, it sends the draft to its correspondent in New York and

receives credit for it, which is the same as sending the money received for drafts which it has sold.

397. In New York, and every other large city, many checks and drafts are received by one bank, payable by other banks in the city. For the sake of convenience, all these checks and drafts are sent by the different banks to one place, called the **clearing-house**, where they are classified and sent to the banks to which they should go, and balances are settled.

398. *Making payments by means of drafts or money orders* is **exchange.** It is really an *exchange of credits.*

399. *Exchange between places in the same country* is **domestic exchange.**

The exchange business of the Middle West is largely carried on through Chicago and St. Louis banks. A similar exchange business is conducted between every great money center and the surrounding section. But the great exchange center of the United States is New York, which is sometimes called the country's clearing-house.

400. It sometimes happens that banks in one city have large sums on deposit with banks in another city, and need currency for immediate use. They may then sell drafts **at a discount** from their face value in order to get the money at once. When the balance is against them, they may sell drafts **at a premium**, which is a certain per cent above their face value.

401. Personal checks are used, like drafts, in making payments at a distance, and a small fee for collection is charged by the banks.

402. *Oral and Written*

1. Mr. William Harris, in South Bend, Ind., desires to send $200 to his nephew Arthur Otis, who is in college in New

Haven, Conn. How much will a New York draft for that sum cost, if the exchange is $\frac{1}{10}\%$?

2. The banks making the above exchange are the Farmers' Bank of South Bend and its correspondent, the Marine Bank of New York, the Exchange National Bank of New Haven and its correspondent, the Industrial Bank of New York. Describe the entire transaction.

3. Write the draft, and indorse it properly.

4. Minneapolis banks have large balances in New York banks. Therefore they are selling New York drafts at $\frac{1}{20}\%$ discount.

a. What is the cost in Minneapolis of a New York draft for $800?

HINT. — $800 − $\frac{1}{20}\%$ of $800 = ?

b. Write the draft in question *a*, the parties being James B. Weaver, the Produce Exchange Bank of New York, and the Minnehaha National Bank of Minneapolis.

c. Charles O. Richards of Minneapolis has collected $3938.03 for John Howe & Co. of Scranton, Pa. Write the New York draft that he can purchase with that sum at the Minnehaha National Bank.

Statement of Relation: $99\frac{1}{20}\%$ of —— = $3938.03.

5. Milwaukee banks have small balances in New York banks. They are selling New York exchange at $\frac{1}{4}\%$ premium.

a. What is the exchange on a New York draft for $7500?

b. The exchange on a draft sold to Cyrus Johnson by the Northeastern Bank of Milwaukee was $20.50. What was the face of the draft?

Statement of Relation: $\frac{1}{4}\%$ of —— = $20.50.

c. Write the draft in question *b*, making the Traders' Bank of New York the drawee.

6. What is the rate of exchange when a draft for $7500 costs $7505?

Statement of Relation: ——% of $7500 = $5.

7. The discount on a draft for $8400 is $7. What is the rate of discount?

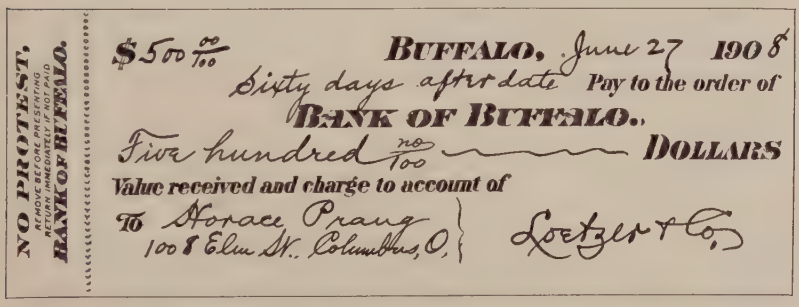
Statement of Relation: ——% of $8400 = $7.

8. When money was scarce in San Francisco, and large balances were held in Chicago, a man in San Francisco bought a Chicago draft of $12,800, paying $12,784 for it. At what rate of discount did he buy the draft?

COMMERCIAL DRAFTS

403. Drafts are frequently used as a means of collecting bills. For example, Horace Prang of Columbus, O., owes Loetzer & Co. of Buffalo, an account of $500, payable Aug. 26, 1908. Loetzer & Co. make out the following:

Time Draft


NO PROTEST.
REMOVE BEFORE PRESENTING
RETURN IMMEDIATELY IF NOT PAID
BANK OF BUFFALO.
$500 00/100
BUFFALO, June 27 1908
Sixty days after date Pay to the order of
BANK OF BUFFALO.
Five hundred no/100 DOLLARS
Value received and charge to account of
To Horace Prang
1008 Elm St. Columbus, O.
Loetzer & Co.

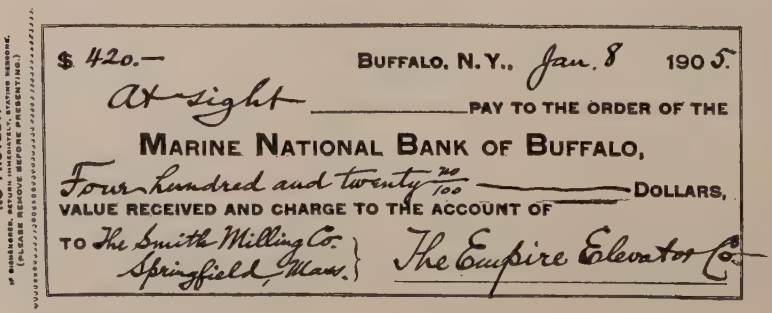
Loetzer & Co. deposit this draft in the Bank of Buffalo, which sends it to some bank in Columbus. This bank presents the draft to Horace Prang, who, if he is willing, writes in red ink across its face, "Accepted, July 1, 1908" (if that is the

day on which it is presented) and signs his name. The draft is now equivalent to Mr. Prang's bank note, payable Aug. 26, indorsed by Loetzer & Co. It is returned to the Bank of Buffalo, which will discount it at once, if Loetzer & Co. are customers in good standing, and credit them with the proceeds, less a small fee for collection.

If the draft were an order to pay "sixty days after *sight*," and accepted by Mr. Prang, he would be entitled to sixty days, after its presentation and acceptance, before paying it. If not paid then, it would be protested, like a bank note.

404. Shippers often use drafts as a means of collecting payment for goods on delivery, or of securing promise of payment at a specified time.

Suppose the Empire Elevator Company of Buffalo is sending a carload of corn, containing 700 bushels, billed at 60 cents a bushel, to the Smith Milling Company of Springfield, Mass. The Elevator Company receives a bill of lading from the railroad company, which the Smith Milling Company must have before it can get permission to take the corn from the car at Springfield. The Elevator Company takes this bill and deposits it in the Marine National Bank of Buffalo together with the following draft:


$ 420.—
BUFFALO, N. Y., Jan. 8 1905.
At sight
PAY TO THE ORDER OF THE
MARINE NATIONAL BANK OF BUFFALO,
Four hundred and twenty 20/100
DOLLARS,
VALUE RECEIVED AND CHARGE TO THE ACCOUNT OF
TO The Smith Milling Co. Springfield, Mass.
The Empire Elevator Co.
NO PROTEST.

The Marine National Bank of Buffalo sends the draft and bill of lading to a bank in Springfield, which presents it to the Milling Company for payment. If the Milling Company pays the draft, it receives the bill of lading, which entitles it to take the corn from the car. The Springfield bank remits the amount, by draft or otherwise, to the Marine National Bank of Buffalo, which credits it to the Empire Elevator Company, less the cost of collection.

If the Smith Milling Company refuses to pay the draft, the Springfield bank notifies the Marine National Bank of Buffalo, which notifies the Empire Elevator Company. Then the Elevator Company must arrange to have the corn returned or disposed of in some other way.

405. *When the drawee has accepted a draft, he* is called the **acceptor** *and the draft* is called an **acceptance.**

406. When a draft is drawn payable a certain number of days or months *after sight*, it is necessary to have a date in the acceptance so as to determine the day of maturity of the draft.

A draft drawn payable *after date* may be properly accepted by the mere signature of the drawee across its face, though the date is also desirable.

407. *A draft payable at sight* (*i.e.* at the time of presentation) is called a **sight draft** ; *a draft payable at a specified time after sight* or *after date* is called a **time draft.**

408. A sight draft may be accepted *payable a certain time after date of acceptance.* It then has the force of a *note* and may be discounted like a time draft or bank note.

409. The *discount* on a time draft, and the *cost of collection*, called the *exchange*, of *any* draft are computed on the *face of the draft.*

410. *The face of a draft less the exchange and the discount* (on a time draft) is called the **net proceeds.**

NOTE. — In states where grace is allowed by law, add three days to the time in finding the maturity and term of discount.

411. *Written*

1. What was the exchange for collecting the draft on the Smith Milling Company, page 232, at $\frac{1}{12}\%$?

2. *a.* Compute the discount on the draft accepted by Horace Prang, page 231, from the day of acceptance to the day of maturity at 6%.

b. If the charge for collection was $\frac{1}{10}\%$, what sum was credited to the account of Loetzer & Co., by the Bank of Buffalo?

3. What is the cost of collecting the following draft at $\frac{1}{6}\%$?

Cleveland, O. Apr. 14, 1904 $1200 $\frac{00}{100}$

On Demand pay to the order of

The Buckeye National Bank

Twelve hundred $\frac{no}{100}$ Dollars

Value received and charge to the account of

To R. W. Slocum
1305 Huron St.
Lansing
Mich

Sylvester Field & Co.

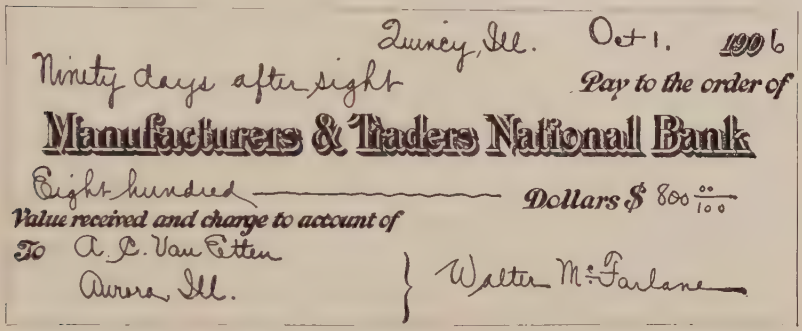
4. William H. Warner of Burlington, Ia., draws on H. H. Franklin of Dubuque, for $1800, at 60 days sight, through the National Bank of Burlington.

a. Write the draft.

b. Compute the discount, the draft having been accepted, and discounted at 6% on the day of acceptance.

c. If the exchange for collection was $\frac{1}{12}\%$, what were the net proceeds of the draft?

5. Find the net proceeds of the following draft on the day of acceptance, computing discount at 5% per annum and exchange at $\frac{1}{8}\%$.


Quincy, Ill. Oct 1. 1906

Ninety days after sight Pay to the order of

Manufacturers & Traders National Bank

Eight hundred ——— Dollars $ 800 00/100

Value received and charge to account of

To A. L. Van Etten
Aurora, Ill.

Walter McFarlane

6. Find the net proceeds of a draft for $1440 payable 60 days after date, the rate of discount being 7% per annum, the day of discount thirty days after date, and the exchange $\frac{1}{8}\%$.

7. The net proceeds of a 60-day draft discounted at 6% on the day of date, exchange at $\frac{1}{8}\%$, were $1977.50. What was the face of the draft?

EXCHANGE BY POSTAL MONEY ORDER

The Post Office Department offers a convenient method of exchange, for small amounts, in the form of *postal money orders.*

412. A **postal money order** is *a written agreement, signed by the postmaster of a certain post office, that the postmaster of another post office will pay a specified sum of money to the person named in the order.*

The following form shows the essential parts of a

Postal Money Order

[Name of office issuing the order] NO.

DATE

THE POSTMASTER AT [Name of office on which order is drawn]

WILL PAY THE SUM OF DOLLARS | CENTS

P DOLLARS P CENTS

words for dollars figures for cents

TO THE ORDER OF [Name of person to whom order is payable]

[Signature of] POSTMASTER

NAME OF REMITTER

413. These orders may be purchased at any **money order post office.** All except the smaller village and rural post offices are money order offices.

The purchaser (called "the remitter," in the order) incloses the order in an envelope, and mails it to the payee named in the order. The payee takes it to the post office named in the order, where he receives in cash the face value of the order.

All money order offices sell postal money orders (for amounts not exceeding $100), payable at money order offices in this country or in foreign countries.

The following table shows the fees that must be paid, in addition to the face, for postal money orders payable in the United States:

Face of Order		Fee
$2.50 or less		3¢
Over 2.50 and not exceeding	$5.00	5¢
Over 5.00 and not exceeding	10.00	8¢
Over 10.00 and not exceeding	20.00	10¢
Over 20.00 and not exceeding	30.00	12¢
Over 30.00 and not exceeding	40.00	15¢
Over 40.00 and not exceeding	50.00	18¢
Over 50.00 and not exceeding	60.00	20¢
Over 60.00 and not exceeding	75.00	25¢
Over 75.00 and not exceeding	100.00	30¢

Fees for foreign orders are about three times as great as for domestic, ranging from 10¢ to $1.00.

414. *Oral*

Using the table of rates on page 236,

1. Find the total cost of a postal money order for—

a. $3.00	*d.* $43.25	*g.* $86.31	*j.* $28.98
b. $4.28	*e.* $89.41	*h.* $72.05	*k.* $90.89
c. $1.75	*f.* $99.99	*i.* $50.10	*l.* $88.95

2. Find the cost of two postal money orders which, together, will pay a bill of $137.55 at Wanamaker's store in Philadelphia.

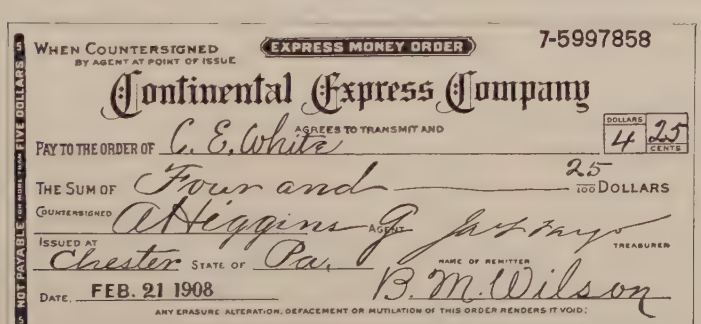
3. Find the cost of postal money orders sufficient to pay a bill of $500.

Make and solve other problems.

EXCHANGE BY EXPRESS MONEY ORDER

415. An **express money order** is *a written agreement by an express company to pay to the order of a person named in the order a specified sum of money.*

The following is the usual form:


WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE
EXPRESS MONEY ORDER
7-5997858
Continental Express Company
AGREES TO TRANSMIT AND
PAY TO THE ORDER OF C. E. White
DOLLARS 4 CENTS 25
THE SUM OF Four and 25/100 DOLLARS
COUNTERSIGNED A Higgins
AGENT
J. [illegible]
TREASURER
ISSUED AT Chester STATE OF Pa.
NAME OF REMITTER B. M. Wilson
DATE FEB. 21 1908
ANY ERASURE, ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID.
NOT PAYABLE FOR MORE THAN FIVE DOLLARS

The fee is the same as that for issuing a postal money order for the same amount. It is called the **exchange** for issuing the order.

416. *An express money order is* **negotiable** *and can be transferred by indorsement, like a check or bank draft.*

417. An express money order, issued by any express company, will be cashed for its full face value at any of the company's offices in this country, or by any other express company, or by any bank.

418. *Oral*

1. Name two similarities between the method of exchange by express money order and that by postal money order.

2. Name two differences.

3. What must be paid in Latrobe, Pa., for an express money order large enough to pay a bill of $27.27 in Los Angeles, Cal.?

Make and solve other problems.

EXCHANGE BY TELEGRAPH MONEY ORDER

419. Exchange by **telegraph money order** is more expensive than that by express or postal money order. It is used only in cases of emergency, when credits must be transmitted without loss of time, so that money paid in one place may be instantly available in another place at some distance from the first.

The method by which this form of exchange is made is as follows:

The person desiring to remit money goes to a telegraph office and pays the money to the person who attends to that branch of the business. A message is then sent, directing the telegraph office, at the place where the money is wanted, to pay the amount to the person designated. Before receiv-

ing the money, that person is required to satisfy the representatives of the telegraph company, by identification or otherwise, that he is the person to whom the money is directed to be paid.

420. The present rate for telegraph money orders is *twice the cost of a ten-word message, plus one per cent of the amount of the order. If the amount of the order is less than* \$25, *the fee is the same as if the order were for* \$25.

421. *Oral*

1. What is the cost in Syracuse, N.Y., of a telegraph money order for \$75, payable in Atlanta, Ga., the cost of a ten-word message being 60¢?

2. What is the cost in Scranton, Pa., of a telegraph money order for \$50, payable in San Francisco, the rate for a ten-word message being \$1.00?

3. What is the cost in Utica of a telegraph money order for \$100, payable in Harrisburg, Pa., the rate for a ten-word message being 40¢?

Make and solve other problems.

FOREIGN EXCHANGE

422. Exchange between places in different countries is **foreign exchange.**

423. The principal gold coin of Great Britain is the **sovereign,** equal to £1. It is equivalent to \$4.8665.

Which of our coins is most nearly like the sovereign?

The **shilling** is a silver coin equal to $\frac{1}{20}$ of a pound, ster-

ling. It is equivalent to about how many cents? It closely resembles what American coin?

The English **penny** is $\frac{1}{12}$ of a shilling. It is equivalent to about how many cents in our money?

424. The principal gold coin of France is the 20-franc piece, nearly equivalent to $4 of our money.

The **franc** is a silver coin equivalent to $.193 of our money. What American coin is most nearly like the franc? The franc is also the principal coin of Belgium and Switzerland.

The **lira** (plural *lire*) of Italy has the same value, and is exchanged evenly for the franc.

A dollar is equivalent to about how many francs? How many lire? How many shillings?

425. The principal coin of Germany is the **mark.** It is a silver coin equivalent to $.238. What American coin is most nearly like the mark? One dollar is equivalent to about how many marks? Four marks are equivalent to how many cents?

426. Foreign drafts, called **bills of exchange,** are always expressed in the money of the country in which they are

payable. Sterling bills (drafts on Great Britain and Ireland) are expressed in pounds, shillings, and pence; drafts on France, Belgium, and Switzerland in francs; on Italy, in lire; and on Germany, in marks (reichmarks).

427. Foreign drafts are usually issued in sets of two, known as the *first* and *second* of exchange. When either of them is paid, the other becomes void.

Set of Exchange

Exchange for £ 250

New York, Feb. 26, 1907

At sight of this first of exchange (second unpaid)
pay to the order of Robert MacDonald

Two hundred fifty Pounds Sterling

Value received, and charge to the account of

To Lombard & Co.
No. 249 London, Eng.

Brown Brothers & Co.

The second draft of the set is like the first, except the interchange of the words *first* and *second*.

FOREIGN EXCHANGE QUOTATIONS

428. The values given on pages 239 and 240 for various coins are the exact equivalents of those coins, in our money. This is called the **intrinsic par** of exchange. The **exchange values** of those denominations, however, fluctuate from day to day, like the prices of corn, wheat, and cotton, and are quoted in the daily papers.

429. Exchange on Great Britain and Ireland is quoted at the *number of dollars* that must be paid for *one pound* of exchange; *e.g.* "Exchange on Liverpool, $4.87\frac{1}{2}$," means that a draft on Liverpool costs at the rate of $\$4.87\frac{1}{2}$ for every pound of its face. Time drafts are quoted at a lower rate than sight drafts, because the drawer, who sells the draft, has the use of the money until the draft matures. There is no discount to be computed, because that is accounted for in the quotation.

"London sight $4.86\frac{3}{4}$; 60 days, $4.85\frac{1}{2}$" means that London sight drafts are sold at $\$4.86\frac{3}{4}$ per pound of their face, while 60-day London drafts are sold at $\$4.85\frac{1}{2}$ per pound of their face.

Exchange on France, Belgium, and Switzerland is quoted at the *number of francs* of exchange that may be bought for *one dollar*; *e.g.* "Exchange on Brussels, $5.17\frac{1}{2}$" means that one dollar must be paid for every $5.17\frac{1}{2}$ francs of the face of the draft.

Exchange on Italy is quoted at the *number of lire* of exchange that can be bought for one dollar.

Exchange on Germany is quoted at the *number of cents* that must be paid for *four marks* of the face of the draft; *e.g.* "Exchange on Hamburg, $96\frac{1}{8}$," means that $\$.96\frac{1}{8}$ must be paid for every four marks of the face of the draft.

The cost of **cable transfers**, or *telegraph money orders* to foreign countries, is computed by finding the exchange value of the order according to the quotation, and adding thereto a certain per cent, usually from $\frac{1}{4}\%$ to 1%, plus the cost of the cable message.

Post remittances, or *foreign postal money orders*, are sold at their exchange value, plus a certain fee for issuing and paying the order.

Thus it will be seen that no general rule can be given for

computation of foreign exchange. The first step in every problem, however, is to determine what the quotation means, according to the explanation given above. When that is determined and clearly fixed in the mind, the student's experience in applying principles to given conditions so as to secure correct results should enable him to solve the problem.

430. *Oral*

1. *Give the meaning of each of the following quotations of exchange:*

a. London 4.87$\frac{1}{4}$
b. Edinburgh 4.86$\frac{3}{4}$
c. London { sight 4.87 / 60 days 4.84$\frac{1}{2}$
d. Paris 5.16$\frac{1}{4}$
e. Geneva 5.18
f. Milan 5.17
g. Antwerp 5.17
h. Berlin 95$\frac{1}{8}$
i. Glasgow 4.87$\frac{1}{2}$
j. Dresden 96
k. Rome 5.18$\frac{1}{4}$
l. Munich 95$\frac{1}{2}$
m. Manchester 4.86$\frac{3}{4}$
n. Frankfort 97
o. Zurich 5.16$\frac{1}{2}$
p. Berne 5.17$\frac{1}{2}$
q. Belfast 4.87$\frac{3}{4}$
r. Florence 5.14$\frac{3}{8}$
s. Bremen 87$\frac{3}{8}$
t. Liverpool { sight 4.86 / 60 days 4.83
u. Brussels 5.19
v. Hamburg 96$\frac{7}{8}$

2. When four marks cost 96 cents, what is the cost of 1 mark? Of 100 marks?

3. What must be paid for a draft for £100, when one pound of exchange costs $4.87?

4. When exchange on Paris is at the rate of 5.19 francs for $1, what is the face of a draft that may be bought for $10?

5. When exchange on France is quoted at 5.20, how may we find the cost of one franc?

6. What is the cost of a draft for 519 francs when 5.19 francs of exchange may be bought for $1?

7. When a draft on Liverpool for £1000 costs $4875, what is the rate of exchange? (What will £1 cost?)

8. What is the rate of exchange when a draft for 100 marks costs $24? (What will 4 marks cost?)

9. What is the rate of exchange when a draft for 1038 francs costs $200? (How many francs will $1 buy?)

10. In what denomination are drafts on England expressed? On France? On Scotland? On Belgium? On Germany? On Italy? On Switzerland? On the United States?

11. What is the rate of exchange when $980 will buy a draft for £200?

12. At what rate of exchange will $50 buy a draft for 258 francs?

13. At what rate of exchange will a draft on Rome for 1040 lire cost $200?

14. When the rate of exchange on Germany is 98, what is the cost per mark of a draft on Cologne?

15. What is the cost per franc of a draft on Brussels, when the rate of exchange is 5.00?

431. *Written*

1. *a.* When exchange on London is quoted at 4.87, what is the cost of a London draft for £250?

b. What is the face of a London draft that can be bought for $4383?

2. *a.* What is the cost of a draft on Paris for 2069.5 francs when exchange is quoted at $5.17\frac{3}{4}$?

b. What is the face of a draft on Berne that can be bought for $689 when the rate of exchange is 5.18?

3. A Liverpool draft for £850 cost $4160.75. What was the rate of exchange?

4. When \$240 will buy a draft for 1242.60 francs, how many francs of exchange will \$1 buy? What is the rate of exchange?

5. When the quotation for London exchange is "sight $4.87\frac{1}{2}$; 30 da. 4.86," what is the difference between the cost of a sight draft for £470 and a 30-day draft for the same amount?

6. What is the cost of a draft on Berlin for 948 marks when exchange is quoted at 98?

4 marks cost \$.98

1 mark cost $\frac{\$.98}{4}$

948 marks cost $\frac{\$.98}{4} \times \frac{948}{1}$ (cancel)

7. A bill of exchange on Frankfort for 8000 marks costs how much when the rate of exchange is $.95\frac{5}{8}$?

8. What is the face of a bill of exchange on Bremen that can be bought for \$2031.75 when the rate is $96\frac{3}{4}$?

9. Brown Brothers & Co., bankers, sent out to their correspondents, Oct. 14, 1907, the following list of quotations:

NO. 447. BROWN BROTHERS & CO. NEW YORK, Oct. 14, 1907

ENGLAND, SCOTLAND & IRELAND	\$4.87	per Pound Sterling
LONDON & LIVERPOOL ONLY	$4.86\frac{1}{4}$	
FRANCE, BELGIUM & SWITZERLAND	5.15	Francs per Dollar
PARIS ONLY	$5.16\frac{1}{4}$	
ANTWERP & BRUSSELS ONLY	5.17	
ZURICH, ST. GALL, BASLE, BERNE & GENEVA ONLY	$5.16\frac{1}{4}$	
ITALY	$5.15\frac{5}{8}$	Lire per Dollar
GENOA, MILAN, NAPLES, FLORENCE, LUCCA & ROME ONLY	$5.14\frac{7}{8}$	
GERMANY	$.95\frac{1}{8}$	Cts. per Four Marks
BERLIN, BREMEN, CHEMNITZ, DRESDEN, FRANKFORT, HAMBURG, HANOVER, MANNHEIM, MUNICH & NUREMBERG ONLY	.95	
HOLLAND	$.40\frac{42}{}$	Cts. per Guilder
AMSTERDAM ONLY	$.40\frac{36}{}$	

AUSTRIA & HUNGARY	.20$\frac{30}{}$	Cts. per Krone
VIENNA, BUDAPEST, AUSSIG, PRAGUE, TEPLITZ & TRIESTE ONLY	.20$\frac{25}{}$	
NORWAY, SWEDEN, DENMARK & ICELAND	.26$\frac{70}{}$	Cts. per Kronor
CHRISTIANIA, COPENHAGEN, GOTEBORG & STOCKHOLM ONLY	.26$\frac{65}{}$	
FINLAND	.19$\frac{45}{}$	Cts. per Finmark
HELSINGFORS & WIBORG ONLY	.19$\frac{40}{}$	
RUSSIA	.51$\frac{3}{4}$	Cts. per Ruble
ST. PETERSBURG, MOSCOW, ODESSA, BAKU, CHARKOFF & KIEF ONLY	.51$\frac{1}{8}$	

POST REMITTANCES at the ordinary rate plus 15c. per payment. CABLE TRANSFERS $\frac{1}{4}$% higher plus cost of message.

By these quotations, find:

a. The cost of a set of exchange on Antwerp for 13,442 francs.

b. The face of a London bill that can be purchased for $1459.50.

c. The cost of a bill of exchange on Dublin for £420 10*s.*

d. The cost of a set of exchange on Glasgow for £200 10*s.* 6*d.*

e. The face of a Berlin draft that can be bought for $190.

f. The cost of a set of exchange on Rome for 20,575 lire.

g. The cost of a cable transfer of 20,650 francs to Paris, computing exchange at the quoted rate, then adding $\frac{1}{4}$% plus the cost of the cable message, which was $5.04.

h. The face of a London bill that can be bought for $2189.25.

i. The face of a bill of exchange on Geneva that can be bought for $200.

j. The cost of a Dresden bill of exchange for 896 marks.

10. A man has saved $400, and desires to send it to his family in Naples. What is the face of the draft which he can buy with that sum, by the above quotation?

11. Make and solve problems of exchange on other countries named in the list of quotations.

THE METRIC SYSTEM

The metric system of weights and measures is a decimal system which originated in France a little more than one hundred years ago. It is the legal system in most of the civilized world except Great Britain and the United States.

In our own country, it is used in the sciences and in some branches of the government business.

Being a decimal system, it is much simpler than the English system which we use; for all reductions from one denomination to another may be made simply by moving the decimal point.

LINEAR MEASURE

432. The standard unit of linear measure in the metric system is the **meter.** It is determined by taking one ten-millionth part (very nearly) of the distance from the earth's equator to either of its poles, measured on a meridian. It is equal to 39.37 inches.

433. *Oral*

1. What denomination in the English linear measure is most nearly like the meter?

2. Draw a line one meter long.

3. Hold your hands one meter apart.

4. A meter is about how many feet long?

5. How many meters long is your schoolroom? Wide? High?

6. About how many meters are there in a rod?

7. About how many meters long is a rifle-range whose length is 500 yd.?

8. Your height is about how many meters?

9. How many meters high can you reach on the blackboard?

How the Table is made

434. Divide a meter into ten equal parts. One of these parts is a **decimeter.** *Dec* is a Latin prefix meaning *tenth.* About how many inches long is a decimeter? Show with your hands the length of a decimeter. What part of a meter is a decimeter?

Divide a decimeter into ten equal parts. One of these parts is a **centimeter.** *Cent* is a Latin prefix meaning *hundredth.* What part of an inch is a centimeter? Show its length. How many centimeters in one meter? What part of a meter is a centimeter?

Divide a centimeter into ten equal parts. One of these parts is a **millimeter.** *Mill* is a Latin prefix meaning *thousandth.* What part of a meter is a millimeter? How many millimeters in a meter? What part of an inch is a millimeter?

Ten meters make one **dekameter.** *Deka* is a Greek prefix meaning *ten.* How many rods in a dekameter? How many feet? How many dekameters long is your schoolroom?

Ten dekameters make one **hektometer.** *Hekto* is a Greek prefix meaning *hundred.* How many meters in one hektometer? How many feet long is a hektometer?

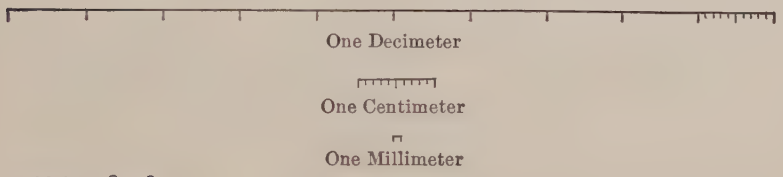
Ten hektometers make one **kilometer.** *Kilo* is a Greek prefix meaning *thousand.* How many meters in one kilometer? How many feet? What part of a mile?

Ten kilometers make one **myriameter.** *Myria* is a Greek prefix meaning *ten thousand.* How many meters in one myriameter? How many feet? How many miles?

These statements may be combined in the following:

Table of Linear Measure

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 kilometer (Km.)
10 kilometers	= 1 myriameter (Mm.)



One Decimeter

One Centimeter

One Millimeter

435. *Oral*

Read the following expressions as meters; thus, seventy *thousand meters*, fifteen *thousand meters*, six *hundred meters*, eighty *meters*, one hundred fifty-two *thousandths meters:*

1. 7 Mm.	**9.** 34 m.	**17.** 5 Dm.	**25.** 6 dm.
2. 15 Km.	**10.** 7 cm.	**18.** 61 Km.	**26.** 47 mm.
3. 6 Hm.	**11.** 69 Hm.	**19.** 384 mm.	**27.** 523 Km.
4. 8 Dm.	**12.** 46 Dm.	**20.** 7856 m.	**28.** 368 Dm.
5. 483 m.	**13.** 931 Km.	**21.** 35 cm.	**29.** 42 Mm.
6. 8 dm.	**14.** 26 Hm.	**22.** 421 mm.	**30.** 58 Km.
7. 67 cm.	**15.** 3 dm.	**23.** 89 Dm.	**31.** 284 Dm.
8. 152 mm.	**16.** 341 mm.	**24.** 58 Hm.	**32.** 700 cm.

Practice reading such expressions as the above in meters, until you can *think* in meters.

Reduction

436.

1 myriameter	=	1 millimeter	=
10 kilometers	=	.1 centimeter	=
100 hektometers	=	.01 decimeter	=
1000 dekameters	=	.001 meter	=
10000 meters	=	.0001 dekameter	=
100000 decimeters	=	.00001 hektometer	=
1000000 centimeters	=	.000001 kilometer	=
10000000 millimeters.		.0000001 myriameter.	

DESCENDING — ASCENDING

437. The following series of numbers read from the top is reduction descending; read from the bottom is reduction ascending. All metric numbers may be reduced in this way.

7.5689132 Mm. =
75.689132 Km. =
756.89132 Hm. =
7568.9132 Dm. =
75689.132 m. =
756891.32 dm. =
7568913.2 cm. =
75689132 mm.

Each of these numbers may be read thus:

Mm.	Km.	Hm.	Dm.	m.	dm.	cm.	mm.
7	5	6	8	9	1	3	2.

438. *Oral and Written*

1. How may a metric number be reduced to higher denominations? To lower denominations?

2. Reduce 12,345,678 mm. to cm.; to dm.; to m.; to Dm.; to Hm.; to Km.; to Mm.

3. Reduce 9.6538714 Mm. to Km.; to Hm.; to Dm.; to m.; to dm.; to cm.; to mm.

4. Reduce 7 Mm. to lower denominations.

5. Reduce 7 mm. to higher denominations.

6. Reduce 6307.1 m. to Km.; to cm.

7. Reduce 31 meters to inches.

8. Write as meters 2 Mm.; 7 Km.; 6 Hm.; 8 Dm.; 5 m.; 3 dm.; 2 cm.; 9 mm. Write them all as one number.

9. Reduce 1 Mm. to feet.

10. Write 7 Mm. and 6 mm. in one number, as meters. Reduce it to higher denominations; to lower denominations.

11. Reduce .075 Km. to cm.

12. Reduce 8 Dm. and 6 m. to Mm.; to mm.

13. Write 75 Km. and 62 dm. in one number as meters; as cm.; as Mm.

14. State the value of each figure in 30769.543 m.

15. A ship sails 100 Mm. in one day. How many miles does it sail?

16. Give the table of Metric Linear Measure.

17. Name the standard unit.

18. How is it determined?

19. What is the scale of the Metric system?

20. *a.* What is the distance in meters between two places if they are 94,488 feet apart?

b. What is the distance in kilometers?

21. A boy in Paris walked 12 Km. in one day. How many miles did he walk?

22. A train in Europe ran 393.7 mi. in 10 hr. That was an average of how many kilometers per hour?

SURFACE MEASURE

439. Draw a square whose side is one meter. How many square meters does it contain? It is how many decimeters on a side? How many square decimeters does it contain? How many square decimeters make one square meter?

How many centimeters long and wide is a square decimeter? How many square centimeters in one square decimeter? Find how many square millimeters in 1 sq. centimeter.

How many sq. meters = 1 sq. dekameter?
How many sq. dekameters = 1 sq. hektometer?
How many sq. hektometers = 1 sq. kilometer?

The answers to the above questions form the following table, which is used for all ordinary surface measurements:

Table of Surface Measure

100 sq. millimeters = 1 sq. centimeter (sq. cm.)
100 sq. centimeters = 1 sq. decimeter (sq. dm.)
100 sq. decimeters = 1 sq. meter (sq. m.)
100 sq. meters = 1 sq. dekameter (sq. Dm.)
100 sq. dekameters = 1 sq. hektometer (sq. Hm.)
100 sq. hektometers = 1 sq. kilometer (sq. Km.)

440. *Oral*

1. Which denomination of our measure is nearest like the square meter?

2. The square dekameter is equivalent to about how many square rods?

3. How many square centimeters in one square meter?

4. How far to the right must the decimal point be moved to reduce square meters to square decimeters?

5. How many places to the left must the decimal point be moved to reduce square meters to square dekameters?

6. To reduce sq. mm. to sq. cm.?

7. To reduce sq. mm. to sq. dm.?

8. How many places to the left must the decimal point be moved to reduce square meters to square kilometers?

441. *Written*

1. Reduce 74.5 square meters to square centimeters.

2. Reduce

a. .2408 sq. mm. to square meters.

b. .0753 sq. m. to square millimeters.

c. 984,769,302 square meters to square kilometers.

d. 24.8 sq. dm. to square centimeters.

e. 48 sq. Km. 73 sq. Dm. to square meters.

3. A table top 2.5 m. long and 95 cm. wide contains how many square meters?

4. How many square meters are there in a floor 8 m. long and 3 m. 75 cm. wide?

5. Find the cost of painting the four walls of a room 4.5 m. long, 3.2 m. wide, and 32 dm. high, at 1.4 francs per square meter.

6. Find in square meters the entire surface of a cube whose edge is 125 cm.

7. How many square meters of carpet will cover a floor 896 cm. long and 50 dm. wide?

8. A city lot is 45 m. long and contains 922.50 sq. m. of land. Find its width in centimeters.

9. At 30¢ per square meter, what will it cost to plaster the sides and ceiling of a room 5.5 m. long, 4 m. wide, and 3 m. 95 cm. high?

10. How many square decimeters of writing surface are there in a tablet containing 90 sheets of paper, each 2 dm. long and 16 cm. wide?

11. Find the area of your schoolroom floor in square meters.

12. Find in square decimeters the area of a square whose edge is 393.7 inches.

LAND MEASURE

442. The **are** (pronounced *air*) and **hectare** are the principal units of land measure.

The *are* is equal to one *square dekameter*, and the *hectare* is equal to *one hundred ares*.

443. *Oral*

1. An are is how many meters long? Wide?
2. How many square meters does the are contain?
3. An are is how many inches long? Feet?
4. The are is about how many rods long?
5. About how many square rods does it contain?
6. About how many ares equal one acre?
7. How many ares does a piece of land as large as the floor of your schoolroom contain?
8. Name all the surfaces you can think of that contain about one are.

444. *Written*

1. *a.* A field 134 m. long and 7 Dm. wide contains how many square meters of land?
   b. How many ares?
   c. How many hectares?
   d. How many square dekameters?
   e. How many square hektometers?
   f. How many square centimeters?
2. *a.* How many square centimeters in an oblong 643 cm. long and 2.5 m. wide?
   b. How many square millimeters?
   c. How many square kilometers?
3. One hectare is equal to how many acres?

VOLUME MEASURE

445. A cube whose edge is one meter long contains how many cubic meters? It is how many decimeters long? Wide? High? How many cubic decimeters does it contain? How many cubic decimeters equal one cubic meter?

A cube whose edge is one decimeter contains how many cubic decimeters? It is how many centimeters long? Wide? High? How many cubic centimeters does it contain? How many cubic centimeters equal one cubic decimeter?

A cube whose edge is one centimeter contains how many cubic centimeters? It is how many millimeters long? Wide? High? It contains how many cubic millimeters? How many cubic millimeters equal one cubic centimeter?

From the answers to the above questions make the following:

Table of Volume Measure

1000 cu. millimeters (cu.mm.) = 1 cu. centimeter (cu. cm.)
1000 cu. centimeters = 1 cu. decimeter (cu. dm.)
1000 cu. decimeters = 1 cu. meter (cu. m.)

446. The unit chiefly used in measuring wood and stone is the **stere** (pronounced *stair*), which is a cube whose edge is one meter. What denomination in the English volume measure is most nearly like the stere? How many cubic meters does the stere contain?

447. *Oral and Written*

1. How may cubic millimeters be reduced to cubic centimeters? To cubic decimeters? To cubic meters?

2. How many places to the right must the decimal point be moved to reduce cubic meters to cubic millimeters?

3. Reduce 7 cubic meters to cubic millimeters.

4. Reduce 5 cubic millimeters to cubic meters.

5. How many steres in one cubic meter?

6. A pile of wood is 30 dm. long, 3 m. wide, and 18 dm. high. *a.* How many cubic meters does it contain?
b. How many steres?
c. How many cubic millimeters?

7. *a.* How many cubic centimeters of air in an empty box 2 m. by 12 dm. by 75 cm.?
b. How many cubic decimeters?

8. How many steres of stone in a wall 30 m. long, 5 dm. thick, and 250 cm. high?

CAPACITY MEASURE

448. The metric capacity measure takes the place of both the liquid and the dry measure of the English system.

The standard unit of capacity measure is the **liter** (pronounced *leeter*), which is a cube whose edge is one decimeter.

449. *Oral and Written*

1. The liter is what part of a meter wide? High? Long?

2. What part of a cubic meter does it contain?

3. About how many inches wide is it? High? Long? About how many cubic inches does it contain?

4. Show with your hands how wide, high, and long a liter is.

5. What denomination of English dry measure corresponds most nearly to the liter?

6. Make a full-sized picture of a liter.

7. What object the size of a liter do you know?

Table of Capacity Measure

450. The table of capacity measure is formed similarly to the other metric tables, and is as follows:

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 liter (l.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 hektoliter (Hl.)
10 hektoliters = 1 kiloliter (Kl.)
10 kiloliters = 1 myrialiter (Ml.)

451. *Oral and Written*

1. How many liters in 1 myrialiter? In 1 milliliter?

2. How many milliliters in 1 myrialiter?

3. Reduce 12,345,678 ml. to higher denominations.

4. Read the number in example 3, giving each figure the name of the denomination it represents.

5. Reduce 154.67 cl. to kiloliters.

6. Reduce .012346 Ml. to deciliters.

7. How many liters equal one cubic meter?

8. A bin is 2.5 m. wide, 6.4 m. long, and 17 dm. deep. How many liters of oats will it hold? How many hektoliters? How many kiloliters?

9. A tank is 3 m. long and 3 m. wide. How many decimeters deep must it be to hold 50 Hl. of water?

10. A stone whose volume is 1 stere, if dropped into a pond, would displace how many liters of water?

MEASURES OF WEIGHT

452. The **gram** is the unit of weight. It is equal to the weight of a cubic centimeter of distilled water at its greatest density. One gram equals 15.432 grains.

Table of Weight

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 kilogram (Kg.)
10 kilograms	= 1 myriagram (Mg.)
10 myriagrams	= 1 quintal (Q.)
10 quintals	= 1 tonneau, or metric ton (T.)

453. *Oral and Written*

1. How many grams in 1 metric ton?
2. How many myriagrams in 1 metric ton?
3. Reduce 1 mg. to metric tons.
4. Reduce 1 T. to milligrams.
5. Reduce 9,876,543,215 mg. to higher denominations.
6. Read the number in example 5, giving each figure the name of the denomination it represents.
7. Recite the table of weight.
8. Spell the name of each denomination.
9. Reduce 7.42 quintals to centigrams.
10. Reduce 543 mg. to myriagrams.
11. How many grains in 1 Kg.?

12. One pound Avoirdupois contains 7000 gr. How many pounds are equivalent to one kilogram?

13. Mr. Smith weighs 100 Kg. How many pounds does he weigh?

14. How many grams does a cubic meter of distilled water weigh?

15. Would a cubic meter of any other substance weigh the same as a cubic meter of distilled water? State your reason.

16. How many kilograms of water will a tank 4 m. $\times$ 3 m. $\times$ 12 dm. hold?

REVIEW QUESTIONS

454. 1. How many tables are there in the Metric System?

2. Name the standard units in the order in which they have been given. Repeat them until you can say them as rapidly as you can talk.

3. Name the prefixes in the same way.

4. Name and describe the unit of capacity measure; of weight; of length; of volume; of surface.

5. Repeat the tables.

6. The stere is the unit of what measure? The meter? The are? The gram? The liter?

7. How can metric numbers be reduced to higher denominations? To lower?

8. How many things are to be committed to memory in the Metric System?

9. What is 39.37? 15.432? 10? These are the only numbers that need be remembered.

DUTIES

455. Under the head of *taxes*, page 220, we discussed the methods of raising money for the support of city, village, township, county, and state governments. These are chiefly methods of *direct taxation;* that is, the taxes are paid directly by all owners of property and are apportioned according to the assessed valuation of the property.

The expenses of the *national* government are great. Vast sums of money are required for the support of the army and navy, payment of pensions to veteran soldiers and sailors, payment of the salaries of the President, Vice-President, senators, representatives, and other officers and employees of the government, building of post offices and other public buildings, improvement of rivers and harbors, keeping of lighthouses and life-saving stations, and for many other purposes. Name other expenses of the national government.

The expenses of the post office department are largely paid by the sale of postage stamps. This is a tax upon the persons buying the stamps, but they receive an immediate and direct return by having their mail carried. There are two other means by which most of the money for government use is obtained; namely,

a. By internal revenue taxes.

b. By duties or customs.

456. **Internal revenue taxes** are taxes levied on certain articles made in this country, chiefly spirits and tobacco products. It is unlawful to sell these articles before the internal revenue tax upon them has been paid, and persons who break the law may be punished by fine or imprisonment.

457. **Duties** or **customs** are taxes levied on certain articles imported into the country from foreign lands.

Most articles, other than those subject to internal revenue taxes, may be produced or manufactured *in this country* with entire freedom and without taxation; but there are many things, both manufactured articles and "raw materials," that cannot be brought into the country without having taxes levied upon them and collected by the government. These taxes, called *duties* or *customs*, are collected at custom houses, located at cities and towns called **ports of entry.** The ports of entry are situated not only along the seacoast and other boundaries of the country, but also along the great river and railroad routes. Can you name some cities that are ports of entry?

458. *Articles on which duty must be paid* are called **dutiable articles.** It is unlawful for dutiable articles to be brought into the country at any other place than a port of entry.

459. *A list of dutiable articles and the rates of duty to be paid upon them* is called a **tariff.** The tariff of the United States is fixed by Congress.

The importer of foreign goods must pay the duty on goods which he imports. Therefore, when he sells the goods, he must ask a price sufficient to cover the cost, the duty paid, and his profit; so that the person who finally buys the goods for his own use really pays the duty upon them. The duty or custom is therefore said to be an *indirect tax* upon the purchaser or consumer.

460. *Duty computed at a certain per cent of the cost of the goods in the country from which they were shipped* is **ad valorem duty**; *e.g.* the duty on \$10,000 worth of laces at 60% *ad valorem* is \$6000.

461. *Duty computed according to the quantity of goods imported* is **specific duty**; *e.g.* the duty on 10,000 lb. of currants at 2 cents *per pound* is \$200.

Some articles are subject to both an ad valorem and a specific duty; *e.g.* the duty on cotton wicking is 15% *ad valorem* and 10 cents *per pound.*

462. **Tare** is an allowance made for the weight of boxes or cases in which goods are packed for shipment.

463. **Leakage** and **breakage** are allowances for loss of liquids shipped in barrels, casks, and bottles.

464. In computing ad valorem duty, take the net foreign invoice valuation (value of the goods in the money of the country from which they were shipped, less all discounts), find its exchange value in United States money, and find the required per cent of that sum. If the valuation contains a fraction of a dollar equal to, or greater than, fifty cents, call it another dollar; if less than fifty cents, omit it; *e.g.* a case of cotton laces invoiced at £100, less 4%, is valued at £96, or $467, and the duty is 60% of $467, or $280.20.

In changing the foreign invoice valuations to dollars, use the following rates, which represent the *intrinsic par* or real comparative values of the various denominations, as adopted by the United States Treasury Department.

Country	Monetary Unit	Value in U. S. Dollars
Great Britain	Pound	$4.866
Germany	Mark	$.238
France, Switzerland, Belgium	Franc	$.193
Italy	Lira	$.193
Austria	Crown	$.203

465. *Oral*

Find the duties on the following invoices:

	Articles	Rate of Duty
1.	500 lb. of figs	2¢ per pound.
2.	$200 worth of cotton-seed meal	20 per cent.
3.	800 lb. macaroni	$1\frac{1}{2}$¢ per pound.
4.	$2000 worth of mandolins	45 per cent.
5.	2 T. of mutton	2¢ per pound.
6.	50,000 white pine shingles	30¢ per 1000.
7.	5000 bu. of apples	25¢ per bushel.
8.	4855 lb. lemons	1¢ per pound.
9.	2500 pineapples	$7 per 1000
10.	$200 worth of straw hats	35 per cent.
11.	$480 worth of artists' proof etchings	25 per cent.
12.	15 cwt. of Italian chestnuts	1¢ per pound.
13.	One ton of hydraulic cement	8¢ per 100 pounds.
14.	10 horses, valued at $300 apiece	25 per cent.
15.	$200 worth of silk gloves	60 per cent.
16.	50 bu. of flaxseed	25¢ per bushel.
17.	2 T. of maple sugar	4¢ per pound.
18.	$150 worth of rubber balls	30 per cent.
19.	$2100 worth of steel plows	20 per cent.
20.	5 T. of car tires	$1\frac{1}{2}$¢ per pound.
21.	800 lb. of frozen salt-water fish	$\frac{3}{4}$¢ per pound.
22.	$1000 worth of sawed mahogany	15 per cent.
23.	600 bottles of Apollinaris water	30¢ per dozen bottles.
24.	5 T. of scoured wool	33¢ per pound.
25.	500 knives invoiced at 40¢ each	5¢ each and 40%.

466. *Written*

In examples **1–15** *compute the duties in dollars:*

1. On $1275 worth of chisels at 45%.

2. On $3842 worth of fur rugs at 35%.

3. On 500 bbl. of rye flour, each containing 196 lb., at $\frac{1}{2}¢$ per pound.

4. On $8374 worth of wool garments weighing 1047 lb., at 44¢ per pound and 60% ad valorem.

5. 35% on 1893 yd. of gingham, invoiced at 13¢ per yard.

6. $2\frac{1}{2}¢$ per square yard on 648 sq. yd. of unbleached cotton cloth.

7. 60% ad valorem and 44¢ per pound on 8 cases of wool stockings, average weight per case 272 lb., invoiced at $2685.

8. 25% ad valorem and $3 apiece on 25 Swiss watches, valued at $65 apiece.

9. On 350 lb. of cologne water, invoiced at 40¢ per pound, the rate being 45% ad valorem and 60¢ per pound.

10. Five tons of corrugated iron plates at $1\frac{1}{10}¢$ per pound.

11. 20% ad valorem and 60¢ per square yard on 500 yd. of inlaid linoleum, 6 ft. wide, invoiced at 60¢ per square yard.

12. On 504 dozen boxes of friction matches at 8¢ per gross of boxes.

13. 60¢ per square yard and 40% ad valorem on 525 yd. of Wilton carpet, 27 in. wide, invoiced at 80¢ per yard.

14. 4¢ per pound and 15% on 1500 lb. of candy, invoiced at 15¢ per pound.

15. 35% on a shipment of fur coats from Kraft and Levin, Berlin, invoiced at 3192 marks, less 4%.

16. Henry Johnson of Denver purchased from the Broadway Damask Co. of Belfast, Ireland, 1168 sq. yd. of linen damask, invoiced at £88, less 3%.

a. Find the net invoice price in dollars.

b. Compute the duty at 30% and 6¢ per square yard.

17. Williams & Co. of Cleveland bought of Moritz Pach of Berlin, 15 wool jackets, weighing 20 Kg., invoiced at 873 marks; 8 wool coats weighing 12 Kg., for 798 marks; and 9 silk coats for 1068 marks. The purchasers were allowed a 4% trade discount on the entire invoice. They paid a duty of 44¢ per pound and 60% ad valorem on the wool garments, and 60% ad valorem, only, on the silk garments.

a. Find in marks the net price of the entire invoice.

b. Find in dollars the net price of the entire invoice.

c. Find in pounds the weight of the wool garments (to tenths).

d. What was the amount of duty paid?

18. Mr. M. J. McCarthy purchased of E. J. Weinfurter, Vienna, Austria, 410 Kg. of candle wicking, invoiced at 2460 crowns, less 5% trade discount.

a. What was the net invoice price in crowns?

b. What was the net invoice price in United States money?

c. What was the duty, computed at 10¢ per pound and 15% ad valorem?

d. What was the total cost of the goods, including the net invoice price, the duty, 15 crowns for cases and packing, and 12.40 crowns for consular certificates?

19. Leighton and McArthur of Rochester bought from Sanderson Brothers and Newbould, of Sheffield, England, 12,518 lb. of steel ingots invoiced at £811 9*s.* 2*d.* What was the entire cost in United States money, including a duty of $4\frac{7}{10}$ cents per pound, freight £6 5*s.* 2*d.*, commissions 18*s.* 9*d.*, consular fees 10*s.* 4*d.* and insurance £1 6*s.*?

20. Fancher and Dunham of Providence purchased of the Compagnie de Vichy of Lyons, France, 120 cases of mineral water, each containing 50 quart bottles, invoiced at 35 francs per case, and 5 cases, each containing 100 pint bottles, invoiced at 45 francs per case. Find in United States money the entire cost, including a duty of 20¢ per dozen pint bottles and 30¢ per dozen quart bottles.

21. What is the duty at 60% on a case of cotton laces containing 1090 pieces purchased from the Thomas Adams Co., Limited, of Nottingham, England, invoiced at $8\frac{1}{2}d.$ per piece with trade discounts of 20% and 5%?

22. When the duty on 32,500 pine shingles amounts to \$9.75, what is the duty per 1000?

23. When a 45% duty on an invoice of goods from France amounts to \$260.55, what is the invoice price, in French money?

24. A shipment of goods from Austria was invoiced at 3500 crowns. What was the ad valorem duty at 15%?

25. Find the amount of a 6% duty on goods invoiced at £200.

EQUATIONS

467. *An expression of the equality of two numbers or quantities* is an **equation**; *e.g.*

$$\$40 = \$40; \qquad 32 \text{ oz.} = 2 \text{ lb.}; \qquad \$20 \times 2 = \$40;$$
$$8 \text{ cents} \div 2 = 4 \text{ cents}; \qquad £1\ 5s. = 25s.$$

468. *The part of an equation at the left of the sign of equality* is the **first member** *of the equation.*

469. *The part of an equation at the right of the sign of equality* is the **second member** *of the equation.*

Name the first member of each of the equations in section **467**; the second member.

FIG. 1

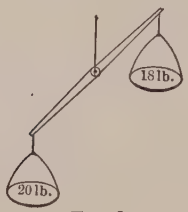


FIG. 2

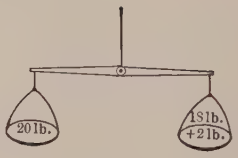


FIG. 3

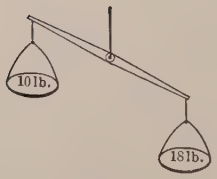


FIG. 4

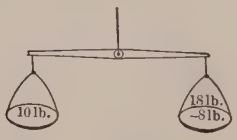


FIG. 5

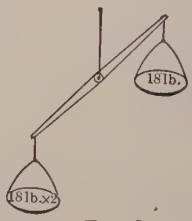


FIG. 6

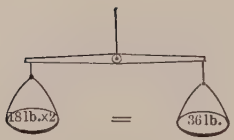


FIG. 7

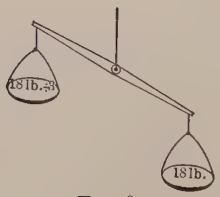


FIG. 8

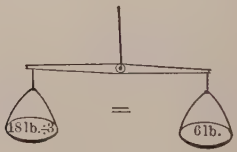


FIG. 9

470. *Oral*

1. Which of the above figures represent equations?

2. Why do the scales balance in Fig. 1?

3. Why do they not balance in Fig. 2?

4. What must be done with Fig. 2 to obtain the balance shown in Fig. 3?

5. What must be done with Fig. 4 to obtain the balance shown in Fig. 5?

6. What must be done with Fig. 6 to obtain the balance shown in Fig. 7?

7. What must be done with Fig. 8 to obtain the balance shown in Fig. 9?

8. Write an equation expressed in dollars. Add \$5 to each member. Is it still an equation? Why?

9. How may we make a true equation from $17 = 14$?

10. How may we make a true equation from $21 = 7$?

11. How may a true equation be made from 15 gal. $\div 3 = 60$ qt.?

12. Complete the following equations:

a. $35 + \text{——} = 45$.
b. $89 - 3 = 80 + \text{——}$.
c. $45 = 15 \times \text{——}$.
d. 17 ft. $= 5$ yd. —— ft.
e. 2 rd. 7 ft. $= 32$ ft. $+$ ——.
f. 4 wk. $=$ —— da.
g. $18 + 3 \times 6 = 30 + \text{——}$.
h. \$99 $\div 11 =$ \$1 $\times$ ——.
i. $86 - 46 = 5 \times \text{——}$.
j. 5 ft. $+ 3$ in. $= 60$ in $+$ ——.
k. 2 hr. $+ 30$ min. $=$ —— min.
l. $\frac{6}{8} = \frac{3}{\text{——}}$.
m. $\frac{42}{14} =$ ——.
n. —— $\times 7 = 60 - 11$.

13. Make an equation. Add 7 to the first member. Is it still an equation? What must be done to the second member to restore the equality?

14. Make an equation of two sums of money. Add 10 cents to the first member. What must be done to the second member in order to preserve the equality?

15. Make an equation of two numbers expressing time. Subtract 15 min. from the second member. What must be done to the first member to preserve the equality?

16. Make an equation of two numbers expressing surfaces. Multiply both members by 10. How is the equality of the two members of the equation affected?

17. Make an equation. Divide both members by the same number. How is the equality of the two members affected?

471. **Axioms**

1. *If the same or equal quantities are added to equal quantities, the sums are equal.*

2. *If the same or equal quantities are subtracted from equal quantities, the remainders are equal.*

3. *If equal quantities are multiplied by the same or equal quantities, the products are equal.*

4. *If equal quantities are divided by the same or equal quantities, the quotients are equal.*

Summary

We may add the same number or equal numbers to both members of an equation, subtract the same number or equal numbers from both members of an equation, multiply both members by the same or equal numbers, or divide both members by the same or equal numbers without destroying the equality.

472. Many problems may be solved more easily by the use of equations than by the usual methods of analysis. In solving problems by means of equations, it is customary to represent the *number which is to be found*, called the **unknown number,** by some letter, usually x, y, or z.

In expressing the equation, if x stands for a certain number, two times the number is represented by $2\,x$, three times the number by $3\,x$, ten times the number by $10\,x$, and so on; that is, $5\,x$ means 5 times x, $7\,x$ means 7 times x, $25\,x$ means 25 times x, $.05\,x$ means .05 of x, and so on.

What is the meaning of $11\,x$? $15\,x$? $\frac{2}{3}\,x$? $7\frac{1}{2}\,x$? $.15\,x$? $2.07\,x$? $.03\,x$? $2\frac{7}{8}\,x$?

473. *Finding the value of the unknown number in an equation* is called **solving the equation.**

We solve an equation by *adding* the same or equal numbers to both members, *subtracting* the same or equal numbers from both members, *multiplying* both members by the same or equal numbers, or *dividing* both members by the same or equal numbers, or by performing several of these operations in succession. In other words, there are *four operations* that we may perform upon the members of an equation *without destroying the equality.*

Examples

1. Solve the equation, $8x = 24,$

Dividing both members by 8, $x = 3.$ *Ans.*

2. Solve the equation, $x + 15 = 45,$

Subtracting 15 from both members, $x = 30.$ *Ans.*

3. Solve the equation, $4x + \$10 = \$38,$

Subtracting \$10 from both members, $4x = \$28,$

Dividing both members by 4, $x = \$7.$ *Ans.*

4. Solve the equation, $16x + \$20 = 6x + \$35,$

Subtracting $6x$ from both members, $10x + \$20 = \$35,$

Subtracting \$20 from both members, $10x = \$15,$

Dividing both members by 10, $x = \$1.50.$ *Ans.*

5. Solve the equation, $\frac{1}{5}x - 18 = 2,$

Adding 18 to both members, $\frac{1}{5}x = 20,$

Multiplying both members by 5, $x = 100.$ *Ans.*

6. Solve the equation, $82x - \frac{7}{11} = 40\frac{4}{11},$

Adding $\frac{7}{11}$ to both members, $82x = 41,$

Dividing both members by 82, $x = \frac{1}{2}.$ *Ans.*

7. Solve the equation, $1.00\frac{1}{4}x = \$84.21,$

Dividing both members by $1.00\frac{1}{4}$, $x = \$84.$ *Ans.*

474. *Written*

Solve the following equations:

1. $5x = 35$.
2. $7x = 18 + 4x$.
3. $13x + 4 = 95$.
4. $18\frac{1}{2}x = 74$.
5. $\frac{1}{12}x = 86$.
6. $\frac{2}{3}$ of $\frac{3}{4}x = 825$.
7. $1.03x = 412$.
8. $.08x = 4.32$.
9. $5.18x = 466.2$ yd.
10. $8.5x + 30$ bu. $= 1135$ bu.
11. $18x - 2 = 88$.
12. $75x - \frac{3}{7} = 224\frac{4}{7}$.
13. $14x + \frac{5}{18} = 560\frac{5}{18}$.
14. $12\frac{3}{7}x = \$957$.
15. $45x = 72$.
16. $.36x + \$1.45 = \$19.45$.

Problems

475. *Written*

1. .16 of the cost of my house was \$320. What did my house cost?

Solution

Let	$x =$ cost of my house.
Then	$.16x = \$320$.
Dividing both members by .16,	$x = \$2000$, cost of my house. *Ans.*

2. A pony and cart cost \$135. The pony cost four times as much as the cart. Find the cost of each.

Solution

Let	$x =$ cost of the cart.
Then	$4x =$ cost of the pony.
Adding,	$5x = \$135$, cost of both.
Dividing both members by 5,	$x = \$27$, cost of the cart,
Multiplying by 4,	$4x = \$108$, cost of the pony. } *Ans.*

3. The sum of two numbers is 199.40. Their difference is 2.70. What are the numbers?

Solution

Let x = the smaller number.
Then $x + 2.70$ = the larger number.
Adding equals to equals, $2x + 2.70$ = the sum of the numbers
or, $2x + 2.70 = 199.40$.
Subtracting 2.70 from both members, $2x = 196.70$.
Dividing both members by 2, $x = 98.35$, the smaller,
Adding 2.70 to both members, $x + 2.70 = 101.05$, the larger. } *Ans.*

4. The area of a rectangle is 5875 square inches. The width is 25 inches. What is the length?

Solution

Let x = the length in inches.
Then $25x = 5875$ (area = length × breadth).
Dividing both members by 25, $x = 235$ inches. *Ans.*

5. A merchant gained 35 % by selling cloth at \$1.89 per yard. What was the cost per yard?

Solution

Let x = cost of 1 yard.
Then $.35x$ = gain on 1 yard.
Adding equals to equals, $1.35x$ = cost + gain, or the selling price
or, $1.35x = \$1.89$.
Dividing both members by 1.35, $x = \$1.40$, cost of 1 yard. *Ans.*

6. What principal on interest for 2 mo. 21 da. at 5 %, will yield \$7.47 interest?

Solution

Let x = the required principal.

Then $x \times \frac{5}{100} \times \frac{81}{360} = \$7.47$. $\frac{\cancel{5}}{100} \times \frac{\overset{9}{\cancel{81}}}{\underset{8}{\cancel{360}}} = \frac{9}{800}$

Therefore, $\frac{9}{800}x = \$7.47$.

Dividing both members by $\frac{9}{800}$, $x = \$664$, principal. *Ans.*

Solve by means of equations:

7. John and Henry earned $38.40 during the summer vacation. Henry earned twice as much as John. How much did each earn?

8. The sum of two numbers is 8347; their difference is 1265. What are the numbers?

9. Elsie, Ruth, and Mabel received $42 in prizes, Elsie receiving $3 as often as Ruth $2 and Mabel $1. What was the amount of each prize?

10. A pole stands $\frac{2}{17}$ in the mud, $\frac{7}{17}$ in the water, and the remainder, which is 32 feet, in the air. How long is the pole?

HINT. — Let $x = \frac{1}{17}$ of the length of the pole.

11. A tree 55 ft. high was broken off so that the part broken off was four times as long as the part left standing. How long was the piece that was broken off?

12. Three men, A, B, and C, engaged in business, B furnishing three times as much capital as A, and C furnishing twice as much as B. If they furnished $8950 in all, how much did each furnish?

13. A man is four times as heavy as his son, and the difference of their weights is 63 Kg.

a. What is the weight of each, in kilograms?

b. In pounds?

14. What number increased by $\frac{1}{7}$ of itself equals 192?

HINT. — Let $x = \frac{1}{7}$ of the number; then $7x =$ the number.

15. What number diminished by $\frac{2}{11}$ of itself equals 162?

16. A man, having a sum of money, earned five times as much, and spent one half of what he then had. He had left $270. How much had he at first?

17. A boy, having some money, earned twice as much and \$.48 more, when he had \$9.78. How much did he earn?

18. One third of a sum of money exceeds one fourth of the sum by \$17. What is the sum?

19. Two fifths of a number is 14 less than five ninths of the number. Find the number.

20. $2\frac{5}{8}$ times a certain number is greater by 45 than three fourths of the number. Find the number.

21. Divide 176 into four parts so that the first shall be four times the second, the third one third of the second, and the fourth one half of the first.

HINT. — Let x = the third part.

22. The sum of three numbers is 1658. The second exceeds the first by 130, and the third exceeds the first by 79. Find the three numbers.

23. Three numbers, when added, amount to 11.89. The second exceeds the first by 3.28 and the third exceeds the second by 1.37. Find them.

24. A farmer has apples, potatoes, turnips, and onions in his cellar. The number of bushels of apples is 13 less than the number of bushels of potatoes; the number of bushels of turnips is 19 less than the number of bushels of apples, and there are 3 more bushels of turnips than of onions. The entire quantity is 72 bushels. Find the number of bushels of each.

25. In a certain class, the number of girls who received honor marks was three more than twice the number of boys who received honor marks. The number of honor pupils was 18. How many were girls, and how many were boys?

26. Seven times a certain sum of money plus \$18 is equal to five times the sum plus \$50. What is the sum of money?

REVIEW AND PRACTICE

476. *Oral*

1. Name the prime numbers from 1 to 100.

2. How may we know, without trial, that 723,468 will not exactly divide 398,650,076,341?

3. There are two decimal places in one factor, three in another, one in another, and four in another. How many decimal places are there in the product of the four factors?

4. If one fifth of an acre of land is worth $20, what is one twentieth of an acre worth at the same rate?

5. $48 \times 25 = ?$ $57 \times 99 = ?$ $560 \times 125 = ?$

6. $61 \div 25 = ?$ $33 \div 125 = ?$ $17 \div .33\frac{1}{3} = ?$

7. $360 \times .16\frac{2}{3} = ?$ $39 \div .25 = ?$ $150 \div .2 = ?$

8. $63 \times 33\frac{1}{3} = ?$ $99 \times 66\frac{2}{3} = ?$ $42 \times .14\frac{2}{7} = ?$

9. $50 + 5 \times 2 = ?$ $88 - 8 \div 4 = ?$ $7 \times 8 + 16 \div 4 = ?$

10. 20 % of $33\frac{1}{3}$ % = what common fraction?

11. Two successive trade discounts of 10 % are the same as what single discount?

12. Test each of the following numbers for divisibility by 2, 3, 4, 5, 6, 8, and 9:

a. 2364 *b.* 486,728 *c.* 72,056,391 *d.* 91,307,865
e. 42,836,076 *f.* 90,010,332 *g.* 8,705,637,411

13. If a man earns $99 in 17 days, how much will he earn in 51 days at the same rate?

14. What is the least number that will exactly contain 2, 3, and 4?

15. What is the greatest number that will exactly divide 60, 96, and 132?

16. What is the cost of 7000 shingles at $5.50 per M?

17. What is the cost of 1500 lb. of mixed feed at \$1.80 per cwt.?

18. Two long tons contain how many more pounds than two short tons?

19. What is the length of a solar year?

20. How many grains are there in 5 lb. Avoirdupois?

21. A quart of spirits of camphor will fill how many 4-ounce bottles?

22. What is the area of a triangle whose base is 2 ft. and whose altitude is 20 in.?

23. What is the altitude of a parallelogram having an area of 96 sq. in. and a base of 2 ft.?

24. A piece of lumber 2″ by 4″, and 6 ft. long, contains how many board feet?

25. *a.* How many shingles are required for 1 square foot of roof, when they are laid 6 inches to the weather?

b. How many are required for one *square* of roofing?

26. What is the cost of a slate roof $20' \times 30'$ at \$10 per square?

27. A grocer sold $66\frac{2}{3}\%$ of a hogshead of vinegar. How many gallons did he sell?

28. $33\frac{1}{3}\%$ of a rod is how many feet?

29. What per cent does a grocer gain on celery bought at 30¢ a dozen heads, and sold at 5¢ a head?

30. What per cent does a merchant gain when he sells two yards of cloth for what three yards cost?

31. A newsboy bought 30 papers and sold them at a profit of 50%. How many papers can he buy with the money received for the papers sold?

32. How much commission does an agent receive for selling \$1200 worth of goods, when the rate of his commission is $16\frac{2}{3}\%$?

33. What is the premium for insuring a \$10,000 stock of goods for one fourth of its value at 2%?

34. Mr. Wheelock's county tax was \$75 when the county tax rate was 5 mills on a dollar. What was the assessed valuation of Mr. Wheelock's property?

35. A tax collector's suretyship bond cost him \$28, at the rate of \$4 per thousand. What was the amount of his bond?

36. On a certain day, New York exchange sold in Kansas City at $\frac{1}{8}\%$ premium. What was the premium on a \$16,000 draft?

37. When exchange on London is quoted at $4.87\frac{1}{4}$, what is the cost of a draft for £100?

38. When exchange on Geneva is quoted at 5.20, what is the cost in Philadelphia of a draft on Geneva for 104 francs? What is the face of a draft that \$100 will buy?

39. What is the face of a Berlin draft that can be bought for \$240 when exchange is quoted at 96?

40. A room 12 meters long is how many feet long? (Think all the way through before you perform any operation.)

41. 300 liters of oats are about how many bushels?

42. 100 liters of kerosene oil are about how many gallons?

43. About how many square meters of carpet are required to cover a floor 2 rods wide and 4 rods long?

44. What is the scale of linear measure in the metric system? Of surface measure? Of volume measure?

45. State your weight approximately, in kilograms.

477. *Written*

Solve the following problems, using equations wherever they will shorten or simplify the work:

1. Find (*a*) the greatest common divisor, and (*b*) the least common multiple of 126, 210, 294, and 462.

2. Reduce $\frac{5}{160}$ to a decimal.

3. Kerosene is $80\frac{2}{5}\%$ as heavy as water. If a gallon of water weighs $8\frac{1}{3}$ lb., how many gallons are there in a ship load of kerosene weighing 3900 tons?

4. In 1890 there were 166,706 miles of railroad in the United States, and in 1900 there were 190,082 miles. What was the per cent of increase?

5. The copper cent, which has not been coined since 1864, weighed 72 grains and was composed of 88% copper and 12% nickel. How many pounds, Avoirdupois, of copper were there in \$100 worth of those coins?

6. Find the number of gallons of water that can be contained in a rectangular cistern 7 ft. by 12 ft. by $5\frac{1}{2}$ ft.

7. On the 29th day of April, 1908, Francis Burns bought of Fred J. Peck, 9 tons of egg coal and 5 tons of chestnut coal at \$6.10 per ton, and 2 tons of pea coal at \$4.25 per ton. Make out the bill and receipt it as the creditor's agent.

8. A boy spent $\frac{3}{5}$ of his money, earned 65 cents, and then had $\frac{5}{6}$ of his original sum. How much money had he at first? (Let x = the money he had at first.)

9. A man owning 135 acres of land, sold 63 A. 87 sq. rd. How much land had he left?

10. Add $40° 37' 19''$, $20° 40' 30''$, and $9° 30' 45''$.

11. Divide $35° 21' 30''$ by 15.

12. How many cords are there in a pile of 4-foot wood 7 ft. high and 40 ft. long?

13. Find the cost, at 36 cents per square yard, of plastering the four walls and ceiling of a store 72 ft. long, 36 ft. wide, and 12 ft. 3 in. high, allowing 375 sq. ft. for openings.

14. What is the cost of carpeting a room 14 ft. 9 in. long and 12 ft. 6 in. wide with Brussels carpet 27 in. wide, costing $1.35 a yard, running the strips lengthwise of the room and making no allowance for waste in matching the pattern?

15. Find the cost of 48 planks, 16 ft. long, 14 in. wide, and 3 in. thick, at $34 per M.

16. What is the altitude of a triangle whose area is 600 sq. ft. and whose base is 60 ft.? (Let $x =$ the altitude and make an equation.)

17. A building lot was sold for $1150, which was an advance of 15% on the cost. If it had been sold for $2210, what would have been the rate per cent of gain?

18. A farm, sold at a loss of 18%, brought $16,400. How many dollars were lost?

19. At what price must cloth costing $3.50 per yard be marked, that the merchant may deduct 20% from the marked price and still gain 20%?

20. One brand of tin plate is made by dipping thin steel plates into molten tin. A coating of tin adheres to the steel, making a sheet of bright tin.

a. If 112 of the plates weigh 98 lb. before being dipped, and 106 lb. after being dipped, what per cent of the tin plate is tin?

b. What per cent of the tin plate is steel?

c. How many pounds of tin will 2800 tin plates contain?

21. The following is a record of receipts and expenses for one year of a 94-acre farm in New York State, owned by Mr. Tallcott, and worked by a tenant who received one half of the net income as his share :

Receipts		Expenses	
Wheat, 107 bu., at	80¢ per bu.	Phosphates	$47
Potatoes, 598 bu., at	60¢ per bu.	Seed	$23
Cabbage, 44 tons, at	$14.40 per T.	Miscellaneous	$94
Hay, $17\frac{1}{10}$ tons, at	$11.00 per T.		
Milk	$239.00		
Veal	$22.00		
Young stock, growth	$50.00		
Nine pigs	$106.00		
Poultry	$92.00		

a. How much did the tenant receive for his year's work?

b. The owner's entire investment consisted of $2700 paid for the farm, $500 for improvements, and $800 for stock. Out of his share of the profits, he paid $35 taxes and insurance, $68 for repairs, and $90 for other items. His net income was what per cent of his investment?

c. The next year, the income from produce (cabbages, wheat, potatoes, etc.) diminished $388. The income from milk and live stock increased $407, and the expenses increased $107. Was Mr. Tallcott's per cent of net income increased or diminished, and how much?

22. How many steres of stone are there in a stone wall 3 m. long, 5 dm. thick, and 250 cm. high?

23. Write 10 dm., 5 m., and 9 mm. as one number.

24. How many liters of water will be contained in a vat which is 3 m. long, 25 dm. wide, and 200 cm. deep?

25. How many kilograms of water will a rectangular tin box hold, if it is 15 dm. long, 25 cm. deep, and 1 m. wide?

26. What is the cost of goods that bring $742.56, when sold at a gain of 7%?

27. A certain kind of dress goods shrinks 4% in sponging. How many yards should be purchased for a suit requiring 12 yd. of sponged cloth?

28. A sloyd class was composed of 20 boys. Each boy made a sled of the following parts: runners, 42 in. long and $4\frac{1}{2}$ in. wide; three crosspieces, each $2\frac{1}{2}$ in. by 12 in.; a top, 12 in. by $28\frac{1}{2}$ in. What was the cost of the lumber at $70 per M, none of it being more than 1 in. thick, and estimating that 20% of all the lumber purchased was wasted in the work?

HINT.—What per cent of the lumber was not wasted?

29. For what sum must I give my note, without interest, due 90 days from date, in order that it may yield $492.50, when discounted at 6% on the day of date?

30. A note for $600, without interest, dated July 1, due 90 days from date, was discounted Aug. 30, at the rate of 7% per annum. Find the proceeds.

31. What principal will give $63 interest in 2 yr. 3 mo. at 8%?

32. At what rate of interest will $600 amount to $692 in 2 yr. 6 mo. 20 da.?

33. Find the interest on $390 for 1 yr. 6 mo. 5 da. at 5%.

34. What per cent of the list price is paid by a purchaser who is allowed discounts of 20% and 10%?

35. The premium on an insurance policy is $33, and the rate $\frac{6}{10}$%. What is the face of the policy?

36. A certain village must raise \$9017 by taxation. There are 670 men who pay a poll tax of \$1 each. The assessed valuation of the property of the village is \$667,760.

a. What must be the tax rate?

b. What is the tax on property assessed at \$7500?

c. What is the entire tax of a man whose property is assessed at \$1475 and who is a resident of the village?

d. What is the assessed valuation of property on which the tax is \$95.50?

37. In order to close out a stock of gloves that cost me \$9.60 a dozen pairs, I am selling them at \$.75 a pair. What per cent do I lose?

38. A collector collected a sum of money, took out his commission of 3%, and sent his principal the remainder, which was \$3636.53. How much did he collect?

39. Marcus Stevens, of Fort Wayne, Ind., bought of Johnson & Co. of Harrisburg, Pa., a bill of goods amounting to \$673. Johnson & Co. shipped the goods and drew on Stevens for the amount, at 60 days sight, through the First National Bank of Harrisburg. The draft was accepted by Stevens, discounted by the First National Bank at 6%, and the proceeds credited to the account of Johnson & Co.

a. Write the draft as it was when discounted.

b. What was the amount credited to Johnson & Co.?

40. Franklin J. Becker, of Nashville, imported 10 cases of machinery from Germany, invoiced at 7700 marks. Find the duty, in United States money, at 45% ad valorem.

41. *a.* What is the duty, at 10¢ per gallon, on 400 barrels of rape-seed oil, each barrel containing 52 gallons?

b. If the oil was invoiced at 69,000 francs, what must be the

cost of a draft on Paris sufficient to pay the bill, the rate of exchange being $5.17\frac{1}{2}$?

STOCKS

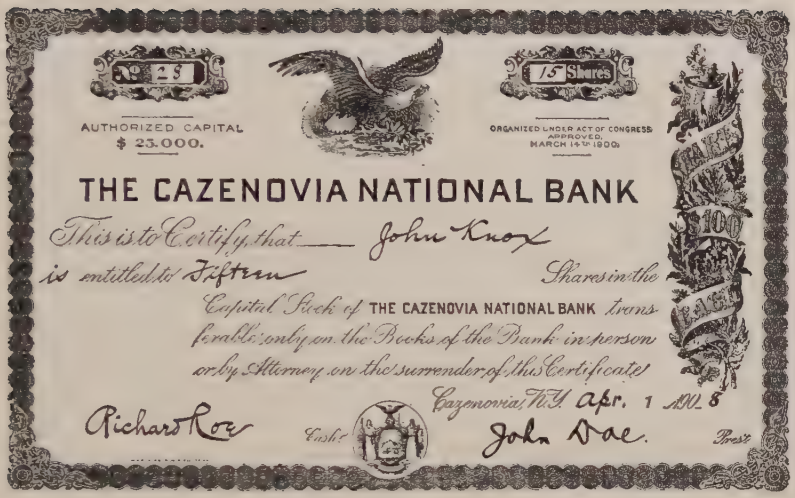
478. It often happens that one man or a small group of men desire to engage in a business that requires more money, or capital as it is called, than they alone are able or willing to invest in it. They obtain more money by organizing a **stock company**. That is, they draw up a subscription paper, describing the business in which they purpose to engage, the signers of which agree, on certain conditions, to pay into the treasury of the company the sums of money set opposite their names in the subscription paper.

For convenience, the entire amount to be raised is divided into a certain number of parts, called **shares**, and each subscriber may subscribe for as many shares as he desires.

The shares of railroad, steamship, telegraph, banking, and manufacturing companies are usually \$100 each. The shares of Western mining companies are usually one dollar each. Sometimes shares are even less than one dollar.

When a sufficient number of shares have been subscribed for, the company is organized, and receives from the state or government a **charter** or **certificate of incorporation** empowering it to transact business as an individual. The shareholders elect certain ones of their number, generally not less than five, to be the **directors** of the company. In voting, each shareholder has as many votes as the number of shares which he owns. The directors elect officers whose duty it is to manage the business.

Each shareholder receives a **certificate of stock**, which is a document, signed by officers of the company, stating the size of each share and the number of shares which he owns. These shares may be bought and sold like any other property.


No. 25
15 Shares
AUTHORIZED CAPITAL
$ 25.000.
ORGANIZED UNDER ACT OF CONGRESS APPROVED MARCH 14th 1900.
THE CAZENOVIA NATIONAL BANK
This is to Certify that John Knox
is entitled to Fifteen Shares in the
Capital Stock of THE CAZENOVIA NATIONAL BANK transferable only on the Books of the Bank in person or by Attorney on the surrender of this Certificate
Cazenovia, N.Y. Apr. 1 1905
Richard Roe Cashr
John Doe. Prest.

CERTIFICATE OF STOCK.

On the back of the above certificate is printed the following form for the transfer of the shares:

For value received _____ *hereby sell, transfer, and assign to* _____ *the shares of stock within mentioned, and authorize* _____ *to make the necessary transfer on the books of the company. Witness my hand and seal this* _____ *day of* _____ 19_____.

_____ [L.S.]

Witnessed by

_____

When this form is properly filled out, the purchaser may surrender the certificate to the company and receive a new one made out in his own name.

479. *Oral*

1. The certificate on page 285 is for how many shares?

2. Each share represents how many dollars of capital stock?

3. What is the entire capital of this bank?

4. It is divided into how many shares?

5. A certain manufacturing company has a capital of $600,000. This is equal to how many shares of $100 each?

6. The capital stock of a certain company is divided into 2000 shares of $50 each. What is the entire amount of its capital? How many dollars of capital stock has a man who owns 40 shares?

7. How many shares of stock are there in a company whose capital stock is $200,000, divided into shares of $25 each? How many dollars of this stock has a man who owns 50 shares?

8. What is the entire capital stock of a company whose capital is divided into 10,000 shares of $100 each? How many dollars of this stock has a man who owns 50 shares?

9. Name some stock companies that transact business in your vicinity.

10. If I own twenty-five 50-dollar shares of Pennsylvania R.R. stock, how many dollars of stock do I own?

11. Make a definition of (*a*) a stock company, (*b*) capital stock, (*c*) a share, (*d*) a certificate of stock.

480. When a stock company succeeds in business so that its income is greater than its expenses, the profits are divided among the stockholders, each one receiving a part of the profits, according to the number of shares of capital stock which he owns.

In some companies, if there are losses in the business, they are apportioned among the stockholders, each one contributing according to the number of shares that he owns.

The *real value* of a share of stock begins to change very soon after it is issued.

If the business of the company is prosperous, so that there are large profits to be divided among the shareholders, people are anxious to buy the shares and are willing to pay more for them than their original or *face value*. If the business is not prosperous, so that there are no profits, but sometimes losses, the shareholders are willing to sell their shares for less than their original or face value.

The abundance or scarcity of money in the great money centers of the country, and the general condition of business, also affect the real or *market value* of shares.

Summary

481. A **stock company** consists of *a number of persons, organized under a general law or by special charter, and empowered to transact business as a single individual.*

482. The **capital stock** *of a company* is the amount named in its charter.

The capital stock nominally represents the *original* investment in the company, but is, in most cases, either greater or less than the present real value of the company's property.

483. A **share** is *one of the equal parts into which the capital stock of a company is divided.*

In this book, a share will be considered as $100 of stock unless otherwise indicated.

484. A **stockholder** is a *person who owns one or more shares of capital stock.*

485. The **par value** *of a share of stock* is *its original or face value;* the **market value** *of a share of stock* is *the price for which the share will sell in the market.*

The market values of leading stocks fluctuate from day to day, and are quoted in the daily papers; *e.g.* "N. Y. C., 131" means that the stock of the New York Central R.R. Co. is selling to-day at $131 a share; "Western Union, 56" means that the stock of the Western Union Telegraph Company is selling at $56 a share.

486. When the market price of stock is the same as the par value, the stock is said to be **at par**; when the market value is greater than the par value, it is said to be **above par,** or **at a premium**; when the market value is less than the par value, it is said to be **below par** or **at a discount**; *e.g.* when the General Electric Company's stock is quoted at 113, it is 13% above par, or at a premium of 13%; when Missouri Pacific R.R. stock is quoted at 47, it is 53% below par, or at a discount of 53%.

The par value never changes. A share of stock that was originally $100 is *always* $100, though its market value may be more or less than $100. The par value of stock, therefore, does not represent value at all, but a certain *quantity* or *part* of the entire capital stock of a company; just as, if you own 100 bushels of wheat, in an elevator containing 100,000 bushels, you own $\frac{1}{1000}$ part of the entire quantity, though it may be worth $125, or only $60. It is always the same quantity of wheat, whatever may be its value.

We should therefore avoid speaking of a share as "$100 *worth* of stock"; it is $100 *of* stock, like 100 yards *of* cloth, or 100 gallons *of* oil.

487. **Dividends** are the *net profits of a stock company divided among the stockholders according to the amount of stock they own;* **assessments** are the *losses apportioned among, and required to be paid by, the stockholders according to the amount of stock they own.*

Both dividends and assessments are computed at a certain per cent of the *par value* of the capital stock; *e.g.* if a company is capitalized at $100,000,

and makes a net profit of $2000 during one year, the profit is 2% of the par value of the stock; the company may therefore declare a dividend of 2%, and pay to each stockholder a dividend of 2% of the par value of his stock.

488. Stock companies often issue two kinds of stock, namely:

Preferred stock, which consists of a certain number of shares on which dividends are paid at a fixed rate, and

Common stock, which consists of the remaining shares, among which are apportioned whatever profits there are remaining after payment of the required dividends on the preferred stock.

489. Stocks are generally bought and sold by **brokers,** who act as agents for the owners of the stock. Brokers receive as their compensation a certain per cent of the *par value* of the stock bought or sold. This is called **brokerage.**

The usual brokerage is $\frac{1}{8}\%$ of the par value; *e.g.* if a broker sells 10 shares of stock for me, his brokerage is $\frac{1}{8}\%$ of $1000, or $1.25.

490. *Oral*

1. How many dollars of stock are represented by fifty $100 shares?

2. Explain the meaning of each of the following quotations: Pacific Transportation Co., $57\frac{1}{4}$; Great Northern, preferred, $117\frac{7}{8}$; American Sugar, $101\frac{3}{8}$; Mexican Central, $14\frac{1}{2}$; Lighting Co., 188; U. S. Rubber, common, 20, preferred, 77.

3. When stock is quoted at 85, what is the market value of 100 shares? What is the par value? Is it at a premium, or at a discount? What per cent?

4. When stock is quoted at $132\frac{1}{2}$, what is the rate of premium at which it sells? What is the market value of two shares?

5. When stock is quoted at 90, what is the rate of discount at which it sells? What is the market value of one share? How many shares may be bought for $450? What will be the cost of 1000 shares?

6. When stock sells at a discount of $21\frac{3}{8}\%$, what is the quotation?

7. What is the market value of one share of stock which is quoted at 120? Of 8 shares? How many shares can be bought for $480? For $1080? For $360?

8. When stock is quoted at 75, what is the market value of one share? Of 4 shares? Of 3 shares? Of 20 shares? How many shares can be bought for $150? For $375? For $7500? For $1500?

9. $1600 will buy how many shares of stock at 80? At 40? At 160?

10. What must be paid for 100 shares of Rapid Transit R.R. stock at $49\frac{1}{4}$?

11. If I invest $4000 in U. S. Rubber Company's stock at 20, how many shares will I receive?

12. How many shares of Union Pacific R.R. stock at 120 can be purchased by a woman who has $3600 to invest?

13. A mining company's stock is divided into $1 shares. What is the market value of 200 shares, when they are quoted at 140?

14. What is the brokerage, at $\frac{1}{8}\%$, on one share of the Columbia Construction Company's stock? If the stock is quoted at $105\frac{7}{8}$, what is the market value of one share? What will one share cost me, including brokerage? If I buy two shares, how much is my investment?

15. What must I pay for 100 shares of railroad stock, at par, including $\frac{1}{8}\%$ brokerage?

16. This morning's paper tells me that Southern Pacific R.R. common stock sold yesterday at $78\frac{1}{8}$. If my broker sold 100 shares of it for me at that figure, and sent me the proceeds, after taking out his brokerage of $\frac{1}{8}\%$, how much per share do I receive? How much do I realize from the sale of the 100 shares?

17. A broker sold 400 shares of Erie R.R. stock at 16. How much did he receive for it? How much was his brokerage at $\frac{1}{4}\%$? How much did the owner of the stock realize after paying the brokerage?

18. Mr. Barrett bought, through a broker, 50 shares of Denver & Rio Grande R.R. stock at $20\frac{7}{8}$, paying $\frac{1}{8}\%$ brokerage for buying. How much did a share cost him? What was his entire investment? How much did the broker receive? How much per share was received by the man who sold the stock, after paying his broker? How much would he have received for 100 shares, at the same rate?

19. A manufacturing company, having a capital of \$100,000, declares a dividend twice a year. From Jan. 1 to July 1 of a certain year, its net profits amounted to \$3000. The profits were what per cent of the capital stock? What rate per cent of dividends could the company declare? What amount of dividends did Mr. Scott receive, if he owned 200 shares? How many shares had a stockholder who received \$30 in dividends? If this company's net profits for the remainder of the year were \$5000, what rate of dividends could it declare for that time?

20. A gas company declared a dividend of 6%, which amounted in all to \$60,000. What was the capital of the com-

pany? How many shares must a stockholder have owned, to receive $120 in dividends? How many dollars of stock?

21. If U. S. Steel, preferred, pays 7% annual dividends, what are the dividends on $10,000 of the stock?

22. I have some bank stock that I bought at 200. How much did a share cost me? The stock paid a 10% dividend this year. What did I receive on a share? What rate per cent of income do I receive on my investment?

23. A railroad stock that was bought at 50 pays a 2% annual dividend. What was the cost of 10 shares? What is the income on 10 shares? The income is what per cent of the investment?

24. The following article appeared in a morning paper Jan. 2, 1908:

"The Board of Directors of the Syracuse Rapid Transit Railway Company, at a meeting held Dec. 30, 1907, declared a dividend of 3 per cent on the common stock of the company, payable Feb. 1 to stockholders of record at the close of business Jan. 10, 1908.

"The common stock of the company was quoted on the Syracuse Stock Exchange yesterday at 79 bid, and 95 asked."

How many 5-cent car fares would it take to pay the dividends on 10 shares of the Rapid Transit common stock? How much would 10 shares cost, if bought at the price bid? How much would be received for 10 shares, if sold at the price asked? What was the difference between the asking price of 100 shares and the price bid? If this company paid 6% dividends on its preferred stock, what was the entire income from 10 shares of preferred and 10 shares of common stock?

25. If I buy stock at $87\frac{1}{2}$, and after keeping it for a time, sell it at $95\frac{3}{4}$, paying $\frac{1}{8}$% brokerage both for selling and buying, how much will I gain on 100 shares?

26. A man bought stock at par and sold it six months later at $89\frac{1}{4}$, paying $\frac{1}{8}\%$ brokerage both for selling and buying. What was his loss on 100 shares?

27. Cut the stock quotations from your daily paper, bring them to school, compare them with quotations of the same stocks as given in these exercises, and find the gains or losses that would have resulted from buying stocks at the quotations here given, and selling at the quotations given in your paper.

28. If you buy stocks through a broker, does the brokerage add to, or take from, the cost of the stocks?

29. If you sell stocks through a broker, does the brokerage add to, or take from, your receipts from the sale?

30. Name all the things that are computed on the par value. Can you think of anything that is not computed on the par value?

491. *Written*

The following quotations are copied from a daily paper. Use them in solving problems **1–10.**

American Cotton Oil	$29\frac{3}{4}$	N. Y. Central	$93\frac{1}{2}$
American Woolen	$15\frac{1}{2}$	National Lead	$39\frac{3}{8}$
American Sugar	$101\frac{5}{8}$	Northern Pacific	$116\frac{7}{8}$
Baltimore and Ohio	$81\frac{1}{8}$	People's Gas	80
Brooklyn Rapid Transit	$38\frac{3}{8}$	Rock Island	$15\frac{1}{2}$
Chicago, Mil. and St. Paul	$105\frac{3}{4}$	Southern Pacific, common	$73\frac{1}{8}$
Chicago, Northwestern	$137\frac{3}{4}$	Southern Pacific, preferred	$107\frac{7}{8}$
Manhattan	125	Western Union	55

1. Find the cost, including $\frac{1}{8}\%$ brokerage, of

a. 150 shares of American Woolen Co.

b. 250 shares of Western Union Telegraph.

c. 300 shares of Manhattan R.R.

d. 200 shares of Rock Island R.R.

2. What will the seller realize, allowing $\frac{1}{8}\%$ brokerage, from the sale of

a. 175 shares of American Sugar Company?

b. 95 shares of Brooklyn Rapid Transit R.R.?

c. 200 shares of Chicago and Northwestern R.R.?

d. 400 shares of Chicago, Milwaukee, and St. Paul R.R.?

3. 350 shares of Southern Pacific common stock are worth how much less than the same quantity of Southern Pacific preferred?

4. How many shares of the People's Gas Company can be bought for \$7211.25, which includes $\frac{1}{8}\%$ brokerage?

5. A man realized \$7290 from the sale of Baltimore and Ohio R.R. stock, paying $\frac{1}{8}\%$ brokerage. How many shares did he sell?

6. How many shares of New York Central R.R. stock must be sold to realize \$28,012.50, brokerage $\frac{1}{8}\%$?

7. My broker sold for me 90 shares of stock of the American Cotton Oil Company, and bought with a part of the proceeds 60 shares of National Lead stock. He then sent me the remainder of the money in the form of a New York draft, deducting $\frac{1}{4}\%$ brokerage for selling, $\frac{1}{8}\%$ for buying, and 25¢ exchange for the draft. What was the face of the draft?

8. If I sell 300 shares of Baltimore and Ohio R R. stock, how much must I put with the proceeds of the sale in order to buy an equal quantity of Northern Pacific, paying $\frac{1}{8}\%$ brokerage for each transaction?

9. How much National Lead stock can be bought for \$3910.50, paying $\frac{1}{8}\%$ brokerage?

10. Find in your daily paper the quotations of some of these stocks and compute the gain or loss on 25 shares bought at the rates given here and sold at to-day's prices.

11. What must be paid for 700 shares of Southern Railway stock at $13\frac{1}{4}$?

12. What must be paid for 550 shares of Wisconsin Central Railway stock at $15\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

13. How many shares of Illinois Central stock at $128\frac{1}{4}$ can be bought for $\$9618.75$?

14. How much Railway Steel Spring stock at $26\frac{7}{8}$ can be bought for $\$18{,}495$, which includes brokerage at $\frac{1}{8}\%$?

15. *a.* What must be paid, including brokerage at $\frac{1}{8}\%$, for 190 shares of D. & H. R.R. stock at $150\frac{1}{2}$?

b. What does the seller realize from the sale if he also pays $\frac{1}{8}\%$ brokerage?

16. When 90 shares of stock are worth $\$10{,}125$,

a. What is the value of one share?

b. What is the quotation?

17. *a.* How many dollars of stock paying $4\frac{1}{2}\%$ dividends must I own in order to receive a dividend of $\$900$?

SUGGESTION. — Let x = the number of dollars; then the statement of relation is $.04\frac{1}{2}x = \$900$.

b. How many shares of stock?

c. How much is it worth at $97\frac{3}{4}$?

18. I received in exchange for an office building 700 shares of a bank stock which was selling in the market at 125 and drawing 8% annual dividends.

a. I received the equivalent of how much money?

b. How many *dollars of stock* did I receive?

c. What is the dividend on this amount of stock?

d. That is what per cent of the value of the stock?

Statement of relation; —— of $\$87{,}500 = \$5600$

or, $\$87{,}500\,x = \$5600$.

19. On the 1st of January, 1908, the Faneuil Hall National Bank paid a dividend of $1\frac{3}{4}\%$.

a. What was the dividend on 75 shares of the stock of this bank?

b. How many shares of stock are held by a stockholder who receives $700 in dividends?

20. At a certain time the stock of the Pennsylvania Telephone Company consisted of 88,497 shares of $50 each. The company paid 6% dividends. What was the entire amount of one dividend?

21. The Rocky Mountain Bell Telephone Company paid $142,170 in dividends on $2,369,500 of stock.

a. What was the rate of dividends?

b. If a stockholder bought his stock at 80, what is the rate of income on his investment?

22. The Maryland Coal Company paid a dividend of $2\frac{1}{2}\%$, June 15, 1908. What was the dividend on $11,000 of the stock?

23. At what price must stock paying 5% dividend be bought that the buyer may receive an income of 6% on his investment?

24. Stock bought at 120 was sold at 80.

a. What was lost on 150 shares?

b. What per cent was lost?

BONDS

492. *A stock company or other body of people, organized under a general law, or by special charter, and empowered to hold property, and to act as an individual,* is a **corporation**; *e.g.* any stock company, a city, an incorporated village, a college, a church, a charitable organization such as a hospital or soldiers' home.

Corporations and national, state, county, and town governments often find it necessary to borrow money in order to meet extraordinary expenditures.

For example, our national government borrowed vast sums of money with which to carry on the Civil War and the Spanish War, and later, to build the Panama Canal.

States borrow money with which to construct public buildings, highways, canals, etc. Cities, towns, and counties borrow money for similar purposes. Railroad and manufacturing companies borrow money with which to extend their business.

Mention something for which your own city, town, or village has borrowed money.

Governments and corporations, borrowing money, sell their interest-bearing notes to any one who will buy them, just as a man sells his note to a bank when he borrows money from the bank. These notes are called **bonds**. They are made payable at some future time, usually several years after date, the interest to be paid annually or semi-annually, at a fixed rate. Bonds, other than those of nations, and of states, counties, towns, cities, villages, or other political divisions of the country, are secured by mortgages on the property of the corporations issuing them.

Bonds are generally issued in denominations of $100, $500, or $1000, just as paper money is issued in denominations of $1, $5, $20, etc. Occasionally bonds are issued in denominations smaller than $100, as was done with the Spanish War bonds, some of which were $20 bonds.

Thus, if a corporation wishes to borrow $50,000, it may issue fifty 1000-dollar bonds, one hundred 500-dollar bonds, or five hundred 100-dollar bonds.

Each bond is numbered so that it may be distinguished from the other bonds of the same issue. Some bonds are so drawn that the owner's name must be registered, with the number of the bond, in the books of the government or corporation issuing them, so that the interest is payable only to the owner or his order, and is sent to him when due.

Other bonds, like the one shown on page 298, have attached to them as many interest coupons as there are interest periods. Each coupon is payable to the bearer, and bears the date when it is due, so that the holder of the

No. 1

500.

The Trap Rock Company

The Trap Rock Company, a corporation, duly created, organized and existing under and pursuant to the laws of the State of New York, for value received hereby promises to pay to the bearer, or to the registered holder hereof on the first week day of July A.D. 1954 at the office of the Columbia Trust Company, in the city of Middletown, State of Connecticut, upon surrender of this Bond

FIVE HUNDRED DOLLARS

in gold coin of the United States of America of equal weight, fineness and standard of weight and fineness existing at the time of payment, and the Trap Rock Company also promises to pay interest thereon from the said date at the rate of Five Hundred Dollars per annum, payable annually from the date hereof, by the delivery at the office aforesaid to the bearer and upon presentation and surrender of the respective annual coupons hereto attached as they severally become due, until maturity of the Bond.

This Bond is subject to the terms and conditions endorsed hereon and shall be governed by the laws of the State of New York.

In Witness Whereof, said Trap Rock Company has caused its corporate seal to be hereto affixed and attested by its Treasurer and its corporate name to be signed to these presents by its President this ____ day of A.D. 1904.

Trap Rock Company,

Attest

Richard Roe
Treasurer

By

John Doe
President

THE TRAP ROCK COMPANY $2500
ON THE FIFTEENTH DAY OF JULY, 1924,
WILL PAY TO THE BEARER
AT THE OFFICE OF THE COLUMBIA TRUST COMPANY IN THE CITY OF MIDDLETOWN, CONNECTICUT, TWENTY FIVE ONE HUNDREDTHS DOLLARS BEING TWELVE MONTHS INTEREST DUE ON THIS DAY.
W. A. Beattie,
TREASURER

THE TRAP ROCK COMPANY $2500
ON THE FIFTEENTH DAY OF JULY, 1914,
WILL PAY TO THE BEARER
AT THE OFFICE OF THE COLUMBIA TRUST COMPANY IN THE CITY OF MIDDLETOWN, CONNECTICUT, TWENTY FIVE ONE HUNDREDTHS DOLLARS BEING TWELVE MONTHS INTEREST DUE ON THIS DAY.
W. A. Beattie,
TREASURER

THE TRAP ROCK COMPANY $2500
ON THE FIFTEENTH DAY OF JULY, 1904,
WILL PAY TO THE BEARER
AT THE OFFICE OF THE COLUMBIA TRUST COMPANY IN THE CITY OF MIDDLETOWN, CONNECTICUT, TWENTY FIVE ONE HUNDREDTHS DOLLARS BEING TWELVE MONTHS INTEREST DUE ON THIS DAY.
W. A. Beattie,
TREASURER

bond may collect his interest when due by merely cutting off the coupon and presenting it at the place named for payment, or by depositing it in his bank for collection. The bond on page 298 had originally thirty interest coupons attached. The last three show in the form given.

Summary

493. Bonds are *the interest-bearing notes of governments and corporations, given under seal.*

494. Registered bonds are *bonds that are recorded by number in the name of the owner, on the books of the government or corporation that issued them.*

495. A **coupon** is *an interest certificate attached to a bond.*

496. Coupon bonds are *bonds to which interest coupons are attached.*

497. The **face** of a bond is the sum mentioned in the bond.

498. Comparisons

1. *Shares of stock* represent the *property* of a corporation, while *bonds* represent *debts* of the corporation; *stockholders* are, therefore, the *owners* of the *property* of the corporation, while *bondholders* are its *creditors.*

2. The income on shares of stock is in the form of *dividends*, and its amount *fluctuates* (except on preferred stock), *depending* on the prosperity of the corporation's business; whereas the income on bonds is in the form of *interest at a fixed rate*, and must be paid, regardless of the condition of the business.

3. The *market value* of bonds, like that of stocks, fluctuates from day to day; they may be *at par*, *at a premium*, or *at a discount.*

4. Bonds are bought and sold through brokers in the same manner as shares of stock, and at the same rates of brokerage.

5. The market values of bonds are quoted in the same way as the market values of shares of stock; *e.g.* "U. S. 5's, 110," means that *one dollar* of United States bonds bearing 5% interest is worth \$1.10.

6. The premium, discount, income, and brokerage on bonds is computed on the *par value.* In this respect, do bonds resemble, or differ from capital stock?

499. *Oral*

1. What is the par value of ten 500-dollar bonds?

2. When selling at 110, what is their market value?

3. What must be paid for five 100-dollar bonds when they are quoted at 120?

4. When bonds are quoted at 80, how many dollars of bonds can be bought for \$400?

5. What is the annual interest on a four per cent 500-dollar bond? On a $4\frac{1}{2}$% 1000-dollar bond?

6. How many dollars of 6% bonds must I own in order to receive an annual income of \$1200 from them? A semi-annual income of \$1200?

7. How many 5% 100-dollar bonds must I own in order to receive from them an annual income of \$750? To receive an annual income of \$1000? To receive a semi-annual income of \$1000?

8. A farmer invested \$9000 in railroad bonds at 90. How many dollars of bonds did he buy? How many bonds did he obtain if they were 500-dollar bonds?

9. A speculator invested \$1050 in 6% bonds at 105. How many dollars of bonds did he buy? What was the annual interest on them?

10. A broker bought for his principal $10,000 of railroad bonds at 89⅞, charging ⅛% brokerage. What did the bonds cost the principal? What did the broker receive for his services?

11. A broker sold $10,000 of bonds for his principal at 89⅞, charging ⅛% brokerage. How much did the principal receive? How much did the broker receive?

12. A $500 bond was sold for $400. The selling price was what per cent of the par value? The bond was sold at what per cent below par? If this was the regular market value, how were that kind of bonds quoted? If it was a 5% bond, what was the annual interest on the bond?

13. A man invested $7800 in bonds at 77⅞ %, paying ⅛% brokerage. How many dollars of bonds did he buy? If they were 4% bonds, what was the annual interest?

14. When the market value of a $1000 bond is $1030, how are the bonds quoted?

15. If a man invests $1200 in 7% bonds quoted at 120, how much money does he receive from them annually?

16. How many 1000-dollar 3% bonds must a man buy to secure an annual interest of $600? What will they cost, if bought at 90?

500. *Written*

1. *a.* What is the market value of $40,000 of U. S. 2% registered bonds due in 1930, when quoted at 104?

b. What is the annual interest?

c. How many dollars of these bonds will $20,800 buy?

d. What is the annual interest on them?

e. What quantity of these bonds will $35,360 buy?

f. What will be the yearly interest on them?

2. At one time, U. S. 4 % coupon bonds were quoted at 120.

a. What was the cost of $21,500 of those bonds?

b. What interest did the government pay annually on them?

c. How many dollars of bonds could be bought for $84,600?

d. What interest did the government pay annually on those bonds?

3. Milwaukee Electric Railway $4\frac{1}{2}$ % bonds once sold at 90.

a. How many dollars of the bonds would $81,000 buy?

b. What must be paid for $19,500 of these bonds?

c. What interest is the railroad required to pay annually on that amount of bonds?

d. What amount of the bonds would $10,800 buy?

e. What interest would the railroad be required to pay annually on that amount of bonds?

f. A man invested $63,000 in these bonds. What interest did the railroad pay him annually?

g. How much must be invested in these bonds to secure the payment of $2700 yearly interest from the railroad company?

4. A man bought $198,000 of Atchison, Topeka, and Santa Fé R.R. 4 % bonds at $96\frac{3}{8}$, paying $\frac{1}{8}$ % brokerage.

a. What did he pay for the bonds?

b. He sold them at $100\frac{1}{8}$, paying $\frac{1}{8}$ % brokerage. How much did he receive for them?

c. How much did he gain by the speculation?

d. With the proceeds of the sale, he bought Allegheny and Western first mortgage 4 % bonds at $98\frac{7}{8}$, paying $\frac{1}{8}$ % brokerage. What amount of bonds did he buy?

5. A man sold 400 shares of stock, yielding $2\frac{1}{2}$ % semi-annual dividends, at $102\frac{1}{8}$, and with the proceeds bought Toledo, St. Louis, and Western R.R. 4 % bonds at $79\frac{7}{8}$, paying $\frac{1}{8}$ % brokerage for each transaction. What amount of bonds did he buy?

6. On the 18th of February, 1908, the 4% bonds of the Adams Express Company were quoted at 88. Make and solve four problems from the data here given.

7. Metropolitan Street Railway 5% bonds once sold at $103\frac{1}{4}$. Make and solve four problems using this fact.

RATIO

501. *The* **ratio** *of two numbers is the quotient obtained by dividing one number by the other, e.g.:*

a. The ratio of 6 to 3 is $6 \div 3$, or 2.

b. The ratio of 3 to 6 is $3 \div 6$, or $\frac{1}{2}$.

c. The ratio of 11 to 7 is $11 \div 7$, or $1\frac{4}{7}$.

d. The ratio of 7 to 11 is $7 \div 11$, or $\frac{7}{11}$.

502. *Oral*

What is the ratio of

1. 15 to 5?
2. 5 to 15?
3. 24 to 8?
4. 8 to 24?
5. 100 to 1?
6. 1 to 100?
7. 30 to 3?
8. 3 to 30?
9. 81 to 27?
10. 27 to 81?
11. 1 to 19?
12. 19 to 1?
13. 99 to 3?
14. 3 to 99?
15. 625 to 25?
16. 25 to 625?
17. 7 to 17?
18. 17 to 7?
19. 36 to 35?
20. 35 to 36?
21. 14 to 42?
22. 42 to 14?
23. 6 to 9?
24. 9 to 6?

503. Division is one method of comparing numbers. By it we determine, not how much greater one number is than another, but *how many times as great;* thus, 15 is three times as great as 5, and 5 is $\frac{1}{3}$ as great as 15.

504. *The numbers compared in determining the ratio of one number to another are the* **terms** *of the ratio; the first term of a ratio is its* **antecedent**; *the second term of a ratio is its* **consequent**;

the sign (:) of ratio is the sign of division with the horizontal line omitted; *e.g.* the ratio of 14 to 2 is expressed, $14 : 2 = 7$; 14 is the antecedent, 2 is the consequent, and 7 is the ratio.

505. *The antecedent and consequent taken together are called a* **couplet.**

506. *The* **inverse ratio** *of two numbers is the quotient of the second divided by the first; e.g.* the inverse ratio of 18 to 3 is $3 \div 18$ or $\frac{1}{6}$. The quotient of the first divided by the second is called the **direct ratio.**

507. *Oral*

Name the antecedent and the consequent and give the ratio of each of the following couplets:

1. 18 : 6	**3.** 16 : 64	**5.** 81 : 9	**7.** 5 : 29	**9.** $\frac{7}{8} : \frac{3}{8}$
2. 24 : 3	**4.** 49 : 7	**6.** 13 : 4	**8.** $3 : \frac{1}{4}$	**10.** $\frac{2}{5} : 4$

508. Since, in a direct ratio, the *antecedent* is always a **dividend,** the *consequent* a **divisor,** and the *ratio* a **quotient,** the *antecedent* must be the **product** of the *consequent* and *ratio.* Therefore, the relations of product and factors will enable us to determine any one of these numbers when the other two are given.

509. *Oral*

Find the value of x in each of the following ratios:

1. $51 : 17 = x$	**4.** $x : 19 = 2$	**7.** $\frac{3}{4} : \frac{1}{2} = x$	**10.** $\frac{3}{3} : \frac{3}{4} = x$
2. $35 : x = 5$	**5.** $95 : x = 5$	**8.** $x : \frac{2}{3} = \frac{3}{7}$	**11.** $\frac{8}{3} : \frac{4}{3} = x$
3. $x : 4 = 3$	**6.** $x : 14 = \frac{1}{2}$	**9.** $\frac{11}{16} : x = 2$	**12.** $2\frac{1}{2} : x = 1\frac{1}{4}$

13. The ratio of the length to the breadth of a table is 3. If the length is 12 feet, what is the breadth? Illustrate by a drawing.

14. The ratio of the length to the breadth of a city lot is 2. If the breadth is 4 rods, what is the length? Illustrate by a drawing.

15. The ratio of the height of a boy to the height of a tree is $\frac{1}{7}$. If the tree is 35 feet high, how tall is the boy? Illustrate by a drawing.

16. What is the ratio of the length of a rod measure to the length of a yard stick?

17. *What is the ratio of*

a. One gallon to one quart?
b. Two gallons to 16 quarts?
c. One bushel to one pint?
d. Five dollars to 25 cents?
e. Eighty cents to one dollar?
f. One gram to one grain?
g. One square meter to one square decimeter?
h. One cubic inch to one gallon?
i. £1 to $1?
j. One mark to one cent?

18. Give two numbers whose ratio is 5.

19. Give two numbers whose ratio is 16.

20. Give two numbers whose ratio is $2\frac{1}{2}$.

21. Give two numbers whose ratio is $1\frac{1}{3}$.

22. The ratio of 25 miles to what distance is $12\frac{1}{2}$?

23. The ratio of what time to 3 months is 4?

24. When the consequent is greater than the antecedent, the ratio is what kind of a number?

25. When the ratio is greater than 1, how do the antecedent and consequent compare?

PROPORTION

15 : 3 compares how with 10 : 2?

\$48 : \$8 compares how with 12 da. : 2 da.?

\$3 : \$21 compares how with 2 men : 14 men?

15 apples : 30 apples compares how with 8 lb. : 16 lb.?

The answers to the above questions may be expressed:

$$15 : 3 = 10 : 2$$

$$\$48 : \$8 = 12 \text{ da.} : 2 \text{ da.}$$

$$\$3 : \$21 = 2 \text{ men} : 14 \text{ men}$$

$$15 \text{ apples} : 30 \text{ apples} = 8 \text{ lb.} : 16 \text{ lb.}$$

Of what is each of the above statements composed?

510. *An equality of ratios* is a **proportion.**

The first of the above proportions is read, "15 *is to* 3 *as* 10 *is to* 2." Read the others. Let each pupil in the class write three proportions. What must be true of two ratios that they may form a proportion?

511. *Complete the following proportions:*

a. 32 : 8 = 28 : ?

b. 16 : ? = 32 : 2

c. 45 : 9 = 10 : ?

d. 33 : 3 = ? : 2

e. 42 : 6 = 14 : ?

f. ? : 3 = 18 : 9

g. \$12 : \$6 = 6 da. : ? da.

h. 12 mi. : 24 mi. = 2 hr. : ? hr.

i. 21 ft. : 3 ft. = ? : 5¢

j. 6¢ : 60¢ = 8 lb. : ?

k. 8 girls : 16 girls = \$32 : ?

l. 3 : ? = 11 : $5\frac{1}{2}$

m. 100 : 1000 = ? : 70

n. 6% : 20% = 9% : ?

o. 8 mo. : 1 year = \$60 : ?

512. *The numbers that form a proportion* are the **terms** *of the proportion.*

513. *The first and fourth terms of a proportion are the* **extremes**; *the second and third terms are the* **means**; *e.g.* in the

proportion $49 : 7 = 350 : 50$, 49 and 50 are the extremes and 7 and 350 are the means.

NOTE. — The sign (::), called the sign of proportion, is sometimes used instead of the sign of equality, which means the same.

514. In any proportion, the first term is the product of the second term and ratio; and the third term is the product of the fourth term and ratio, thus,

$$35 : 7 = 15 : 3$$

may be written, $7 \times 5 : 7 = 3 \times 5 : 3$,

and any proportion may be written,

2d term × ratio : 2d term = 4th term × ratio : 4th term.

Whence, the product of the means = 2d term × 4th term × ratio, and the product of the extremes = 2d term × ratio × 4th term.

How does the product of the means compare with the product of the extremes?

515. *Oral*

In the following proportions, verify the principle established above, that *the product of the means is equal to the product of the extremes*, thus, In the proportion, $15 : 5 = 12 : 4$,

The product of the means is 5×12, or 60.

The product of the extremes is 15×4, or 60.

1. $9 : 3 = 6 : 2$ **3.** $3 : 60 = 6 : 120$ **5.** $7 : 2 = 28 : 8$

2. $63 : 21 = 3 : 1$ **4.** $14 : 28 = 2 : 4$ **6.** $3 : 9 = 9 : 27$

516. *Written*

1. Complete the proportion, $88 : 24 = 264 : x$, by finding the value of x.

Solution

$88x = 24 \times 264$. Why?

$x = \frac{\overset{3}{\cancel{24}} \times \overset{}{264}}{\underset{11}{\cancel{88}}} = 72$. Therefore, $88 : 24 = 264 : 72$. *Ans.*

2. Complete the proportion, $92 : x = 69 : 12$.

Solution

$69x = 92 \times 12$. Why?

$$x = \frac{\overset{4}{\cancel{92}} \times \overset{4}{\cancel{12}}}{\underset{23}{\cancel{69}}} = 16.$$ Therefore, $92 : 16 = 69 : 12$. *Ans.*

Complete the following proportions:

3. $50 : 2 = 125 : x$

4. $4 : 17 = x : 34$

5. $24 : x = 18 : 30$

6. $x : 10 = 21 : 35$

7. $55 : 20 = x : 28$

8. $x : 51 = 65 : 39$

9. $455 : 273 = x : 66$

10. $x : 240 = 209 : 264$

11. $\$110 : \$88 = x : 28$

12. 10 A. $: 35$ A. $= \$25 : x$

13. 10 yd. $: 50$ yd. $= \$20 : x$

14. $81 : 84 = x$ bu. $: 132$ bu.

15. $x : 5 = \$\frac{3}{4} : \$3\frac{3}{4}$

16. $\$x : \$4 = \frac{1}{3} : \frac{2}{9}$

17. 888 ft. $: 74$ ft. $= x : 111$ hr.

18. $\frac{2}{7} : \frac{8}{9} = \frac{13}{70} : x$

PROBLEMS SOLVED BY PROPORTION

517. *Oral*

1. In the proportion, $20 : 80 = 3 : x$, how does 80 compare with 20? How must the value of x compare with 3?

2. In the proportion, $x : 18 = 23 : 46$, how does 23 compare with 46? How does the value of x compare with 18?

3. If the proportion, $? : ? = 3 : 90$, is completed by supplying a first term and second term, how must the second term compare with the first term?

4. In any proportion, if the fourth term is greater than the third, how must the second compare with the first? If the fourth term is less than the third, how must the second compare with the first?

Turn to § **516** and, without referring to your answers, tell whether the value of x, in each proportion, is greater or less than the other term in the same ratio.

518. *Written*

1. If 12 yards of cloth cost \$14, what will 132 yards cost at the same rate?

Since the ratio of 12 yards to 132 yards is the same as the ratio of \$14 to the required number of dollars, the numbers in this problem may form a proportion.

Let x represent the required number of dollars and let it be the fourth term, thus,

$$? : ? = 14 \text{ yd.} : x \text{ yd.}$$

Then, since 132 yards will cost *more* than 12 yards, the fourth term will be *greater* than the third term; therefore the second term must be greater than the first term, and the proportion is

$$12 \text{ yd.} : 132 \text{ yd.} = \$14 : \$x.$$

Solving, $12x = 132 \times 14$. Why?

$$x = \frac{\overset{11}{\cancel{132}} \times 14}{\cancel{12}} = 154.$$

Therefore, 132 yards will cost \$154. *Ans.*

There are many ways of stating a proportion for the solution of a problem, but it is well to adopt some one of them, and use it whenever a problem is to be solved by proportion.

The following outline has been found helpful:

1. *Let the fourth term be x, the required number.*
2. *Let the third term be the given number that denotes the same kind of quantity as the required answer.*
3. *Determine, by reading the problem, whether the answer will be greater or less than the third term, and arrange the other two given numbers accordingly, as the first and second terms of the proportion.*
4. *Solve the proportion.*

Solve the following problems by proportion:

2. At the rate of 5 tons for \$31, how many tons of coal can bought for \$217?

3. If a man can earn \$217 in 43 days, how much can he earn in 301 days?

4. Traveling at the rate of 49 miles in 196 minutes, in how many minutes will a trolley car run 7 miles?

5. What must be paid for 5700 cubic feet of gas when 3800 cubic feet cost \$3.61?

6. What will 8 tons of coal cost, when $17\frac{1}{2}$ tons cost \$78.75?

7. How far will a train run in 7 hours, at the rate of 656 Km. in 8 hours?

8. What will it cost to buy a new arithmetic for each pupil in a class of 19 pupils, when 24 arithmetics cost \$13.20?

9. A messenger boy rode his bicycle 126 miles in 7 days. How far would he ride in 29 days at the same average rate per day?

10. Write the numbers 27, 18, 26, 39, so as to form a proportion.

11. A farmer sowed 6 bushels of grain on $4\frac{4}{5}$ acres of land. At the same rate, what quantity of seed is required for $13\frac{1}{2}$ acres?

12. If $26\frac{1}{4}$ gal. of oil can be extracted from $\frac{3}{4}$ T. of cotton seed, how much oil can be produced from 375 lb. of seed?

13. Paul earns 75¢ a day; his father earns \$3.75 a day. In how many days will Paul earn as much as his father earns in 61 days?

14. In a mile foot-race, A gained on B at a uniform rate of 17 ft. in 15 sec. If A finished in 4 min. 45 sec, he was how many feet ahead of B?

15. C and D bought for $18.75 a load of hay weighing $1\frac{1}{4}$ tons. 1200 lb. of the hay was put into C's barn and the remainder into D's. How much should D pay?

16. If 33 bushels of wheat will make 7 barrels of flour, how many bushels are required for $2\frac{1}{3}$ barrels at the same rate?

17. If $\frac{3}{8}$ of a tract of land is sold for $3900, what is $\frac{4}{5}$ of the tract worth at the same price per acre?

18. If 315 l. of water fell on the roof of my house during a rainstorm of two hours, how long must it rain at the same rate in order that enough water may run from the roof to fill a rectangular cistern 35 dm. long, 3 m. wide, and 75 cm. deep?

19. *a.* When exchange on Berlin is at the rate of 4 marks for 97 cents, what must be paid in Baltimore for a Berlin draft for 3476 marks?

b. What is the face of a draft that may be bought for $176.54?

20. *a.* When exchange on Antwerp is at the rate of 15.525 francs for $3, what must be paid for a draft for 646.75 francs?

b. What is the face of a draft that can be bought for $850?

21. A contractor engaged to construct a sewer two miles long for $58,080. How much has he earned when he has completed 2112 feet of the sewer?

22. If the interest on a sum of money for one year is $360, what is the interest on the same sum for 15 months, at the same rate?

23. If $800 yield $48 interest in a certain time, how large a sum will yield $216 in the same time at the same rate?

24. If stock bought at 80 yields 6% income on the money invested, what per cent would it yield if bought at 120?

25. If a sum of money will buy provisions to last 250 soldiers for 30 days, the same sum will purchase provisions to last 75 soldiers how long?

26. How many yards of carpet 27 inches wide are required to cover as much floor space as are covered by 26 yards of carpet 1 yard wide?

27. If a train runs 140 mi. in 4 hr. 30 min., what is the rate per hour?

28. How many men must be employed to accomplish in 35 days what 55 men can accomplish in 21 days?

29. Frank's net profit from a flock of 24 hens for one year was \$17.60. How many hens must be added to the flock in order that the yearly profit, at the same rate, may be \$44?

PARTITIVE PROPORTION

519. *Separating a number into two or more parts that have a given ratio* is called **partitive proportion**; *e.g.* if the number 55 is divided into four parts, having the ratio of 1, 2, 3, and 5, the parts are 5, 10, 15, and 25; for $1 : 2 = 5 : 10$, $2 : 3 = 10 : 15$, $3 : 5 = 15 : 25$.

520. *Written*

1. Separate 25 into two parts having the ratio of 2 to 3.

Solution

Let $2x$ represent one part.

Then $3x$ will represent the other part.

(1) Adding, $5x = 25$, the sum of the two parts.

(2) Dividing (1) by 5, $x = 5$

(3) Multiplying (2) by 2, $2x = 10$ } *Ans.*

(4) Multiplying (2) by 3, $3x = 15$ }

Or,

Take $\frac{2}{5}$ and $\frac{3}{5}$ of 25.

2. Divide \$87 into four parts having the ratio of 1, 2, 5, and 7.

Solution

Let x, $2x$, $5x$, and $7x$ represent the four parts.

(1) Then, adding, $15x = \$87$

(2) Dividing (1) by 15, $x = \$5\frac{4}{5}$

(3) Multiplying (2) by 2, $2x = \$11\frac{3}{5}$

(4) Multiplying (2) by 5, $5x = \$29$

(5) Multiplying (2) by 7, $7x = \$40\frac{3}{5}$

Ans.

Or,

Take $\frac{1}{15}$, $\frac{2}{15}$, $\frac{5}{15}$, and $\frac{7}{15}$ of \$87.

3. Divide 91 into two parts that shall be to each other as 3 to 4.

4. Divide as indicated:

	Number Divided	Number of Parts	Ratio of Parts
a.	1200	2	11, 13
b.	3690	3	2, 7, 1
c.	\$923	4	4, 5, 1, 3
d.	3179	4	1, 2, 3, 5
e.	418 bu.	4	4, 5, 7, 3
f.	624 miles	5	2, 7, 1, 2, 1
g.	\$2640	5	9, 8, 7, 6, 3
h.	430	5	1, 2, 6, $\frac{2}{5}$, $\frac{3}{5}$
i.	18,000	2	97, 83
j.	337	3	1, 4, 7

5. Joe and Harry earn \$25 a month. Harry earns \$3 while Joe is earning \$2. How much per month does each earn?

6. Mr. Olsen and his two sons together received \$192 on pay day, Mr. Olsen receiving \$4 as often as each of his sons received \$2. How much did each receive?

7. An orchard contained twice as many pear trees as peach trees and four times as many apple trees as pear trees. If the three kinds of trees numbered 99, how many were there of each kind?

8. A kind of medicine is composed of licorice, ipecac, and muriate of ammonia in the ratio of 10, 3, and 2. In three pounds (Avoirdupois) of this medicine there are how many grains of each of the three ingredients?

PARTNERSHIP

521. *When two or more individuals own and conduct a business in common* they are called **partners**, *and their association in business* is called a **partnership.**

A partnership is different from a stock company in that each partner has a voice in the actual management of the business, and is personally liable for all the debts of the firm.

The profits and losses of a partnership are shared by the partners according to the amount of capital that each has invested in the business, unless by contract they agree otherwise.

522. *Written*

1. A, B, and C formed a partnership, furnishing $800, $1000, and $1200 capital, respectively. They gained $1500. Divide the gain among the partners in proportion to their capital.

2. Mr. Wilson and Mr. Mead entered into partnership. Mr. Wilson's capital was $3000, and Mr. Mead's $2000. They gained $1500. What was each partner's share of the gain?

3. Jones & Smith were partners for a year, with a capital of $3000 and $5000 respectively. They gained $2000. Find each one's share of the gain.

4. Three men form a partnership. A invests $1250, B $2000, and C $1550. They gain $1200. What is each man's share of the gain?

5. Three men hired a coach to convey them to their homes. A's home was 20 miles away, B's 24 miles, and C's 28 miles. They paid $24 for the coach. What ought each to pay?

6. A cargo of wheat valued at $4500 was entirely destroyed. One third of it belonged to A, two fifths to B, and the remainder to C. What was each one's share of the loss, there being an insurance of $3600?

7. A man fails in business owing $15,000, and his available means amount to only $9000. How much will two of his creditors receive, to one of whom he owes $3000 and to the other $4500?

8. A and B gain in business $2500, of which A's share is $1000 and B's $1500. What part of the capital does each furnish, and what is the investment of each if their joint capital is $16,000?

9. A, B, and C own $600 worth of timber land, which they divide in proportion of 3, 5, and 7. Find the value of each part.

10. A, B, and C bought a business for $6000, A furnishing $2500 of the capital, B $1500, and C the remainder. If the value of the business increases to $8000, and C buys out A and B, how much should he pay each of them?

11. A man failing in business owes $10,800, and has property worth $7200 to be divided among his creditors in proportion to their claims. How much will be received by a creditor whose claim is $180?

12. A junior partner owns a $\frac{1}{16}$ interest in a business, the annual net profits of which are $90,000. He also receives a salary of $2500 a year.

a. What is his annual income?

b. A good concern offers to buy his interest in the business for $100,000, giving in payment a good real estate mortgage paying 5% interest, and retaining him in the business at a salary of $3000 a year. Would his income be increased or diminished by accepting this offer, and how much?

523. When the capital of the partners is not employed for the same time.

Written

1. A and B formed a partnership. A furnished \$500 for 8 months and B \$600 for 10 months. They gained \$360. What was each partner's gain?

Solution

A \$500 for 8 mo. = \$4000 for 1 mo.
B \$600 for 10 mo. = 6000 for 1 mo.
\$10000

A's share = $\frac{4}{10}$ of \$360, or \$144.
B's share = $\frac{6}{10}$ of \$360, or \$216.

The use of \$500 for 8 months is equivalent to the use of \$4000 for 1 month; and the use of \$600 for 10 months is equivalent to the use of \$6000 for 1 month. Consider A's capital to be \$4000 and B's \$6000. A's share of the gain = $\frac{4}{10}$; B's share of the gain = $\frac{6}{10}$.

2. A commenced business with \$10,000 capital. Four months later B put in \$10,500. Their profits at the end of a year were \$5100. What was each man's share of the gain?

3. Three persons loaned sums of money, at the same rate, for which they received \$1596 interest. The first loaned \$4000 for 12 mo., the second \$3000 for 15 mo., and the third \$5000 for 8 mo. How much interest did each receive?

4. A, C, and H form a partnership. A puts in \$8000, C \$5000, H \$10,000. A's capital remains in the business 8 mo., C's 9 mo., H's 12 mo. The net gain is \$6900. Find each man's share of the gain.

5. A and B were in partnership for 2 years. A at first invested \$2000, and B \$2800. At the end of 9 months A took out \$700, and B put in \$500. They lost in the two years \$3740. Apportion the loss.

6. A's capital was in business 6 months, B's 7 months, and C's 11 months. A's gain was \$600, B's \$1400, and C's \$990. Their joint capital was \$7800. What was each man's capital?

7. A put \$600 in trade for 5 months, and B \$700 for 6 months. They gained \$228. What was each man's share?

8. April 1, 1905, A goes into business with a capital of \$6000; July 1, 1905, he takes in B as a partner with a capital of \$8000; and Oct. 1, 1906, they have gained \$2900. Find the gain of each.

9. A merchant failed in business, owing A \$3000, B \$1500, C \$2400, and D \$600. His assets are \$5000, and the expense of settling up his business will be \$500. What will each creditor receive?

10. A and B were in partnership. B furnished \$18,000 for a year, and his share of the gain was \$1296. A invested his capital for 9 months, and his share of the gain was \$1620. What was A's capital?

REVIEW AND PRACTICE

524. *Oral*

1. What is the meaning of "Baltimore and Ohio, $85\frac{3}{8}$"?

2. What is the cost of 10 shares of railroad stock at $89\frac{7}{8}$; brokerage $\frac{1}{8}\%$?

3. How many dollars of bonds will \$10,500 buy, when they are at 5% premium?

4. What is the income from 10 shares of Lighting Company stock when it pays an annual dividend of 4%?

5. How many dollars of 3% government bonds must I own in order to receive \$30 a year in interest?

6. What is the ratio of 480 to 48? Of 48 to 480?

7. Complete the proportion $x : 16 = 5 : 20$.

8. Divide 60 into parts having the ratio of 1, 2, and 3.

9. Two boys, A and B, bought some oranges for 45 cents. In sharing them, A took two oranges as often as B took three. How much of the cost should each pay?

10. The ratio of a boy's age to his father's age is the ratio of 1 to 7. If the father is 32 years old, what is the boy's age?

11. What is the difference between bonds and capital stock?

12. Draw a horizontal line on the blackboard. Draw a vertical line cutting off 25 % of the horizontal line. Draw a line $\frac{2}{3}$ as long as the first one.

13. A grocer sold some damaged goods for $\frac{2}{3}$ of their cost. What per cent did he lose?

14. A farmer sold 90 % of his crop of potatoes and had 45 bushels left. How many bushels did he raise?

15. Give the common fractions equivalent to the following per cents: 50 %, $33\frac{1}{3}$ %, 25 %, 20 %, $16\frac{2}{3}$ %, $66\frac{2}{3}$ %, 75 %, $62\frac{1}{2}$ %, $87\frac{1}{2}$ %, $12\frac{1}{2}$ %, 10 %.

16. Frances missed $\frac{1}{10}$ of the words in a spelling lesson. What per cent of them did she spell correctly?

17. On what base are profit and loss computed?

18. Goods costing \$30 were sold for \$40. What per cent was gained?

19. Goods costing \$40 were sold for \$30. What per cent was lost?

20. I paid a bill of \$50, receiving 2 % discount for cash. How much did I pay?

21. I saved \$15 by paying cash for goods, thereby obtaining a discount of 5 %. What was the original amount of the bill? What was the net amount?

22. The list price of a set of books was \$80. The net price was \$60. What was the rate of discount?

23. Successive discounts of 10% and 10% are equivalent to what single discount?

24. Which of the following numbers are composite: 31, 49, 51, 87, 97, 39, 51, 71?

25. What is the bank discount on a note of \$100 for 90 days at 6%?

26. A man paid \$7.50 premium for insuring his household goods, the rate being 75¢ per hundred dollars. What was the face of his policy?

27. A merchant had his stock of goods insured for \$10,000 for three years, the rate being 1%. The agent who transacted the business for the insurance company received 25% of the premium. What was the amount of the agent's commission?

28. Without a rule, draw a line 5 decimeters long. Measure and correct it.

29. Put your finger on the door 40% of the distance from the top to the bottom.

30. Describe a board foot.

31. How many feet of lumber are there in a scantling 3″ by 4″, and 10 feet long?

32. How many quart cans of varnish will cover as much surface as twenty cans holding a gallon each?

33. What will a man receive for a 60-day note for \$200, without interest, if he has it discounted at date, money being worth 6%?

34. A 90-day note, dated April 1, 1908, matured when?

35. What is the meaning of each of the following exchange quotations: Paris 5.19; Brussels 5.20; Bremen $95\frac{1}{2}$; London 4.868?

36. When exchange on London is quoted at $4.86\frac{1}{2}$, what is the cost of a bill of exchange on London for £100?

37. How may we tell, without dividing, whether a number is divisible by 25 or not?

38. How may we know, without actual trial, that 24,374 will not exactly divide 2,903,076,543?

39. How many liters are equivalent to one cubic meter?

40. Name some object that is as large as a liter.

525. *Written*

1. A man paid a certain sum for a harness, five times as much for a carriage, two times as much for a horse as for the carriage, and then had left as much as he paid for the harness. He had $340 at first. What did each article cost?

2. What is the cost of 250 shares of railroad stock at $120\frac{3}{8}$, brokerage $\frac{1}{8}$?

3. A man invested $31,600 in mining stock at $78\frac{7}{8}$, brokerage $\frac{1}{8}\%$.

a. How many shares did he buy?

b. What was his income when the stock paid a dividend of $4\frac{1}{2}\%$?

4. A man sold railroad bonds at $93\frac{1}{8}$, paying $\frac{1}{8}\%$ brokerage. How many dollars of bonds must he sell to realize $18,600?

5. By proportion, find the cost of 780 barrels of flour, when 130 barrels cost $780.

6. $23\frac{1}{8}$ is the ratio of 42 to what number?

7. 69 is the ratio of what number to 793?

8. A man failed in business owing $17,500. He had property worth $10,000, which was used in part payment of his

debts, the creditors sharing according to the amounts owing to them. How much did a creditor receive to whom the debtor owed $3750?

9. A man pays $120 for three years' insurance on his buildings, the policies amounting to $\frac{4}{5}$ of the value of the buildings, and the rate being 60¢ per hundred for three years.

a. How much insurance does he carry?

b. What is the value of his buildings?

10. The tax rate one year in a village was $12\frac{1}{2}$ per $1000 of assessed valuation.

a. What was the assessed value of property which paid a tax of $125?

b. What was the entire tax budget, if the total valuation was $4,000,000?

11. An article was sold for $4.50 after successive discounts of 40% and 10% had been made. Find the list price.

12. A merchant can buy at one place a bill of goods listed at $1900, receiving successive discounts of 27% and 13%. At another place he can buy the same goods at the same list price with a single discount of 40%. Which is the better rate for the purchaser and how much better?

13. A note for $900 payable at a bank 90 days after date, without interest, was discounted 30 days after date, at the legal rate.

a. Write the note, dating it at your place of residence.

b. Compute the proceeds.

14. At what rate of interest will $400 earn $70 in 2 yr. 6 mo.?

15. A load of hay weighing 1 T. 2 cwt. cost $19.80. At the same price per ton, what was the cost of 1500 lb. of hay? Solve by proportion.

16. A, B, and C bought a piece of property for \$50,000, A furnishing \$12,500, B \$17,500, and C the remainder. They sold the property for \$69,000. Find each man's share of the gain.

17. A rectangular cellar measures 33 ft. by 21 ft. and 8 ft. deep, inside measure. The wall is of concrete $1\frac{1}{2}$ ft. thick. Find the number of cubic yards of concrete, allowing 4 cubic yards for openings.

18. Suppose Connellsville coal to be composed of the following substances: carbon, $60\frac{1}{2}\%$; sulphur, 1%; moisture, $1\frac{1}{4}\%$; ash, 8%; the remainder, volatile combustible matter. In one long ton of such coal there are:

a. How many pounds of carbon?

b. How many pounds of moisture?

c. How many pounds of volatile combustible matter?

19. *a.* If a miner receives 42 ¢ per ton for mining coal, mines 6 tons per day, 5 days in a week, 52 weeks in the year, and pays \$6 per month for rent of his house, how much per year has he for other purposes?

b. What per cent of his money does he pay for rent?

20. Coke is made from bituminous coal by heating it in ovens. This process is called "burning" coke. If three tons of coal will make two tons of coke, how much less will the Keystone Coal and Coke Company receive for 50,000 tons of coal by selling it at \$1.05 per ton, than by coking it and selling the coke at \$2.10 per ton?

21. A merchant imported from London 1000 sq. yd. of linoleum invoiced at 3*s.* 6*d.* per square yard.

a. What was the cost in English money?

b. What was the cost in United States money, computing the exchange value of £1 at \$4.866?

c. What was the duty at 20¢ per square yard and 20% ad valorem?

d. If the freight and other charges amounted to $28.14, what was the total cost per yard?

e. At what price per yard must the merchant sell it to make a profit of 40%?

22. Divide $17,500 among A, B, C, and D so that their shares shall be in the ratio of 4, 3, 2, and 11.

23. A, B, and C purchased an office building for $450,000. The net income from rents, after paying all expenses, was $22,500 per year, in which each man shared according to his share of the investment, B receiving $7500, A $12,500, and C the remainder. How much money did each contribute toward the purchase price?

24. How long must a sum of money be on interest to gain $350 interest if it gains $140 in 11 months?

25. How many men would be required to earn in 55 days as much money as 77 men can earn in 35 days, if all receive the same wages per day?

INVOLUTION

$2 \times 2 \quad = ?$	$3 \times 3 \quad = ?$
$2 \times 2 \times 2 \quad = ?$	$3 \times 3 \times 3 \times 3 = ?$
$2 \times 2 \times 2 \times 2 \quad = ?$	$5 \times 5 \quad = ?$
$2 \times 2 \times 2 \times 2 \times 2 \quad = ?$	$5 \times 5 \times 5 \quad = ?$
$2 \times 2 \times 2 \times 2 \times 2 \times 2 = ?$	$5 \times 5 \times 5 \times 5 = ?$

4 is what of 2 and 2?
8 is what of 2, 2, and 2?
81 is what of 3, 3, 3, and 3?
25 is what of 5 and 5?
2 is what of 4? Of 8? Of 16? Of 32? Of 64?

3 is what of 9? Of 81?

5 is what of 25? Of 125? Of 625?

How do the factors of 4 compare with each other? Of 8? Of 16? Of 32? Of 64? Of 9? Of 81? Of 25? Of 125? Of 625?

526. *The product of equal factors* is a **power.** Which of the numbers given above are powers?

527. *The product of two equal factors* is a **square**; *e.g.* 4 is the square of 2; 9 is the square of 3; 25 is the square of 5.

The area of a square surface is the product of its length and breadth. Since these are equal, the area of a square is the *square* of either dimension. For example, the area of a square whose side is 7 ft. is 49 sq. ft. 49 is the *square* of 7. Any number that is the product of two equal factors is called a square because it may be supposed to represent a square surface whose side is represented by one of the two equal factors.

528. *The product of three equal factors* is a **cube**; *e.g.* 8 is the cube of 2; 27 is the cube of 3; 125 is the cube of 5.

The contents of a cubical solid are equal to the *cube* of one of its dimensions. For example, 125 cu. in. are the contents of a cube whose edge is 5 in. The product of three equal factors is called a cube because it may always represent the contents of a cube whose edge is one of the three equal factors.

529. *The product of four equal factors* is called a **fourth power**; *the product of five equal factors* is called a **fifth power**, and so on; *e.g.* the fourth power of 3 is 81, the fifth power of 2 is 32. A number is sometimes called the *first power* of itself.

530. *An* **exponent** *is a figure placed above and at the right of a number to show which power of the number is to be taken; e.g.* in the expressions 11^2 and 5^4, the 2 shows that the square of 11 is to be taken, and the 4 shows that the fourth power of 5 is to be taken. $11^2 = 121$, is read, *The square of* 11 *is* 121. $5^4 = 625$, is read, *The fourth power of* 5 *is* 625.

531. *Finding the powers of numbers* is **involution.**

532. *Oral*

1. Give rapidly the values of the following expressions: 1^2; 2^2; 3^2; 4^2; 5^2; 6^2; 7^2; 8^2; 9^2; 1^3; 2^3; 3^3; 4^3; 5^3; 6^3; 7^3; 8^3; 9^3; 10^3; 12^2; 20^2; 40^2; 50^2; 90^2; 900^2; 2^4; 2^6; 3^4; 3^5; 5^4; $(\frac{1}{2})^2$; $(\frac{2}{3})^2$; $(\frac{2}{7})^2$; $(1\frac{1}{2})^2$; $(\frac{1}{2})^4$; $(\frac{1}{3})^4$; $(\frac{2}{3})^3$; $(1\frac{1}{3})^2$; $.3^2$; $.5^3$; $.2^3$; $.2^4$; $.1^2$; $.01^2$; $(\frac{1}{100})^2$.

2. What is the area of a square whose side is 3 ft.?

3. What is the area of a square whose side is 12 in.?

4. What is the area of a square whose side is $5\frac{1}{2}$ yd.?

5. What are the contents of a cube whose edge is 12 inches? 3 feet? 2 inches? 10 inches? $\frac{1}{2}$ inch? $\frac{3}{4}$ inch?

6. What is the fourth power of $\frac{2}{5}$?

7. 81 is the square of what number? 100 is the square of what number? $\frac{1}{4}$ is the square of what number? $\frac{4}{9}$ is the square of what number?

8. What number raised to the fourth power equals 81?

9. What number raised to the fifth power equals 32?

10. What is the cube of 4? Of 1? Of 0? Of $\frac{1}{3}$? Of $\frac{1}{10}$?

11. What is the square of .5? Of 1.2?

Written

533. *Find the powers indicated:*

1.	15^2	**7.**	13^5	**13.**	$(\frac{1}{2}\frac{1}{3}\frac{6}{1})^2$	**19.**	$(15\frac{1}{8})^2$	**25.**	$(.7\frac{3}{5})^2$
2.	33^2	**8.**	108^3	**14.**	2.7^3	**20.**	$(17\frac{2}{3})^3$	**26.**	$(24\frac{1}{3})^3$
3.	98^2	**9.**	25.3^2	**15.**	$(\frac{5}{11})^5$	**21.**	$(.08)^3$	**27.**	$(\frac{6}{7})^5$
4.	87^2	**10.**	4.06^2	**16.**	2.1^4	**22.**	$(1.07)^2$	**28.**	$(12\frac{1}{5})^3$
5.	18^3	**11.**	$.835^2$	**17.**	$(\frac{2}{1}\frac{8}{1}\frac{}{7})^2$	**23.**	$(2.1\frac{1}{2})^2$	**29.**	$(1000)^3$
6.	24^2	**12.**	4.05^3	**18.**	$.0035^2$	**24.**	$(.012)^4$	**30.**	$(.33\frac{1}{3})^6$

534. FINDING THE SQUARE OF A NUMBER EXPRESSED BY TWO FIGURES

$$\begin{array}{rcrcr} 37 = & & 30+7 & = & t+u \\ 37 = & & 30+7 & = & t+u \\ \hline 259 = & & 30\times 7+7^2 & = & t\times u+u^2 \\ 111 & = & 30^2+30\times 7 & = & t^2+t\times u \\ \hline 1369 = & & 30^2+2\times 30\times 7+7^2 & = & t^2+2\times t\times u+u^2 \end{array}$$

From the above illustration we may observe

a. That any number expressed by two significant figures may be separated into two parts, one of which is a certain number of tens, and the other a certain number of units.

b. That the square of a number expressed by two figures may be found by adding the *square of the tens*, *twice the product of the tens and units*, and *the square of the units;* thus,

$43 = 40 + 3$

$43^2 = 40^2 + 2\times 40\times 3 + 3^2 = 1600 + 240 + 9 = 1849$

535. *Oral*

Find the value of:

1. 21^2	**5.** 31^2	**9.** 45^2	**13.** 25^2	**17.** 65^2	**21.** 33^2
2. 22^2	**6.** 46^2	**10.** 52^2	**14.** 34^2	**18.** 55^2	**22.** 84^2
3. 41^2	**7.** 38^2	**11.** 91^2	**15.** 73^2	**19.** 42^2	**23.** 31^2
4. 44^2	**8.** 92^2	**12.** 82^2	**16.** 61^2	**20.** 43^2	**24.** 95^2

25. What is the area of a square meadow whose breadth is 62 rods?

EVOLUTION

$4 = 2\times 2$	$49 = 7\times 7$	$36 = 6\times 6$
$9 = 3\times 3$	$625 = 5\times 5\times 5\times 5$	$343 = 7\times 7\times 7$
$8 = 2\times 2\times 2$	$961 = 31\times 31$	$169 = 13\times 13$
$125 = 5\times 5\times 5$	$81 = 3\times 3\times 3\times 3$	$10{,}000 = 10\times 10\times 10\times 10$

536. *Oral*

1. 2 is what of 4? Of 8?
2. 3 is what of 9? Of 81?
3. 7 is what of 49? Of 343?
4. 5 is what of 25? Of 125? Of 625?
5. How do the factors of 49 compare? Of 169? Of 961? Of 81? Of 10,000? Of 36?

537. *One of the equal factors that produce a number* is a **root** *of that number; e.g.* 2 is a root of 4, of 8, and of 16; 5 is a root of 125 and of 625.

538. *One of the two equal factors that produce a number* is the **square root** *of that number; e.g.* 2 is the square root of 4; 3 is the square root of 9; 5 is the square root of 25.

539. *One of the three equal factors that produce a number* is the **cube root** *of that number; e.g.* 2 is the cube root of 8; 3 is the cube root of 27; 5 is the cube root of 125.

540. Other roots are known as the **fourth root, fifth root, sixth root,** etc., according to the number of equal factors which produce the corresponding power; *e.g.* 2 is the fourth root of 16, the fifth root of 32, and the sixth root of 64; 3 is the fourth root of 81, the fifth root of 243, and the sixth root of 729.

541. *The* **radical sign** ($\sqrt{\ }$) *placed over a number indicates that a root of the number is to be taken.*

542. *A small figure placed within the radical sign to indicate which root is to be taken* is called the **radical index.** When the square root is to be taken, the index is omitted, the radical sign only being used; *e.g.* $\sqrt{625}$ indicates that the square

root of 625 is to be taken; $\sqrt[4]{256}$ indicates that the fourth root of 256 is to be taken; $\sqrt[3]{1728}$ indicates that the cube root of 1728 is to be taken.

543. *A number whose indicated root can be exactly obtained* is a **perfect power**; *e.g.* $\sqrt{9}=3$, $\sqrt[4]{256}=4$, $\sqrt[5]{32}=2$; 9, 256, and 32 are perfect powers.

544. *A number whose square root can be exactly obtained* is a **perfect square**; 25, 144, 100.

545. *A number whose cube root can be exactly obtained* is a **perfect cube**; *e.g.* 8, 64, .027, 1728.

546. *Finding the roots of numbers* is **evolution.**

547. *Oral*

Read the following expressions and state the value of each:

1. $\sqrt{4}$	10. $\sqrt[3]{1728}$	19. $\sqrt{36}$	28. $\sqrt{8100}$	37. $\sqrt{.01}$
2. $\sqrt{49}$	11. $\sqrt{144}$	20. $\sqrt[3]{1}$	29. $\sqrt{1600}$	38. $\sqrt{.81}$
3. $\sqrt[3]{27}$	12. $\sqrt[4]{625}$	21. $\sqrt{1}$	30. $\sqrt{4900}$	39. $\sqrt{.64}$
4. $\sqrt[3]{125}$	13. $\sqrt{81}$	22. $\sqrt[2]{169}$	31. $\sqrt{14400}$	40. $\sqrt{.09}$
5. $\sqrt[4]{16}$	14. $\sqrt{100}$	23. $\sqrt{25}$	32. $\sqrt{3600}$	41. $\sqrt{625}$
6. $\sqrt[4]{81}$	15. $\sqrt[3]{343}$	24. $\sqrt{196}$	33. $\sqrt{6400}$	42. $\sqrt{.0625}$
7. $\sqrt{144}$	16. $\sqrt{121}$	25. $\sqrt{400}$	34. $\sqrt{.16}$	43. $\sqrt{1.44}$
8. $\sqrt[3]{1000}$	17. $\sqrt[3]{64}$	26. $\sqrt{900}$	35. $\sqrt{.04}$	44. $\sqrt[3]{27}$
9. $\sqrt[5]{32}$	18. $\sqrt[4]{10000}$	27. $\sqrt{2500}$	36. $\sqrt{.25}$	45. $\sqrt[3]{.027}$

46. What two equal fractions multiplied together will produce $\frac{1}{4}$? $\frac{1}{9}$? $\frac{1}{25}$? $\frac{1}{36}$? $\frac{4}{9}$? $\frac{16}{25}$? $\frac{9}{64}$?

47. The area of a square field is 100 square rods. How long is it?

48. What is the width of a square page whose area is 81 square inches?

49. Give the value of $\sqrt{\frac{1}{49}}$; $\sqrt{\frac{4}{49}}$; $\sqrt{\frac{16}{81}}$; $\sqrt{\frac{9}{64}}$; $\sqrt{\frac{1}{144}}$.

50. 7 is the square root of what number? 3? 11? 12? $\frac{1}{2}$? $\frac{1}{8}$? $\frac{2}{11}$? $\frac{3}{5}$? $\frac{7}{12}$?

51. Of what number is 7 one of the three equal factors?

52. Of what number is 12 one of the two equal factors?

53. Find one of the two equal factors of 121.

54. What is the product of three factors 7?

55. The cube root of 64 is how many times the square root of 64?

56. What is the number whose square root is 1? 2? 3? 4? 5? 6? 7? 8? 9?

57. Find the number whose square is 225.

58. Find the number whose square root is 169.

59. Name all the integers whose squares are less than 100.

60. Name all the integers whose square roots are less than 10.

61. The cube of 4 is the square of what number?

62. The square root of 25 is the cube root of what number?

63. One of the five equal factors that produce a number is called what?

SQUARE ROOT

548. When a number is a perfect square and contains but two or three figures, its square root may be obtained easily by **inspection**; that is, we may obtain the square root *mentally*, using no written work. But to obtain the square root of a large number, we generally require a direct method that may be expressed in writing. For example, let it be required to find the square root of 5329.

In discovering such a method let us first consider how a square is made from a given square root.

Copy the following table, filling in the results:

$1^2=$	$10^2=$	$100^2=$	$1000^2=$
$2^2=$	$20^2=$	$200^2=$	$2000^2=$
$3^2=$	$30^2=$	$300^2=$	$3000^2=$
$4^2=$	$40^2=$	$400^2=$	$4000^2=$
$5^2=$	$50^2=$	$500^2=$	$5000^2=$
$6^2=$	$60^2=$	$600^2=$	$6000^2=$
$7^2=$	$70^2=$	$700^2=$	$7000^2=$
$8^2=$	$80^2=$	$800^2=$	$8000^2=$
$9^2=$	$90^2=$	$900^2=$	$9000^2=$
	$99^2=$	$999^2=$	$9999^2=$

From the results found, we may generalize as follows:

The *square* of a number contains twice as many places, or twice as many less one, as the number itself contains, and

The square root of a number contains as many places as the square contains periods of two figures each, counting from the right, the left-hand period sometimes containing but one figure.

$\sqrt{5329}$, then, contains how many places?

Let t represent the tens' figure and u the units' figure of the root; then the root may be represented by $t+u$ and its square by $(t+u)^2$.

Multiplying as in section **534**,

$$\begin{array}{r} t+u \\ t+u \\ \hline t\times u+u^2 \\ t^2+t\times u \\ \hline (t+u)^2=t^2+2\times t\times u+u^2 \end{array}$$

This may be illustrated graphically as follows:

Let $t+u$ represent the parts of a line, thus, $\underline{\quad t \quad + \quad u \quad}$.

Construct a square on this line, thus:

This square contains a square whose area is t^2, another whose area is u^2, and two parts, each having an area equal to $t \times u$. The sum of all these parts is $t^2 + 2 \times t \times u + u^2$, which agrees with the square of $t + u$ as found above.

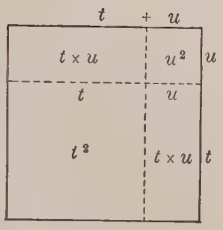


Fig. 1

Since $t + u$ may represent any number expressed by two figures, any square whose square root is expressed by two figures may be supposed to be the area of a square whose side is the required root. This square is always composed of two oblongs and two squares, similar to those in Fig. 1.

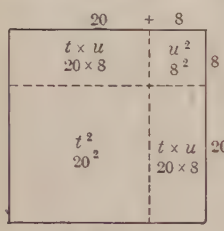


Fig. 2

For example, $28^2 = 784$, and $\sqrt{784} = 28$.

Let 28, or $20 + 8$, be represented by a line, $\underline{\qquad 20 \qquad + \quad 8 \qquad}$.

Its square, 784, is represented by the square, Fig. 2.

Whence we see that $\sqrt{784} = \sqrt{20^2 + 2 \times 20 \times 8 + 8^2}$,

or, $20 + 8$, or 28.

Returning to the example with which we began,

$$\sqrt{5329} = \sqrt{t^2 + 2 \times t \times u + u^2}, \text{ or } t + u.$$

$53 \cdot 29 (70 + 3 = 73$ *Ans.*

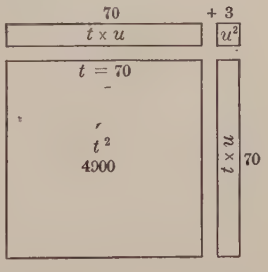
$70^2 \quad = 4900$

$70 \times 2 = 140 \mid 429$

$429 \div 140 = 3 \mid 429$

$140 + 3 = 143 \mid 000$

$143 \times 3 = 429 \mid$



By trial, we find that the greatest number of tens whose square is not greater than 5329 is 7 tens, or 70.

Let 70 be the side of a square. Its area is 70×70, or 4900.

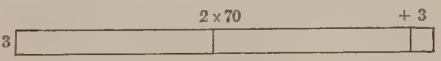


Fig. 3

We may call this 4900 sq. ft., sq. in., or any other kind of square units.

Subtracting 4900 from 5329, we find that there are 429 square units remaining. If we make additions to two sides of the square, we must make additions whose combined length is 70×2, or 140 units. If 429 square units are added, the width of the addition must be as many units as $429 \div 140$, or 3 units, with a small remainder. In order to make a complete square, we must again add a small square 3 units long and wide. The entire length of the three additions is $2 \times 70 + 3$, or 143 units, and the width is 3 units, as shown in Fig. 3. Their combined area is 143×3, or 429 square units, the exact number necessary to complete a square containing 5329 square units. The entire length of one side of this square is $70 + 3$, or 73 square units.

The following form shows the usual convenient arrangement of the work and the steps required:

```
 53·29)73  square root
  49
 ----
143|429
   |429
   ----
```

Find the greatest square (of tens) not greater than 53 (hundred). It is 49 (hundred). Its square root is 7 (tens).

Write 7 (tens) in the root, and subtract 49 (hundred) from 53 (hundred).

Bring down 29.

Multiply 7 (tens) by 2, and write the product, 14, at the left of 429 for a *trial divisor*. (This is 14 tens, or 140, but we omit the cipher because we shall have another figure to takes its place.) $429 \div 140 = 3$, with a small remainder.

Write 3 in the root, annex 3 to 14, making 143. Multiply 143 by 3, the new figure in the root, and write the product, 429, under 429, the trial dividend. If we subtract, there is no remainder, which shows that 73 is the exact square root of 5329.

If after the new root figure has been annexed to the trial divisor and the result multiplied by the new root figure, a product is obtained that is greater than the *trial dividend*, we must retrace our work and take the next lower figure in place of the new root figure, both in the root and in the divisor.

When the given number contains only three figures, we first find the greatest square not greater than the *left-hand figure*. For example,

```
 7·29)27
   4
 ----
47|329
  |329
  ----
```

How may we test the correctness of our answers?

Prove that 16 is the square root of 256.

Prove that $\sqrt{95{,}481} = 309$. Prove that $\frac{2}{3} = \sqrt{\frac{4}{9}}$.

549. *Written*

Find the indicated roots and test your work:

1. $\sqrt{6724}$	**6.** $\sqrt{1521}$	**11.** $\sqrt{1444}$	**16.** $\sqrt{1369}$	**21.** $\sqrt{1024}$
2. $\sqrt{2809}$	**7.** $\sqrt{6561}$	**12.** $\sqrt{7396}$	**17.** $\sqrt{2116}$	**22.** $\sqrt{841}$
3. $\sqrt{2025}$	**8.** $\sqrt{8281}$	**13.** $\sqrt{9025}$	**18.** $\sqrt{576}$	**23.** $\sqrt{324}$
4. $\sqrt{3844}$	**9.** $\sqrt{3721}$	**14.** $\sqrt{2209}$	**19.** $\sqrt{1225}$	**24.** $\sqrt{676}$
5. $\sqrt{5476}$	**10.** $\sqrt{3249}$	**15.** $\sqrt{6241}$	**20.** $\sqrt{361}$	**25.** $\sqrt{441}$

26. Find the side of a square whose area is 3969 sq. ft.

27. A square field contains 10 acres. How many rods long and wide is it?

28. A rectangular floor is twice as long as it is wide. Make a drawing to represent it. Its area is 2178 square feet. Find its dimensions.

29. Find one of the two equal factors that produce 7056.

30. A certain park is in the form of a rectangle 12 rods wide and 108 rods long. What are the dimensions of a square field that contains the same number of acres of land as this park?

31. A square park has an area of 529 sq. rd.

a. What are the dimensions?

b. What are the dimensions of a square park whose area is nine times as great?

32. The product of two equal factors is 5776. Find the factors.

33. The square of a certain number is 6241. Find the number.

550. When a number whose square root we are to find contains more than four figures, we know, by section **548**, that its square root contains more than two figures.

We may find the number of figures in the root by pointing off the given square into periods of two figures each, beginning at the right, if the number is an integer.

We may find the left-hand figure of the root by taking the square root of the greatest square not larger than the left-hand period in the square. This figure may represent the number of hundreds, or thousands, or units of any order above thousands, according to the number of periods in the square. Let us call this the *known* part of the root, and the figures yet to be found the *unknown* part of the root. As we find the successive figures of the root, the number of known figures will increase, while the number of unknown figures will diminish.

In every case, we may represent the *known part* of the root by k and the *unknown part* by u. Thus the square root of any number may be expressed by $k + u$ (the *known* part plus the *unknown* part), and the number itself may be represented by $k^2 + 2 \times k \times u + u^2$. This is always true, although the known part of the root is always increasing, and the unknown part is always diminishing, as we obtain the successive figures of the root. Likewise, the successive remainders may be represented by $2 \times k \times u + u^2$; and we may always find the value of the new figure, u, approximately, by dividing the remainder, with the new period annexed, by $2 \times k$ (twice the part of the root already found).

For example, let it be required to find $\sqrt{40030929}$.

Solution

40·03·09·29 | 6327 square root.
36

123 | 403 $= 2k \times u + u^2$ ($k = 60$; $2k = 120$)
369 ($403 \div 120 = 3$; $120 + 3 = 123$)

1262 | 3409 $= 2k \times u + u^2$ ($k = 630$; $2k = 1260$)
2524 ($3409 \div 1260 = 2$; $1260 + 2 = 1262$)

12647 | 88529 $= 2k \times u + u^2$ ($k = 6320$; $2k = 12640$)
88529 ($88529 \div 12640 = 7$; $12640 + 7 = 12647$)

Observe that the values of k (the known part) and u (the unknown part) change each time a new figure is obtained.

In the foregoing solution, all figures are omitted until they are needed. For example, the value of the first k is really 6000 (see Fig. 4), but we omit the last two ciphers, and call it 60, which is all that we need of the number now.

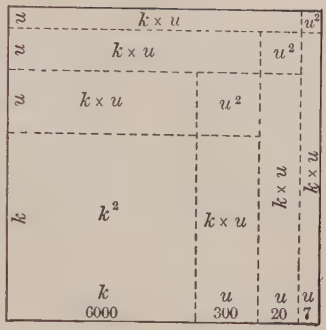


Fig. 4

Summary

To find the square root of an integer:

1. *Point off the integer into* **periods** *of two figures each, beginning at the right.*

2. *Find the* **greatest perfect square** *that is not greater than the left-hand period. Subtract it from the left-hand period and write its square root at the right of the given integer for the first figure of the root.*

3. *Bring down the next period.*

4. *Multiply the part of the root already found (assuming that a cipher is annexed), by 2, and write the product at the left of the remainder for a* **trial divisor.**

5. *Divide the remainder (with period annexed) by the trial divisor. Write the quotient in the root, and also annex it to the trial divisor, making the divisor complete.*

6. *Multiply the complete divisor by the new figure in the root. Subtract the product from the last remainder (with period annexed) and proceed as before until all the periods of the square have been used.*

7. *When the remainder (with period annexed) will not contain the trial divisor, place a cipher in the root, bring down another period, and annex a cipher to the trial divisor for a new trial divisor.*

551. *Written*

Find the square root:

1. 8836	6. 60,025	11. 235,225	16. 792,100
2. 585,225	7. 41,616	12. 16,184,529	17. 30,250,000
3. 137,641	8. 822,649	13. 5,322,249	18. 64,480,900
4. 80,089	9. 164,836	14. 826,281	19. 43,560,000
5. 101,761	10. 95,481	15. 788,544	20. 49,084,036

THE SQUARE ROOT OF A DECIMAL

552. *Oral*

1. Find the square of .2; .3; .8; .9; .01; .05; .07; .12; .08; .001; .005; .011; .008.

2. When we square a decimal of one place, how many decimal places do we obtain in the square? Of two places? Of three places? Of four places?

3. The number of decimal places in the square compares how with the number of decimal places in its square root?

4. The number of decimal places in the root compares how with the number of decimal places in its square?

5. Can a perfect square have one decimal place? Three decimal places? Seven decimal places? Five decimal places?

6. Can any number be multiplied by itself so as to obtain a number consisting only of a figure in units' place and a figure in tenths' place?

553. The above discussion forms the basis of the following

Summary

To find the square root of a decimal:

1. *Beginning at the decimal point, point off the decimal, both to the left (in a mixed decimal) and to the right, into periods of two figures each.*

2. *Find the square root as with integers.*

3. *Point off one decimal place in the root for every two decimal places in the square.*

NOTE 1. — If the given decimal contains an odd number of decimal places, a cipher must be annexed to complete the right-hand period.

NOTE 2. — The square root of a decimal or an integer that is not a perfect square may be found correct to any desired number of decimal places by annexing decimal periods of ciphers and continuing the work of extracting the square root.

554. *Written*

1. *Find the square root of:*

a. .0625	*d.* .0256	*g.* .00005625	*j.* 24.3049
b. .1225	*e.* .007921	*h.* 158.76	*k.* 6130.89
c. .8836	*f.* .092416	*i.* 29.0521	*l.* .000121

2. *Find, correct to two decimal places, the square root of:*

a. .256	*d.* 62.5	*g.* 3.	*j.* 4.096
b. .5	*e.* 45	*h.* 67.3	*k.* 31.3
c. 13	*f.* .75	*i.* 172.341	*l.* .016

555. THE SQUARE ROOT OF A COMMON FRACTION

$$\left(\frac{3}{7}\right)^2 = \frac{3}{7} \times \frac{3}{7} = \frac{3 \times 3}{7 \times 7} = \frac{3^2}{7^2} = \frac{9}{49}.$$

From the above illustration, tell how a common fraction may be squared.

How may we find the *square root* of $\frac{9}{49}$? Of $\frac{25}{64}$? Of $\frac{16}{81}$? Of $2\frac{1}{4}$? Of $\frac{8}{18}$?

Summary

To find the square root of a common fraction:

1. *Reduce the given fraction to lowest terms.*

2. *Extract the square root of the numerator and of the denominator.*

3. *If either numerator or denominator is not a perfect square, change the common fraction to a decimal and find the square root correct to the required number of decimal places.*

To find the square root of a mixed number:

1. *Change the mixed number to an improper fraction.*
2. *Find the square root by the method given above.*

556. *Oral*

1. Find the square root of: $\frac{9}{16}$; $\frac{25}{49}$; $\frac{64}{81}$; $\frac{121}{100}$; $\frac{16}{36}$; $\frac{63}{700}$; $2\frac{14}{25}$; $\frac{20}{45}$; $\frac{18}{72}$; $\frac{1}{25}$; $1\frac{9}{16}$.

557. *Written*

Find the square root of:

1. $\frac{256}{841}$
2. $\frac{12996}{50041}$
3. $\frac{42225}{75669}$
4. $\frac{2304}{2809}$
5. $\frac{8978}{17672}$
6. $\frac{253392}{369663}$
7. $\frac{729000}{6177796}$
8. $64\frac{43}{81}$
9. $26\frac{1}{64}$
10. $1\frac{1056}{2556}$
11. $5\frac{599}{1225}$
12. $\frac{17}{29}$
13. $16\frac{4}{9}$
14. $\frac{3}{7}$
15. $\frac{15}{22}$
16. $25\frac{1}{9}$
17. $\frac{43}{6}$
18. $5\frac{1}{4}$
19. $9\frac{81}{100}$
20. $\frac{455}{81}$

Perform the operations indicated:

21. $\sqrt{3.26 \times .0063}$
22. $\sqrt{\frac{5}{8} + \frac{6}{7}}$
23. $\frac{3}{\sqrt{5184}} + \sqrt{\frac{20736}{129600}}$
24. $\sqrt{4489} \times \sqrt{961}$
25. $\sqrt{35721} \div \sqrt{729}$
26. $\sqrt{3.532 \div 6.28}$
27. $\sqrt{625 + 1296}$
28. $\sqrt{625} + \sqrt{1296}$
29. $\sqrt{25 \times 16 \times 81}$

30. $\sqrt{961-529}$

31. $\sqrt{25}\times\sqrt{16}\times\sqrt{81}$

32. $\sqrt{324}\times\sqrt{441}$

33. $\sqrt{961}-\sqrt{529}$

34. $\dfrac{33\times\sqrt{41}}{\sqrt{41}\times 165}$

35. $\dfrac{7\times\sqrt{1764}}{\sqrt{169}\times 7}$

EVOLUTION BY FACTORING

558. The square root of a perfect square, the cube root of a perfect cube, or any root of the corresponding perfect power may be found by factoring.

To determine the method of evolution by factoring, and the reason for it, let us study the relation between the factors of a number and the factors of the square of that number.

$42=2\times3\times7$; therefore $42^2=(2\times3\times7)^2=$

$2\times3\times7\times2\times3\times7$, or 1764.

We observe that every factor of 42 occurs twice in the square of 42. Likewise, every factor of any number occurs twice in the square of that number, three times in its cube, four times in its fourth power, and so on.

Conversely, $\sqrt{1764}=\sqrt{2\times2\times3\times3\times7\times7}=2\times3\times7$, or 42.

Likewise $\sqrt{225}=\sqrt{3\times3\times5\times5}=3\times5$, or 15.

$$\sqrt[3]{216}=\sqrt[3]{2\times2\times2\times3\times3\times3}=2\times3, \text{ or } 6.$$

Summary

1. *The square root of a perfect square may be found by factoring the square and multiplying together one out of every pair of equal prime factors found in it.*

2. *The cube root of a perfect cube may be found by factoring the cube and multiplying together one of every three equal prime factors found in it.*

How may the fourth root of a perfect fourth power be found? How may the fifth root of a perfect fifth power be found?

559. *Written*

Find, by factoring, the values of the following:

1. $\sqrt{3600}$	6. $\sqrt{1089}$	11. $\sqrt{126 \times 14}$	16. $\sqrt{2 \times 75 \times 6}$
2. $\sqrt{100}$	7. $\sqrt{784}$	12. $\sqrt{98 \times 8}$	17. $\sqrt{18 \times 45 \times 10}$
3. $\sqrt{441}$	8. $\sqrt{.1296}$	13. $\sqrt{32 \times 18}$	18. $\sqrt{\frac{324}{5625}}$
4. $\sqrt{1225}$	9. $\sqrt{20.25}$	14. $\sqrt{40 \times 10}$	19. $\sqrt{48400}$
5. $\sqrt{484}$	10. $\sqrt{.2401}$	15. $\sqrt{45 \times 125}$	20. $\sqrt{11025}$

APPLICATIONS OF SQUARE ROOT

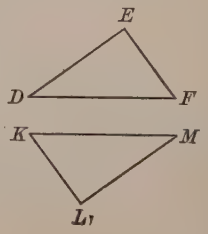
560. *A triangle that contains a right angle* is a **right triangle.**

561. *The side opposite the right angle in a right triangle* is the **hypotenuse** *of the right triangle.*

562. *The two sides that form the right angle of a right triangle* are the **legs** *of the right triangle.*

RIGHT TRIANGLE

563. *When a right triangle rests upon one of its legs, the leg upon which it rests* is called the **base** *and the other leg* is called the **perpendicular** *of the right triangle.*



In triangle ABC, which lines are the legs? In triangle DEF? In triangle KLM?

In triangle DEF, which line is the hypotenuse? In triangle KLM?

564. By geometry it is proved that

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.

The truth of this proposition may be shown in many ways, one of which is the following:

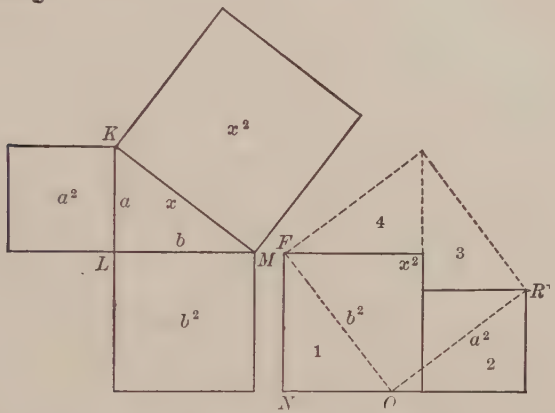


FIG. 1 FIG. 2

Let KLM be a right triangle of any shape and b^2 and a^2 of Fig. 2 equal respectively to b^2 and a^2 of Fig. 1. Take the point O, in Fig. 2, so that the line NO will be equal to the line KL, in Fig. 1, and draw OF and OR. In every case the triangles 1 and 2 may be placed in the position of 3 and 4, making a square equal to x^2 of Fig. 1. Verify this for yourself by cutting the figures from paper, using various lengths for a and b.

565. From the foregoing proposition it follows that

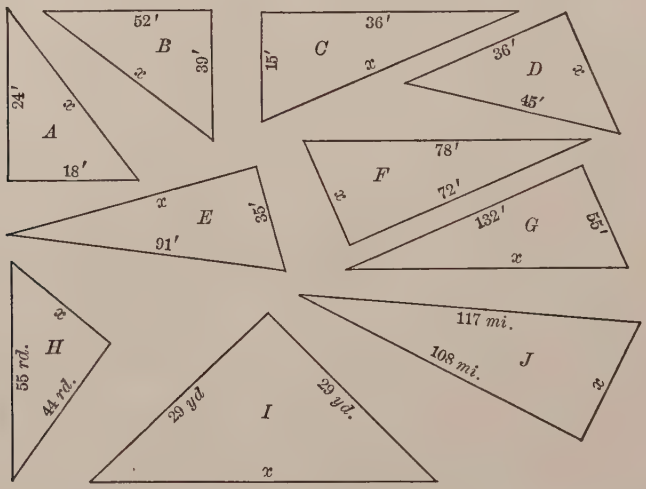
When the legs of a right triangle are known, the hypotenuse may be found by adding the squares of the two legs and extracting the square root of the sum; and that

When either leg and the hypotenuse are known, the other leg may be found by subtracting the square of the known leg from the square of the hypotenuse and extracting the square root of the difference.

566. *Written*

NOTE. — Approximate roots should be carried to two decimal places.

1. Find the value of x in figures A, B, C, D, E, F, G, H, I, and J.



2. A rectangular park is 32 rods by 24 rods. A walk extends diagonally across the park, connecting opposite corners. How long is the walk? (Make a drawing.)

3. One side of a rectangular field is 68 rods. The diagonal distance between opposite corners is 85 rods. Find the other three sides.

4. One side of a rectangle is 69 feet. The diagonal of the rectangle is 115 feet. Find the perimeter of the rectangle.

5. The area of a square is 169 square inches. *a.* What is the length of one side? *b.* What is the length of its diagonal? *c.* Draw the square, exact size, on the blackboard, and verify your work by measuring the diagonal.

6. Find the perimeter of a square whose area is 4489 sq. ft.

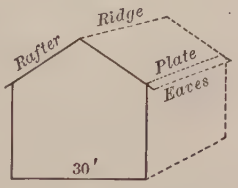
7. Find the diagonal of a square whose area is 324 square inches. Verify your work by drawing the square, exact size, and measuring the diagonal.

8. *a.* What is the area of a square whose perimeter is 228 centimeters? *b.* Find its diagonal, correct to millimeters.

9. Draw a rectangle whose length is twice its width. Suppose that its area is 450 square inches. *a.* What is its width? *b.* What is its length? *c.* What is its diagonal?

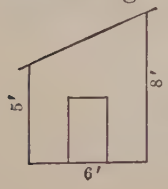
10. Three city streets intersect in such a way as to inclose a right triangle, ABC. The right angle is at B. The side AB is 8.4 meters and the side BC is 11.2 meters. If two boys start at B and walk around the triangle in opposite directions at the same speed, on which side will they meet, and how far from A and from C?

11. This cut represents the gable end of a barn. The ridge of the roof is 11 ft. 3 in. higher than the plates on which the rafters rest. The rafters extend 18 in. beyond the plates. How long must the rafters be made?



12. Rafters that extend 14 in. over the plates are 21 ft. 2 in. long, and the ridge is 12 ft. above the level of the plates. How wide is the building?

13. How long a ladder is needed to reach a window 24 feet from the ground, when the foot of the ladder is 10 feet from the side of the building?



14. This cut represents the end of Fred's chicken house. The roof extends 6 inches over each side. Find the slant height of the roof, correct to the nearest hundredth of a foot.

15. *a.* Measure the length and breadth of your schoolroom. Compute the diagonal of the floor; verify by measurement.

b. Beginning at one end of this diagonal, measure the height of the room. What kind of an angle is formed by the diagonal and the line last measured? Compute the distance from the top of that line to the farther end of the diagonal.

16. If a chalk box is 6 in. long, 4 in. wide, and 4 in. high, what is the distance from an upper corner through the center of the box to the opposite lower corner?

17. Find the perimeter of a right triangle whose legs are 7 ft. and 5 ft.

18. What is the side of a square field containing 10 acres?

HINT. — Reduce 10 A. to square rods. Why?

19. A baseball diamond was 90 ft. square. The ball was batted directly over second base and caught by a fielder who stood 90 ft. from second base. How far from the home plate did he stand?

20. What is the side of a square field containing $2\frac{1}{2}$ acres?

21. What is the diameter of the largest wheel that will go through a rectangular window 42 inches by $31\frac{1}{2}$ inches?

22. What is the length of the longest straight stick that can be inclosed in a box 4 in. by 3 in. by 7 in.?

23. A 30-acre rectangular field, three times as long as it is wide, is bounded on one side and one end by the highway. How much distance will a traveler save by going in a direct line diagonally across this field, from corner to corner, instead of following the highway?

MENSURATION

Review measurement of surfaces and rectangular solids, pages 96–100.

PLANE FIGURES

567. *A* **plane surface** is *a surface such that if any two points in it are connected by a straight line, the straight line will lie wholly in the surface; e.g.* a table top, the surface of a window pane. Test these and other surfaces by a thread held taut.

568. *A portion of a plane surface bounded by lines* is a **plane figure**; *e.g.* a square, a triangle, a circle.

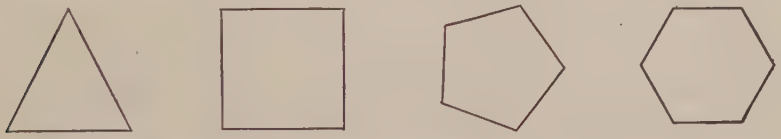
569. *A plane figure bounded by straight lines* is a **polygon.**

A polygon of three sides is called what? A polygon of four sides?

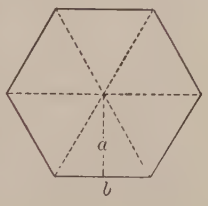
570. *A polygon of five sides* is a **pentagon**; *of six sides* a **hexagon**; *of seven sides*, a **heptagon**; *of eight sides*, an **octagon.**

AREAS OF REGULAR POLYGONS

571. *A polygon whose sides are equal and whose angles are equal is a* **regular polygon**; *e.g.*



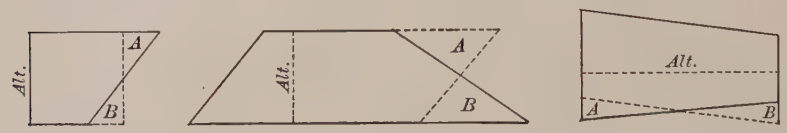
572. *The area of any regular polygon may be found by dividing the polygon into as many equal triangles as the polygon has sides, and multiplying the area of one triangle by the number of triangles; e.g.*



The area of this regular hexagon is six times the area of one of the triangles, or six times one half of the product of a and b.

AREAS OF TRAPEZOIDS

573. *A quadrilateral having two and only two sides parallel* is a **trapezoid.**



TRAPEZOIDS

In each of the above figures, how does the part A compare with the part B?

How does the area of the trapezoid compare with that of the parallelogram which is made from the trapezoid? How is the area of the parallelogram found?

Observe that in each figure the base of the parallelogram is equal to one half of the sum of the parallel sides of the trapezoid.

Summary

The area of a trapezoid is equal to one half of the sum of the parallel sides multiplied by the altitude.

574. *Written*

1. Draw a trapezoid whose altitude is 13 inches and whose parallel sides are 17 inches and 19 inches. Find its area.

2. Find the area of a trapezoid whose parallel sides are 20 feet and 25 feet, and whose altitude is 15 feet.

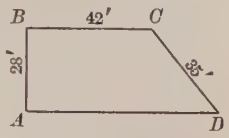
3. A field in the form of a trapezoid has two parallel sides of 30 rods and 35 rods; the distance between them is 20 rods. How many acres of land does the field contain?

4. A board is 1 inch thick, 12 feet long, 11 inches wide at one end and a foot wide at the other end. How many board feet does it contain?

5. A vineyard in France is in the form of a trapezoid, of which the two parallel sides are 185 meters and 155 meters, and the altitude is 130 meters.

a. It has an area of how many ares?

b. How many hectares?



6. Find the area of trapezoid $ABCD$.

7. The parallel sides of a trapezoid are 41 cm. and 55 cm. Its area is 1296 sq. cm. What is its altitude?

Let x = the altitude.

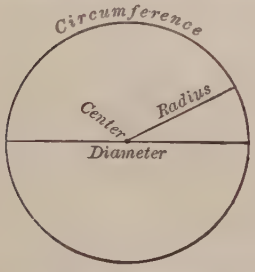
8. The area of a trapezoid is 560.5 sq. ft. The altitude is 19 ft. The difference of the parallel sides is 5 ft.

a. Find the sum of the parallel sides.

b. Find the length of each of the parallel sides.

STUDY OF THE CIRCLE

575. *A plane figure bounded by a curved line, all points of which are equally distant from a point within, called the center,* is a **circle.**



576. *The boundary line of a circle* is the **circumference.**

577. *A straight line passing through the center of a circle and terminating in the circumference* is the **diameter.**

578. *A straight line drawn from the center to the circumference of a circle* is its **radius.**

579. It is proved, by geometry, that *the circumference of every circle is* 3.1416 *times its diameter.*

580. *Oral*

1. The radius of a circle is what part of its diameter?

2. What is the radius of a circle whose diameter is 80 cm.?

3. What is the diameter of a circle whose radius is 35 cm.?

4. What is the circumference of a circle whose diameter is 1 foot?

5. What is the circumference of a circle whose diameter is 100 inches?

6. What is the circumference of a circle whose radius is 5 inches?

7. What is the diameter of a circle whose circumference is 31.416 inches?

8. What is the radius of a circle whose circumference is 3.1416 meters?

Written

1. What is the circumference of a circle whose diameter is 50 inches?

2. What is the radius of a circle whose circumference is 182.2128 feet?

3. What is the diameter of a circle whose circumference is 7854 miles?

4. The radius of the earth is approximately 4000 miles. What is its approximate circumference?

5. The diameter of my bicycle wheels is 28 inches.

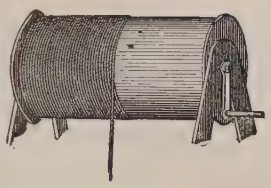
a. How many feet will I travel during 700 rotations of a wheel?

b. How many meters will I travel?

c. How many rotations will a wheel make in traveling 1 mile?

6. A horse is tethered to a stake by a rope 50 ft. long. What is the circumference of the circle over which he can graze?

7. The wire cable of a hoisting apparatus winds upon a cylindrical steel drum 20 inches in diameter and 3 feet long. How many feet of cable will the drum hold, when wound full, if the cable is $\frac{1}{2}$ inch in diameter?



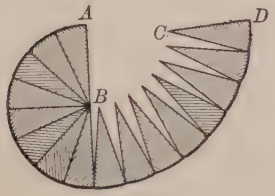


FIG. 1

581.

Observe that Fig. *ABCD* is a parallelogram.

Its altitude is what of the circle?

Its base is what of the circle?

The triangles of the circle are what part of the parallelogram?

How may we find the area of the parallelogram? Of the circle?

FIG. 2

Summary

The area of a circle is equal to one half of the product of its circumference by its radius.

By geometry it is proved also that the area of a circle is equal to .7854 of the square of its diameter, or 3.1416 times the square of its radius.

How may we find the area of a circle when the radius is given? when the diameter is given? when the circumference is given?

582. *Written*

In examples **1–12** *find the area of a circle from the term given, letting D, R, and C stand for diameter, radius, and circumference, respectively:*

1. D = 40 in.
2. D = 102 m.
3. R = 25 cm.
4. R = 2 ft. 6 in.
5. C = 9.4248 in.
6. C = 3.1416 mi.
7. D = 124 rd.
8. R = 35 cm.
9. C = 25.1328 ft.
10. D = 60 Km.
11. C = 31.416 yd.
12. R = 2 mi.

13. A horse tethered by a 50-foot rope in an open field can graze over how many square feet of land?

14. A cow is tied by a rope 100 ft. long at the corner of a rectangular pasture inclosed by a fence.

a. What part of an acre of ground can she graze over?

b. If she is tied to the fence at the middle of one side of the pasture, how much land can she graze over, the pasture being more than 200 ft. long and wide?

15. On a city map the center of the city is indicated by a dot, and a circle is drawn to include all that part which is not more than half a mile from the center, another to include all that is not more than a mile from the center, and so on.

a. What part of a square mile is inclosed by the half-mile circle?

b. How many square miles are inclosed by the 2-mile circle?

c. By the mile circle?

d. By the 3-mile circle?

e. By the $1\frac{1}{2}$-mile circle?

f. By the $2\frac{1}{2}$-mile circle?

g. By the 4-mile circle?

16. If $D^2 = 841$, *a.* what is D? *b.* What is C?

17. If $D^2 = 225$, *a.* what is C? *b.* What is A?

18. What is the diameter of a circle whose area is 63.6174 sq. ft.? *Statement of Relation:* $.7854 \times D^2 = 63.6174$.

19. Find the circumference of a circle whose area is 12.5664 square meters.

20. Find in meters the radius of a circle whose area is 38.4846 square decimeters.

SOLIDS

NOTE. — In the study of solid figures a full set of models should be in constant use.

583. *A* **solid** *is anything that has length, breadth, and thickness.*

Anything that occupies space is a solid. Any portion of space may be considered as a solid.

A solid figure is bounded by surfaces. By what are plane figures bounded?

584. *The side, or face, on which a solid may be supposed to rest* is called its **base.**

STUDY OF PRISMS

585. *A solid having two bases which are equal parallel polygons, and whose other sides are parallelograms,* is a **prism.**

586. Prisms are named according to the *number of sides* which their bases have, as **triangular, quadrangular, pentagonal, hexagonal,** etc.

587. *A prism whose bases and other faces are rectangles* is a **rectangular prism.**

588. *A prism whose bases are squares and whose other faces are equal rectangles* is a **square prism.**

How may the surface of any prism be found?

589. *The* **volume** *of a rectangular prism is equal to the product of its three dimensions.*

590. *The* **volume** *of any prism is equal to the area of the base, multiplied by the altitude.*

591. *Written*

1. Find the entire surface of a prism whose bases are squares 13 inches on a side and whose altitude is 2 feet.

2. Find the entire surface of a rectangular prism whose dimensions are 30 in., 3 ft., and 4 ft. 6 in.

3. Find the contents of a prism whose base is 6 ft. square and whose altitude is 90 in.

4. What is the volume of a rectangular prism whose dimensions are 2 ft., 1 ft. 6 in., and 38 in.?

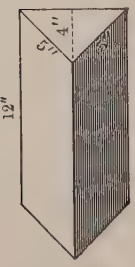


Fig. 1

5. What is the volume of a hexagonal prism the area of whose base is 748 square inches and whose altitude is 3 feet?

6. *a.* Find the entire surface of the prism in Fig. 1.

b. Find the volume of the prism in Fig. 1.

7. The volume of a square prism is 7776 cu. cm. Its altitude is .24 m. Find the length and breadth of its base.

Let x = side of the base.

STUDY OF THE CYLINDER

Note. — This treatment is intended to apply to the right circular cylinder only.

592. *A* **cylinder** *is a solid having two equal parallel circular bases and a convex surface, all points of which are equally distant from a straight line joining the centers of the bases; e.g.* a round lead pencil; a gas or water pipe; a music roll; a curtain rod.

593. Bring a cylindrical tin box to school. Cut a piece of paper that will exactly fit the convex surface of the box. What kind of figure is the paper? Its length is what of the cylinder? Its width? Its area?

Summary

The convex surface of a cylinder is the product of its altitude and circumference.

594. *Written*

What is the convex surface:

1. Of a cylinder whose circumference is 47 in. and whose altitude is 2 ft.?

2. Of a cylinder whose altitude is 10 ft. and whose diameter is 10 in.?

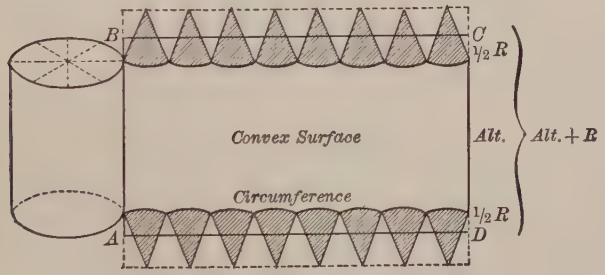
3. Of a cylinder whose altitude is 20 ft. and whose radius is 1 ft.?

4. Of a cylinder whose altitude is 1 ft. and whose radius is 2 ft. 6 in.?

595. Review Figs. 1 and 2, p. 349.

In the above figure observe that *the entire surface of a cylinder is equal to the convex surface, plus the sum of the surfaces of the two bases*, or to *the area of the rectangle ABCD.*

How may the area of the rectangle *ABCD* be found?



Summary

The entire surface of a cylinder is equal to the product of the circumference by the sum of the altitude and radius.

596. *Written*

Find the entire surface of a cylinder:

1. Whose diameter and altitude are 3 ft. and 50 ft.

2. Whose radius and altitude are 1 ft. and 10 ft.

3. Whose circumference and altitude are respectively 25.1328 in. and 12 in.

4. Whose circumference and altitude are 31.416 meters and 20,000 millimeters.

5. Whose diameter and altitude are 20 in. and 20 in.

597. Observe that a cylinder (Fig. 1) may be divided into any number of

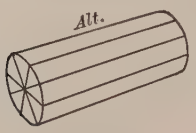


Fig. 1

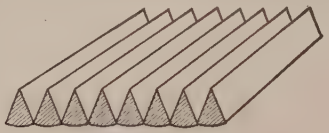


Fig. 2

equal sections (Fig. 2), each of which is approximately a triangular prism. The volume of all of these sections combined is equal to one half that of a rectangular prism (Fig. 3) whose dimensions are the circumference, altitude, and radius, respectively, of the cylinder.

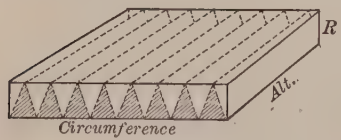


Fig. 3

How may we find the volume of the rectangular prism? of the cylinder?

Summary

The volume of a cylinder is equal to one half of the product of its circumference, altitude, and radius.

One half of the product of the circumference and radius = what? In what other form, then, may the above summary be stated?

598. *Written*

1. *Find the volumes of cylinders, having given dimensions as follows:*

a. Alt. 8 in., D. 5 in.

b. Alt. 3 ft., D. 2 ft.

c. Alt. 1 m., R. 4 dm.

d. Alt. 7 ft. 2 in., D. 9 in.

e. Alt. 30 ft., D. 20 in.

f. Alt. 25 dm., cir. 37.6992 m.

g. Alt. 1 ft., cir. 3.1416 yd. *i.* Alt. 10 ft., cir. 7.854 in.

h. Alt. 80 ft., cir. 49.912 ft. *j.* R. 85 cm., alt. 5 m.

2. How many gallons of water will fill a cylindrical pail 11 in. deep and 9 in. in diameter? (Indicate the work first.)

3. The reservoir of my student lamp is a cylinder 7 in. high and $3\frac{1}{2}$ in. in diameter. How much more or less than a quart of oil will it hold?

4. A cylindrical cistern is 6 ft. in diameter and 7 ft. deep. How many barrels of water will it hold? (Indicate the work first.)

5. How many cubic feet of compressed gas can be stored in a steel cylinder 4 ft. long and 9 in. in diameter?

6. How many cubic feet of wood are there in a log of uniform diameter, whose circumference is 7.854 ft. and whose length is 18 ft.?

7. A farmer has a cylindrical silo 12 ft. in diameter and 30 ft. high. How many cubic feet of ensilage can he store in it?

8. How many cubic feet of iron are there in an iron wire 10,000 ft. long and $\frac{1}{4}$ of an inch in diameter?

9. On the roof of Mr. Gowing's cottage is a cylindrical water tank into which water is pumped from the lake below. It is $5\frac{1}{2}$ ft. deep and $3\frac{1}{2}$ ft. in diameter.

a. How many gallons of water will it hold?

b. How deep is the water in the tank when it contains 100 gallons? (Let x = the depth, and form an equation.)

10. Make and solve five problems about cylinders.

STUDY OF THE CONE

599. *A* **cone** *is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the* **vertex.**

600. *The* **altitude** *of a cone is the perpendicular distance from the vertex to the center of the base.*

601. *The* **slant height** *of a cone is the distance from the vertex to any point in the circumference of the base.*

602. The convex surface of a cone may be considered as made up of any number of equal triangles, each triangle having for its altitude the slant height of the cone, and for its base one of the equal parts of the circumference of the base of the cone.

The sum of the bases of the triangles is what of the base of the cone? The triangles that form the convex surface of the cone are equal to what part of the area of the rectangle *ABCD*?

Fig. 1

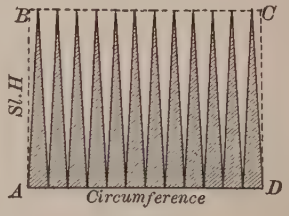


Fig. 2

The base of the rectangle (Fig. 2) is what of the cone (Fig. 1)?

The altitude of the rectangle is what of the cone?

The area of the rectangle is found how?

The convex surface of the cone is what part of the area of the rectangle?

Summary

The **convex surface** *of a cone is equal to one half of the product of the circumference of the base by the slant height.*

603. By section **581** *the* **area of the base** *of a cone is equal to one half of the area of a rectangle whose dimensions are the circumference and radius of the base of the cone.*

Show this by a drawing.

604. Adding this to the convex surface, *the* **entire surface** *of a cone is equal to one half of the product of the circumference by the sum of the slant height and the radius of the base.*

This may be shown by adding to the rectangle, Fig. 2, a rectangle of equal length, with an altitude equal to the radius of the base of the cone.

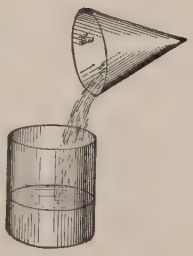
605. *Written*

1. The altitude of a conical spire is 12 ft. Its base is 10 ft. in diameter. Find (*a*) its slant height; (*b*) its convex surface; (*c*) its entire surface.

2. The circumference of the base of a cone is 188.496 in. The slant height is 6 ft. 6 in. Find (*a*) the radius of the base of the cone; (*b*) the altitude of the cone; (*c*) the convex surface of the cone; (*d*) the entire surface of the cone.

3. Make and solve other problems on the cone.

606. *It is proved by geometry that the volume of a cone is equal to one third of the volume of a cylinder of the same base and altitude.*



This may be verified by filling a hollow tin cone with water and pouring it into a cylinder of the same base and altitude. When the cone has been emptied once, the depth of the water in the cylinder is what part of the height of the cylinder?

607. It follows from the above statement that *the volume of a cone may be found by taking one sixth of the product of its altitude, the circumference of its base, and the radius of its base;* or, *by multiplying the area of its base by one third of its altitude.*

608. *Written*

1. Find the volume of a cone whose altitude is 42 in. and the area of whose base is 7 sq. ft.

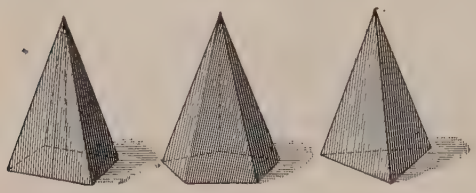
2. What is the volume of a cone the radius of whose base is 20 in. and whose altitude is 3 ft.?

3. Find the volume of a cone whose circumference is 219.912 centimeters and whose altitude is 1 meter.

4. Make and solve other problems on the cone, using dimensions given for cylinders in section **598**.

STUDY OF REGULAR PYRAMIDS

609. *A* **regular pyramid** is *a solid whose base is a regular polygon, and whose other faces are equal triangles meeting at a point called the* **vertex.**



610. Pyramids are named from their bases, as triangular pyramids, square pyramids, hexagonal pyramids, etc.

611. *The* **altitude** *of a regular pyramid* is *the distance from its vertex to the middle of its base.*

612. *The* **slant height** *of a regular pyramid* is *the altitude of one of its triangular faces.*

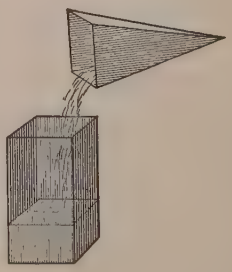
613. *The* **lateral surface** *of a pyramid* is *the combined surface of all of its triangular faces.*

614. How may the surface of each triangular face be found? Of all of them? How may the entire surface be found?

Summary

The lateral surface of a regular pyramid is the product of the perimeter of its base by one half of its slant height. The entire surface is the sum of the lateral surface and the base.

615. It is proved by geometry that *the volume of a regular pyramid is equal to one third of the volume of a regular prism having the same base and altitude.*



How is the volume of a prism found? Of a pyramid having the same base and altitude as the prism?

616. *Written*

1. What is the lateral surface of a regular triangular pyramid whose slant height is 25 feet and one side of whose base is 15 feet?

2. The roof of a tower is in the form of a pyramid whose base is 9 ft. square and whose altitude is 6 ft.

a. Find its slant height.

b. Find its lateral surface.

c. Find its volume.

3. *a*. Find the entire surface of a drawing model in the form of a square pyramid whose altitude is 6 in. and the side of whose base is 4 in.

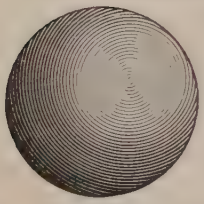
b. Find its volume.

4. Find the lateral surface of an octagonal church spire each side of whose base is 5 ft. and whose slant height is 40 ft.

5. Find the volume of a pyramid whose altitude is 16.5 m. and the area of whose base is 170,000 sq. cm.

STUDY OF THE SPHERE

617. *A* **sphere** *is a solid bounded by a surface, every point in which is equally distant from a point within called the center.*



618. *The* **diameter** *of a sphere is a straight line passing through its center and terminating in its surface.*

619. *The* **radius** *of a sphere is a line drawn from its center to any point in its surface.*

620. *The* **circumference** *of a sphere is the circumference of a circle whose radius and center are those of the sphere.*

621. It is proved by geometry that *the surface of a sphere is the product of its diameter and circumference.*

This is the same as the square of the diameter multiplied by 3.1416. Explain.

It is also four times the square of the radius multiplied by 3.1416. Explain.

622. *Written*

1. Find the surface of a sphere whose diameter is 100 ft.

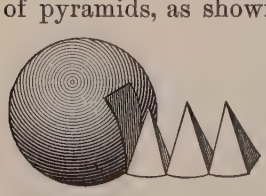
2. Assuming the earth to be a sphere (it is nearly a sphere) and its radius to be 4000 miles, what is its area?

3. What is the surface of a sphere whose circumference is 37.6992 Km.?

4. What is the surface of a sphere whose circumference is 39.27 inches?

5. Assuming the diameter of the moon to be 2150 miles, what is its area?

623. A sphere may be supposed to be made up of a number of pyramids, as shown in the cut.



By sections **590** and **615**, the volume of each of these pyramids is equal to the area of the base multiplied by one third of the altitude. How does the sum of the bases of the pyramids compare with the area of the sphere? The altitude of each pyramid is what of the sphere? How, then, may we find the contents of a sphere?

Summary

The volume of a sphere is equal to one third of the product of its radius and area.

This is the same as $\frac{4}{3}$ of the cube of the radius multiplied by 3.1416. Explain.

624. *Written*

1. Find the volume of a sphere: *a.* Whose diameter is 200 feet. *b.* Whose radius is 100 meters. *c.* Whose circumference is 157.08 cm. *d.* Whose circumference is 78.54 inches. *e.* Whose radius is 1 foot.

2. Assuming the diameter of the moon to be 2000 miles (it is nearly 2150 miles), what is its volume?

3. Assuming the diameter of the earth to be 8000 miles, what is its volume?

4. *a.* Find the volume of a cone whose altitude and the diameter of whose base are each 20 inches.

b. Using the answer to question *a*, find the volume of a cylinder having the same dimensions.

c. Find the volume of a sphere whose diameter is 20 inches.

d. The volume of the sphere is how many times that of the cone?

e. The volume of the cylinder is how many times that of the cone? of the sphere?

f. Find the volume of another cone, sphere, and cylinder whose diameters and altitudes are all equal. Compare them. Geometry proves that this relation always exists.

SIMILAR SURFACES

NOTE. — Review proportion.

625. *Figures that have the same shape, though they may differ in size*, are **similar**; *e.g.* all circles are similar; all regular poly-

SIMILAR FIGURES SIMILAR FIGURES

gons of the same number of sides are similar; two rectangles are similar if the length and breadth of each have the same ratio.

626. It is proved by geometry that *if two figures are similar, any two lines of one figure have the same ratio as the corresponding two lines of the other figure;* and *a line of one figure has the same ratio to the corresponding line of the other figure that any other line of the first figure has to the corresponding line of the other figure.*

For example, in the figures shown in section **625**,

$$AB : AC = A'B' : A'C' \qquad EF : FG = E'F' : F'G'$$
$$AB : A'B' = BC : B'C' \qquad DG : D'G' = FG : F'G'$$

If the side AB equals 21 ft., the side AC 12 ft., and the side $A'B'$ 14 ft., we may find the length of the side $A'C'$ by the following proportion:

$$21 \text{ ft.} : 12 \text{ ft.} = 14 \text{ ft.} : x \text{ ft.}$$

Find the value of x.

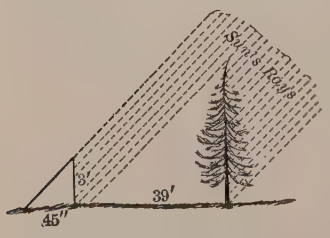
627. *Written*

1. In Figs. 1, 2, 3, and 4:

a. If $AB = 14$ ft., $A'B' = 28$ ft., and $BC = 11$ ft., what is the length of $B'C'$?

b. If $EF = 15$ rd., $FG = 10$ rd., and $E'F' = 18$ rd., what is the length of $F'G'$?

c. If $DG = 27$ mi., $D'G' = 33$ mi., and $FG = 18$ mi., what is the value of $F'G'$?



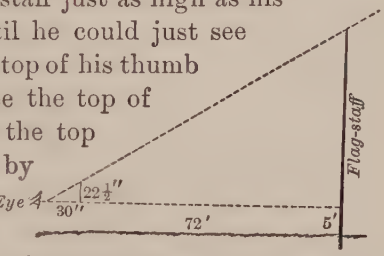
2. A man, desiring to know the height of a tree which stood on level ground, drove a stick into the earth in a vertical position, and it measured 3 ft. above ground. Its shadow measured 45 in. At the same moment the tree cast a shadow 39 ft. long. How tall was the tree?

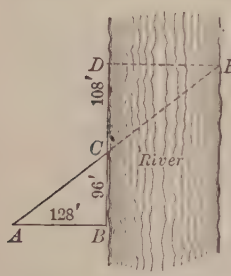
3. A rectangular field is 70 rd. long and 50 rd. wide; what is the length of a similar field whose width is $12\frac{1}{2}$ rd.?

4. One side and the diagonal of a quadrilateral are respectively 18 ft. and 44 ft. Find the corresponding side of a similar quadrilateral whose diagonal is 110 ft.

5. A boy found the height of a flagstaff as follows:

He found that he could hold a cane upright just 30 in. away from his eye. He placed his thumb $22\frac{1}{2}$ in. from the top of the cane, pinned a card on the flagstaff just as high as his eye, and walked backward until he could just see the paper by looking across the top of his thumb where he held the cane, and see the top of the flagstaff by looking across the top of the cane. He then found by measurement that he stood 72 ft. from the flagstaff while taking the observation, and that the card was 5 ft. from the ground. How high was the flagstaff?





6. Two boys, wishing to know the width of a river and having no boat, constructed the right triangle ABC by driving three stakes. They sighted from A, across C, to the opposite bank, at E, and drove a stake at D, so as to make the right triangle CDE. They then measured AB, BC, and CD, and found DE. How wide was the river?

628. It is proved by geometry that *the areas of similar surfaces are to each other as the squares of any two corresponding lines.*

Thus, on page 361, if the side AB of Fig. 1 is 21 ft., the side $A'B'$ of Fig. 2, 14 ft., and the area of Fig. 1, 96 sq. ft., we may find the area of

Fig. 2 by making the proportion

$$21^2 : 14^2 = 96 : x.$$

Solving,
$$x = \frac{\overset{2}{\cancel{14}} \times \overset{2}{\cancel{14}} \times \overset{32}{\cancel{96}}}{\underset{3}{\cancel{21}} \times \underset{3}{\cancel{21}}} = \frac{128}{3} = 42\tfrac{2}{3} \text{ sq. ft.} \quad \textit{Ans.}$$

If the area of Fig. 3 is 48 sq. ft. and of Fig. 4, 120 sq. ft., and the side DE of Fig. 3 is 6 ft., the side $D'E'$ of Fig. 4 may be found by making the proportion,

$$48 : 120 = 6^2 : x^2.$$

Solving,
$$x^2 = \frac{\overset{15}{\cancel{120}} \times 6 \times 6}{\underset{8}{\cancel{48}}} = 90.$$

Since
$$x^2 = 90,$$

$$x = \sqrt{90}, \text{ or } 9.48+ \text{ ft.} \quad \textit{Ans.}$$

629. *Written*

1. The side of a triangle is 7 inches and its area 23 square inches. The corresponding side of a similar triangle is $10\frac{1}{2}$ inches. Find its area.

2. The corresponding sides of two similar rectangles are 19 rods and 152 rods. The area of the second is 5670 square rods. What is the area of the first?

3. A circle is 4 inches in diameter; another is 8 inches in diameter. What is the ratio of their areas?

4. A circle has an area of 16 square feet; another has an area of 64 square feet. What is the ratio of their diameters?

5. The area of a rectangle 12 feet long is 84 square feet. What is the area of a similar rectangle 6 feet long?

6. Two similar fields have areas of 12 acres and 8 acres respectively; the larger is 32 rods wide. How wide is the smaller?

7. The altitudes of two similar triangles are 20 ft. and 10 ft.; the area of the smaller is 80 sq. ft. What is the area of the larger?

8. The areas of two similar rectangles are 8 acres and 72 acres respectively. The diagonal of the first is 51 rods. What is the diagonal of the second?

9. An oval mirror is 32 inches long and has an area of 600 square inches. What is the area of a similar mirror whose length is 40 inches?

10. Make a problem to find the area of one of two similar figures.

LONGITUDE AND TIME

630. *A* **meridian** is *an imaginary line extending directly north and south, on the surface of the earth, from pole to pole.* It is a semi-circumference of the earth.

631. *A* **prime meridian** is *a meridian taken as a starting place for the measurement of distances east and west so as to determine the location of places on the earth's surface.*

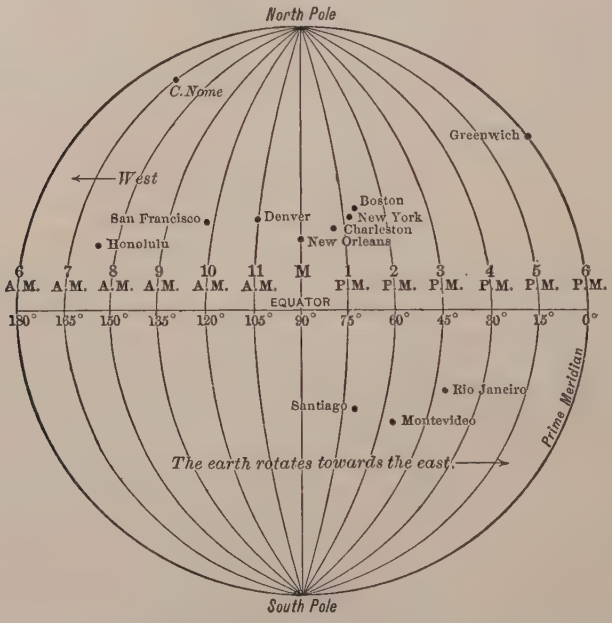
By common consent, the meridian passing through the Royal Observatory at Greenwich, Eng., is generally taken as the prime meridian.

632. *Distance east or west from the prime meridian, measured in degrees, minutes, and seconds* is **longitude.**

Degrees, minutes, and seconds west of the prime meridian are called **west longitude**; *east of the prime meridian* **east longitude.**

Longitude is measured by arc measure. Why? The number of meridians that may be represented on a globe or map is unlimited. Every place on the face of the globe may be supposed to have its meridian. But all places which lie on the same meridian have the same longitude although they may be thousands of miles apart. For example, Boston, Mass., and Santiago, Chile, have nearly the same longitude, though widely separated.

Lay your book on the desk, and imagine that the sun is in the ceiling directly above the middle of this drawing of a hemisphere. The drawing shows the half of the earth's surface that the sun shines upon. The other half is dark. If it is the 21st of March or September, it is now sunset at the prime meridian, noon at the meridian of 90° west longitude, and sunrise at the meridian of 180° west longitude.



The earth makes one rotation toward the east in 24 hours. During one rotation all the meridians will pass under the sun, on to sunset, midnight, sunrise, and noon, finally reaching the same position that they now have. Every place on the earth's surface has passed under the sun, and 360° of longitude have passed under the sun. Therefore the number of degrees of longitude passing under the sun in one hour is $360 \div 24$, or 15°.

Imagine this drawing to be a sphere rotating toward the east. The sun remains overhead; therefore the numbers representing the hours of the day remain fixed, and the meridians pass under them.

Greenwich and all places on its meridian pass into night. In one hour the 15° meridian will be at six o'clock, the 105° meridian at noon, and so on.

In six hours the 90° meridian will be just passing the six o'clock mark, the 180° meridian will be at noon, and Greenwich will be directly opposite, at midnight.

In twelve hours the 180° meridian will have passed entirely across to 6 P.M., and the meridian of Greenwich will be just coming into sight at 6 A.M. The meridians then in view will all be in east longitude and will be numbered from the prime meridian at the left, toward the right, from 0 to 180° east longitude. That is, the meridians are numbered both east and west from the prime meridian to the meridian opposite, which is 180°. No place can have more than 180°, either east or west longitude.

633. The 180° meridian, with slight modifications, has been chosen as the **International Date Line.** Passing chiefly through the Pacific Ocean, it touches no important body of land.

Whenever a ship crosses this line, going *westward*, its calendar is set forward one day; going *eastward*, its calendar is set back one day.

634. *Oral*

Use drawing of hemisphere in obtaining answers.

1. When it is noon at New Orleans, what is the time at Denver? at Cape Nome? at Greenwich?

2. When it is noon at Denver, what is the time at New Orleans? at Greenwich? at Cape Nome?

3. When it is noon at Greenwich, what is the time at New Orleans? at Denver? at Cape Nome?

4. When it is noon at Santiago, what is the time at Boston? at Montevideo? at Rio Janeiro?

5. When it is noon at San Francisco, what is the time at Honolulu? at Charleston?

6. When it is 3 P.M. at New York, what is the approximate time at Santiago? at Montevideo? at Rio Janeiro?

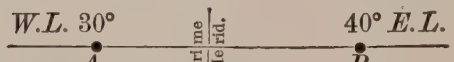
7. When it is 5 A.M. at Charleston, what is the approximate time at San Francisco? at Honolulu? at Greenwich?

8. When it is 7 A.M. at Denver, what is the approximate time at San Francisco? at New York? at Greenwich?

9. The difference in time between two places is 2 hr. What is the difference in longitude?

10. The difference in longitude between two places is 90°. What is their difference in time?

11. When it is 9 A.M. at your home, what is the time at a place 45° farther west? at a place 20° farther east?

12. 

A is 30° west longitude, and B is 40° east longitude. How many degrees of longitude are there between the meridian of A and that of B?

What is the difference in time between A and B?

635. *Written*

1. Cape Town is in longitude 18° 28′ 40″ E., and Hamburg is in longitude 9° 58′ 25″ E.

a. What is their difference in time?

b. When it is 10 A.M. at Cape Town, what is the time at Hamburg?

c. When it is 3 min. 17 sec. before 4 A.M. at Hamburg, what is the time at Cape Town?

a.

	18°	28′	40″
	9	58	25
15)	8°	30′	15″

34 min. 1 sec. *Ans.*

The difference in longitude is 8° 30′ 15″. Since the scale of the table of time is like that of the table of arc measure, and since 15° of longitude pass under the sun in 1 hr. of time, 15′ in 1 min. of time, and 15″ in 1 sec. of time, the number of hours, minutes, and seconds difference in

time is $\frac{1}{15}$ as great as the number of degrees, minutes, and seconds difference in longitude.

	hr.	min.	sec.	
b.	10	0	0	Since Hamburg is farther west than Cape Town, its time is earlier than the time at Cape Town.
		34	1	
	9	25	59	

or 25 min. 59 sec. past 4 A.M. *Ans.*

	hr.	min.	sec.	
c.	3	56	43	The time at Cape Town is later than the time at Hamburg. Why?
		34	1	
	4	30	44	

or 30 min. 44 sec. past 4 A.M. *Ans.*

2. When it is 31 min. $30\frac{1}{3}$ sec. past 1 P.M. at Washington, D.C., it is half past 10 A.M. at San Francisco. What is the longitude of Washington, if the longitude of San Francisco is 122° 25′ 41″ W.?

hr.	min.	sec.	
13	31	$30\frac{1}{3}$	
10	30	0	
3	1	$30\frac{1}{3}$	Diff. in Time.
		15	
45°	22′	35″	Diff. in Long.
122°	25′	41″	W.L.
45	22	35	
77°	3′	6″	W.L. *Ans.*

The day begins at midnight. Hence, 1 P.M. is 13 hr. after the beginning of the day.

For reasons given in example 1, the number of degrees, minutes, and seconds difference in longitude is 15 times as great as the number of hours, minutes, and seconds difference in time, or 45° 22′ 35″.

Since Washington has later time than San Francisco, it must be farther east, therefore nearer the prime meridian. Hence, it has a less longitude. 122° 25′ 41″ minus 45° 22′ 35″ is 77° 3′ 6″.

3. Rome is 12° 27′ 14″ E.L. and Philadelphia 75° 9′ 45″ W.L. What is their difference in longitude? (See Ex. **12**, p. 368.)

Since Philadelphia and Rome are on opposite sides of the prime meridian, their difference in longitude is the sum of their longitudes.

In examples **4–27** the number given is either difference in time or difference in longitude between two places. In every case find the one not given.

4. 5 hr. 1 min. 17 sec.
5. 1 hr. 18 min. 44 sec.
6. 37 min. 20 sec.
7. 2 hr. 48 sec.
8. 8 hr. $2\frac{1}{3}$ min.
9. 47° 19′ 30″
10. 18° 41′
11. 9° 45″
12. 12° 7′ 30″
13. 58′ 15″
14. 113° 30′ 10″
15. 107° 1′ 40″
16. 8 hr. 7 min. $22\frac{1}{3}$ sec.
17. 1 hr. 1 min. 49 sec.
18. 160° 14′ 50″
19. 28° 40′
20. 10 hr. 14 min. 27 sec.
21. 46° 18′
22. 1 hr. 2 min. $14\frac{2}{3}$ sec.
23. 48′ 15″
24. 1′ 49″
25. 7 hr. 50 sec.
26. 42° 19′ 5″
27. 170° 55′

28. One place is in 68° W.L. and another in 53° 15′ W.L. What is their difference in time?

29. Two places are in 120° 47′ and 13° 50′ east longitude respectively. What is their difference in time?

30. One place is in 83° 5′ west longitude and another in 7° 16′ 15″ east longitude. What is their difference in time?

31. It is 12 o'clock, midnight, at a certain place.
a. What is the time at a place 12° 15′ farther east?
b. What is the time at a place 47° 18′ farther west?

32. When it is 2 P.M. at Paris, 2° 20′ 15″ E.L.,
a. What is the time at Melbourne, 144° 57′ 45″ E.L.?
b. What is the time at Albany, 73° 44′ 45″ W.L.?

33. What is the time at Cincinnati, 84° 26′ W.L., when it is 11.50 A.M. at St. Louis, 90° 15′ 15″ W.L.?

34. If I sail from Philadelphia, 75° 9′ 45″ W.L., with my watch set at the exact local time, and, after sailing a certain distance, find that my watch is 1 hr. 28 min. 40 sec. slower than the exact local time at that place, assuming that my watch has kept perfect time, what longitude has the ship reached?

35. A horse trotted a mile in 2 min. 15 sec.

a. During that time, the race track, on which the horse was traveling, moved how many minutes and seconds in its rotation about the earth's axis?

b. Estimating a degree of longitude at that place to be equal to 50 miles, how many miles did the race track move while the horse was trotting a mile?

36. *a.* A railroad train moving at the rate of 24 miles an hour, including stops, travels how far in a day?

b. The track on which the train runs moves how many miles a day, assuming a degree of longitude at that latitude to be 50 miles?

STANDARD TIME

636. The railroad companies of this country and Canada have agreed upon a division of the country into four **time belts**, extending north and south. All places in each belt take the time of the meridian which passes through or near the middle of the belt. This time is called **standard time**. The belts are as follows: *Eastern*, *Central*, *Mountain*, and *Pacific*.

A similar system of standard time is used in other parts of the world.

The standard meridian for the Eastern belt is the 75th, for the Central belt the 90th, for the Mountain belt the 105th, and for the Pacific belt the 120th.

These standard meridians are 15 degrees apart: when it is noon in the Eastern belt, it is 11 A.M. in the Central belt, 10 A.M. in the Mountain belt, and 9 A.M. in the Pacific belt.

In going westward from one time belt into another, the traveler sets his watch back one hour. In traveling eastward he sets his watch ahead one hour.

When it is noon on the standard meridian of a time belt, it is called noon at all places in the belt.

Standard time is not the true solar or local time, except for places situated on the standard meridians. Yet it can vary but little more than thirty minutes from the true time, and its uniformity is a convenience.

Standard time is used not only by the railroads, but also by people generally. The exact time is telegraphed daily to all sections of the country from the Naval Observatory at Washington.

637. *Oral*

1. When it is 5 P.M. Mountain time, what is the time in the Pacific belt?

2. When it is 11 A.M. Pacific time, what is the Central time?

3. In traveling from San Francisco to New York, how many times do I change my watch, and do I set it ahead or back?

4. When it is 4 A.M. at Augusta, Me., what is the standard time at St. Louis?

5. When it is 1 P.M. Mountain time at Denver, what time is it at Washington, D.C.?

6. What is the Pacific time at San Francisco when it is 5 P.M. at Chicago?

638. *Written*

1. What is the local time at Quebec, 71° 12′ 15″ W.L., when the standard time at that place is 7.30 A.M.?

2. What is the difference between local time and standard time in Chicago, whose longitude is 87° 36′ 42″ W.?

3. When it is 6 P.M., standard time, at San Francisco, 122° 25′ 41″ W., what is the local time?

REVIEW AND PRACTICE

639. *Oral*

1. Express in words: 4009; 350.01259; CXLVIII; MCMX.

2. For what is the decimal point used?

3. Moving a figure three places to the left has what effect on its value? Two places to the right?

4. Moving the decimal point two places to the right has what effect on the value of the number in which it is placed? One place to the left?

5. State three principles of Roman notation.

6. Describe two methods of testing results in subtraction.

7. Which term in subtraction corresponds to the sum in addition? It is the sum of what?

8. Which terms in multiplication are factors?

9. What is the shortest way to multiply an integer by 100? To multiply an integer by 7000?

10. $3675 \times 100 = ?$ $600 \times 7000 = ?$ $98 \times 100 = ?$

640. *Written*

In examples **1–4**, *add and test results:*

1.	2.	3.	4.
235	8397	$18.79	$69.
49	65	4.65	72.35
807	482	82.04	670.48
9063	39	9.00	8359.20
584	910	501.83	2517.03
5369	8765	7.62	932.45
70810	1974	9.30	8534.06
52479	193	18.49	92.08
1379	8370	43.86	801.64
95468	246	97.53	17.32
3007	98	68.12	84.63
88894	4839	835.27	91.02

5. From 900,003.2 take 100.01.

6. Multiply 374 by 268 and read the partial products.

7. 468,316 is the product of 68, 71, and what other factor?

8. Find the value of $4837 + 32 \times 1800 - 1728 \div 72$.

9. Find the value of $(4837 + 32) \times (1800 - 1728) \div 72$.

10. Find the prime factors of 36,465.

641. *Oral*

1. How many acres of land can be bought for $18,200, if every two acres cost $182?

2. Test for divisibility by 2, 4, 3, 5, 25, and 9, each of the following numbers: 2352; 86,543,400; 793,422; 123,797.

3. Name the prime numbers from 1 to 100.

4. Name two composite numbers that are prime to each other.

5. When is a fraction in lowest terms?

6. When is a number in simplest form?

7. What common fraction is equal to .50? .33$\frac{1}{3}$? .12$\frac{1}{2}$? .60? .75? .66$\frac{2}{3}$? .80? .40? .90?

8. What is the shortest way to multiply an integer by 700?

9. Multiply 24.651 by 100; by 1000.

10. Name four aliquot parts of 50.

642. *Written*

1. Which of the following numbers are prime: 137; 361; 247; 381; 215; 897?

2. Find the L. C. M. of 63, 66, and 77.

3. Find two numbers whose sum is 835, and whose difference is 473.

4. What is the greatest common divisor of 396 and 468?

5. Simplify $\frac{37\frac{1}{2}}{13\frac{2}{7}}$.

6. Simplify $\frac{14}{37} \times \frac{74}{98} \div (\frac{6}{11} + \frac{4}{7})$.

7. A man made his will, giving his son \$3210, which was $\frac{3}{5}$ of his estate; to his daughter $\frac{1}{10}$ of his estate; and the remainder to his wife.

a. How much did the daughter receive?

b. How much did the wife receive?

8. When $\frac{3}{4}$ of a yard of cloth costs \$2.40, how many yards can be bought for \$19.20?

9. What fraction of $24\frac{3}{4}$ is $6\frac{3}{4}$?

10. A boy spent $\frac{3}{5}$ of his money and then earned 65 cents. He then had $\frac{5}{6}$ of his original sum. How much had he at first?

643. *Oral*

1. What is the easiest way to divide an integer by 100? To divide a decimal by 1000?

2. What is the easiest way to divide a number by 25? by 125?

3. Name a denominate number that is not compound. Name a denominate number that is compound.

4. What is the cost of 3000 shingles at \$5.00 per M?

5. Name some article that weighs about one pound; about two pounds; about three pounds; about fifty pounds.

6. Without measuring, draw a line six feet long on the blackboard. Draw another line two thirds as long. Measure and correct your drawings.

7. My watch chain of 14 k. gold is worn out, and the jeweler will allow me 56¢ per pennyweight for it. If it weighs 10 pwt., how much will I be allowed for it? The value of the gold is in proportion to its fineness. How much would my chain be worth if it were 10 k. gold?

8. What is the cost of 10 quires of paper at the rate of 80¢ per ream?

9. An arc of 30° is what part of a circumference?

10. *a*. How many seconds are there in an hour?

b. What is the difference in time between two places, one of which is 15° W.L., and the other 45° E.L.?

644. *Written*

1. *Add:*

$243.76
58.19
23.79
1.64
82.57
937.48
64.37
9.84
83.06
72.00
9.73
64.58
7.86
.98
3.41
28.62
9.18
.72
519.08

2. A man owning $\frac{7}{8}$ of a boat sold $\frac{3}{4}$ of his share for $1785. What was the value of the boat at that rate?

3. $\frac{3}{4}$ of a number exceeds $\frac{2}{3}$ of the number by $482\frac{1}{3}$. What is the number?

4. A miller bought wheat at $65\frac{3}{4}¢$ per bushel and sold it at $75\frac{1}{2}¢$ per bushel, gaining in all $117. How many bushels did he buy and sell?

5. Factor 17,280.

6. What fraction of a bushel is 3 pk. 7 qt. 1 pt.?

7. What fraction of a gallon of water can be held in a tin box 4 in. square and 3 in. deep?

8. 240 rd. is what fraction of a mile?

9. Reduce 35,816 in. to higher denominations.

10. What is the cost of digging a cellar 25′ by 36′ by $4\frac{1}{2}$′, at $50¢$ per cubic yard?

645. *Oral*

1. A flagstone is 5 ft. long and 3 ft. wide. How thick must it be to contain 5 cu. ft. of stone?

2. How many cubic yards are there in a block of stone 27 ft. long, 6 ft. wide, and 3 ft. thick?

(Think the problem through before you perform any operation.)

3. A piece of cloth is 36 yd. long and 2 ft. wide. How many square yards of cloth does it contain?

4. What is the length of one degree of a circumference which measures 360 inches?

5. What is the length of one degree of a circumference which measures 720 miles?

6. From April 21 to June 15 is how many days?

7. Two quarts of alcohol will fill how many 4-ounce bottles?

8. A 10-acre field contains how many square rods?

9. What is the altitude of a parallelogram whose area is 132 sq. ft. and whose base is 12 ft.?

10. What is the area of a triangle whose base is 4 yd. and whose altitude is 6 ft.?

646. *Written*

1. A wall 77 ft. long, 6 ft. high, and 12 in. thick is built of bricks costing $9 per M. What was the entire cost of the bricks if 22 bricks were sufficient to make a cubic foot of wall?

2. The altitude of a triangle is 16 ft. 6 in., and the base 30 ft. 6 in. What is the area?

3. The altitude of a triangle is 60 ft. and the area 3600 sq. ft. What is the base?

HINT. — Let x = the base, and make an equation.

4. Find the cost of a carpet for a floor 15 ft. square, if the carpet is $\frac{3}{4}$ yd. wide and costs $1.25 a yard, making no allowance for waste.

5. Find the cost of a steel ceiling for a room 18 ft. 6 in. by 28 ft. 6 in., at the rate of 16 cents per square foot.

6. How much milk is contained in 83 cans, each holding 8 gal. 2 qt. 1 pt.?

7. How much coal is there in 9 loads of 2 T. 250 lb. each?

8. Find the value of a pile of 4-foot wood, 40 ft. long and 5 ft. high, at $5.50 per cord.

9. Find the total weight of three loads of hay containing 1 T. 2 cwt. 78 lb., 1 T. 3 cwt. 39 lb., and 19 cwt. 89 lb., respectively.

10. A 5-gallon oil can lacks 3 qt. 1 pt. of being full. What is the value of the oil in the can at 12¢ per gallon?

647. *Oral*

1. An inch board containing 6 ft. of lumber is 6 in. wide. How long is it?

2. A block of wood 1 ft. square and 9 in. thick contains how many board feet?

3. Draw a full-size picture of a board foot.

4. A box 5 in. by 4 in. by 9 in. contains how many cubic inches?

5. A rectangular tin can 4 in. square has a volume of 96 cu. in. What is its other dimension?

6. If one man can mine 6 tons of coal in a 10-hour day, how many tons can he mine in an 8-hour day, at the same rate?

7. In what denominations is volume expressed?

8. In what denominations is capacity expressed?

9. Knowing the number of cubic inches in a gallon, how may we find the number of cubic inches in a liquid quart?

10. Knowing the number of cubic inches in a bushel, how may we find the number of cubic inches in a dry quart?

648. *Written*

1. A garden plot 30 ft. long contains 450 sq. ft. of land. What is the cost of inclosing it with wire fence at 27 cents a yard?

2. Find, to the nearest tenth, the number of bushels of grain that can be stored in a bin 6 ft. long, $3\frac{1}{2}$ ft. wide, and 5 ft. high.

3. What is the weight of a load of wheat that exactly fills a

wagon box that is 14 ft. long, 3 ft. wide, and 20 in. deep, the weight of a bushel of wheat being 60 lb.? (Answer correct to tenths' place.)

4. A rectangular cistern is 22 ft. long and 7 ft. wide. When it contains 32 barrels of water, how deep is the water?

5. Make out a bill of four items for goods bought at a dry-goods store. Foot and receipt the bill.

6. Make out a statement of an account at a hardware store, using four debit items and two credit items.

7. A farmer sold a load of hay weighing 1850 lb. at $15 a ton, and with a part of the money received bought 1 T. 5 cwt. of coal at $ 6.20 per ton. How much money had he left?

8. Find the exact number of days from Dec. 9, 1907, to June 30, 1908.

9. A wheel $9\frac{1}{6}$ ft. in circumference will make how many revolutions in going 11 mi.?

10. Reduce $\frac{17}{128}$ to a decimal.

649. *Oral*

1. What rate per cent is equal to $\frac{1}{10}$; $\frac{1}{8}$; $\frac{1}{5}$; $\frac{3}{4}$; $\frac{5}{8}$?

2. Find 20% of 500 lb.; $33\frac{1}{3}$% of 60 bu.; $16\frac{2}{3}$% of $18.

3. What decimal is equivalent to $\frac{1}{2}$ of 1%?

4. What per cent is equivalent to .25? to .025? to .0025?

5. A gain of $10 on goods costing $20 is what per cent gain?

6. A gain of $10 on goods sold for $20 is what per cent gain?

7. What is the selling price of goods that cost $200 and were sold at 10% advance?

8. What is the cost of goods that bring $50 when sold at a gain of 25%?

9. What is an agent's commission on ten books which he sells for $4 apiece and receives 40% commission?

10. When an agent sells goods at a commission of 20%, what does his principal receive for goods that the agent sells for $200?

650. *Written*

1. *Add:*

23.75
8.679
42.897
835.406
42.973
9.009
80.896
7.234
3.876
98.423
1.89
.907
2.496
53.875

2. What was the cost of goods that brought $1120.20 when sold at 20% profit?

3. Potatoes sold at 10¢ per half peck yield a profit of $33\frac{1}{3}$%. Find the cost per bushel.

4. What is the per cent of loss on a house bought for $5000 and sold for $4900?

5. A merchant paid $900 for 200 bbl. of flour. The freight cost him 45¢ a barrel and the cartage 5¢ a barrel. At what price per barrel must he sell the flour to gain 21%?

6. What is the cost of goods sold for $585 at a loss of $2\frac{1}{2}$%?

7. A commission merchant sold a consignment of 700 doz. eggs at $18\frac{1}{2}$¢ and one of 900 doz. at $21\frac{1}{2}$¢. What was the amount of his commission at $4\frac{1}{2}$%?

8. An agent remitted to his principal $2695.10 as the net proceeds of the sale of a consignment of goods, having retained his commission of 5%, and $12.40 for expenses incurred. What was the amount of his sales?

9. The Kansas City agent of a Philadelphia manufacturer receives an annual salary of $2000 and a commission of 2% on all his sales. His sales for the month of January amounted to

$7329. If he did as well for the remainder of the year, what was his total income?

10. A wagon listed at $200 was bought by a dealer at 20 and 10% off, and sold by him at 5 and 10% off from the same list price.

a. How much did he gain?

b. What per cent did he gain?

651. *Oral*

1. My furniture has been insured 12 years at the rate of $\frac{3}{4}\%$ premium on a three-year policy. How much have I paid on a $1000 policy?

2. What agreement does a man make when he indorses a note in blank?

3. What is the bank discount on a 60-day note for $100 without interest, if discounted at date at the rate of 6% per year? If discounted 30 da. after date?

4. What would I receive for my note for $100 for 90 da., without interest, if I sold it to the bank on the day of date, the discount rate being 6% per year?

5. Why do banks protest notes when they become due?

6. When the tax rate is 12 mills on the dollar, what is my tax on property assessed at $1000?

7. What is the meaning of "Exchange on London, $4.86\frac{1}{4}$"? "Exchange on Paris, $5.17\frac{1}{2}$"? "Exchange on Hamburg, $97\frac{1}{8}$"?

8. What American coin is most nearly like the mark? the franc? the sovereign?

9. When the tax rate is .01, what is the assessed valuation of property on which the tax is $120?

10. When the exchange value of 1 mark is 24¢, what is the quoted rate of exchange on Germany?

652. *Written*

1. The report of a savings bank shows the following resources. Find the total.

Bonds and mortgages	$5,979,120.95
Bonds of states	388,312.50
Boston city bonds	372,937.50
New York City bonds	956,059.45
Buffalo city bonds	39,800.00
Syracuse city bonds	1,178,637.50
Bonds of other cities	200,092.50
Onondaga county bonds	65,975.00
New York county bonds	106,150.00
Bonds of towns	192,514.25
School district bonds	12,315.50
Railroad bonds	2,924,466.83
Banking house	200,000.00
Other real estate	161,777.91
Cash in banks	312,919.22
Cash on hand	88,421.35
Interest accrued	229,800.62

2. When the county tax rate is .004376, what is the county tax on property assessed at $5000?

3. John Brown owes Fred Haskins $200. Haskins draws on Brown for that amount, making the draft payable at sight to the First National Bank. Write the draft.

4. A factory worth $49,677 is insured for $\frac{2}{3}$ of its value, at $1\frac{3}{4}\%$. What is the premium?

5. $420 premium on a fire insurance policy of $56,000 is what rate?

6. A city whose population is 22,000 has an assessed valuation of $11,000,000. Mr. Carpenter owns a house in that city valued at $2800. What was his share of the tax for building a new high school costing $75,000?

7. Find the amount of $867.35 for 1 yr. 3 mo. 27 da. at 9%.

8. What principal at 6% will amount to $272.50 in 1 yr. 6 mo.?

9. How long will it take $360 to gain $53.64 at 6%?

10. A man bought a bill of lumber for $850, Jan. 1, 1907, giving his note with interest at 6%. He paid $100 May 1, and $150 Aug. 16. What was due at settlement, Nov. 1, 1907, by the United States rule?

653. *Oral*

1. *a.* Draw a line one meter long without a measure. Measure and correct it.

b. Draw a line 80% of a meter long.

c. Draw a line 20% as long as the one in *b*.

d. The line in *c* is what per cent as long as the line in *a*?

2. *a.* Without using a measure, draw a square meter. A square decimeter.

b. Draw a line dividing the square meter into two parts, one of which is four times as large as the other.

c. How many square decimeters are there in each of these parts?

3. Estimate the number of square meters in the floor of your class room.

4. Name some object whose volume is about one cubic decimeter. Its size is like that of what unit of capacity measure?

5. *a.* One kilogram is about how many pounds?

b. A man bought a load of coal weighing 1000 Kg. About how many pounds did it weigh?

6. What is the duty on \$100 worth of mahogany boards at 15%?

7. A box 5 dm. long, 3 dm. wide, and 2 dm. deep will hold how many liters of oats?

8. A cubic decimeter of water weighs how many grams?

9. What is the value of 100 shares of bank stock quoted at $103\frac{1}{2}$?

10. How many dollars of city bonds can be bought for \$104,000, when they are selling at 4% premium?

654. *Written*

1. *Add:*

4763
8257
6039
5872
1397
685
9107
5236
1476
8838
947
86
9432
7943
8688

2. Find, in hectoliters, the capacity of a bin which is 9 m. long, 1 m. wide, and 175 cm. high.

3. How many kilograms of water will fill a rectangular vat which is 5 m. long, 4 m. wide, and .50 cm. deep?

4. What is the duty, at 35%, on a shipment of fur coats invoiced at 2150 marks, less a trade discount of 4%? (1 mark = $.238.)

5. Find, by means of equations, three numbers, of which the first is smaller by 106 than the second, the third larger by 22 than the second, and the sum of the three is 495.

6. A man in St. Paul wishes to send $386 to his family in Berlin. What is the face of the draft which he can buy with that sum, exchange being at $96\frac{1}{2}$?

7. A merchant in Galveston owes a bill of £47 10*s.* in Glasgow. What must he pay for a draft for that amount when exchange is at 4.872?

8. On Jan. 1, 1908, the stock of the Wampanoag Mills was quoted at $92\frac{1}{2}$. What must be invested in this stock, including brokerage at $\frac{1}{8}$%, to secure 238 shares?

9. The Central Coal and Coke Company paid a dividend of $1\frac{1}{2}$% on its common stock, Jan. 15, 1908.

a. What is the dividend on 200 shares?

b. How many shares must I own in order to receive a dividend of $900?

10. What must I invest in $4\frac{1}{2}$% city bonds at par to obtain an annual interest of $675?

655. *Oral*

1. Draw a vertical line on the blackboard, cutting off $33\frac{1}{3}\%$ of the board. Draw another line, cutting off 25 % of what remains. What fraction of the entire board is cut off?

2. When the dividend on 5 shares of railroad stock is \$25, what is the rate of dividend?

3. What is the annual interest on ten 500-dollar 4 % bonds?

4. What is the ratio of 75 to 3?

5. What is the number whose ratio to 45 is $\frac{1}{2}$?

6. $7 : ? = \frac{1}{11}$; $? : 18 = 3$; $? : ? = 6$.

7. $2 : 4 = 7 : ?$ $3 : 8 = 1 : ?$ $3 : ? = ? : 12$.

8. Divide \$25 among three boys in the ratio of 1, 2, and 2.

9. Divide 77 into two parts having the ratio of 5 to 6.

10. Three numbers are in the ratio of 1, 2, and 3. The first number is 7. Find the others.

656. *Written*

1. *Add*

\$385.24
17.89
3.20
976.
831.19
209.37
65.88
7.90
98.36
521.83
829.17
743.65
812.79

2. Solve by proportion: What is the cost of a 200-acre farm at the rate of 25 acres for \$1324?

3. What sum of money will yield as much interest in 4 yr. 6 mo. as \$9000 will yield in 9 mo.?

4. How long will it take 435 men to earn as much money as 145 men can earn in 4 yr. 3 mo.?

5. When a post 4 ft. 6 in. high casts a shadow 3 ft. $4\frac{1}{2}$ in. long, how high is a tree that casts a shadow 40 ft. 6 in. long?

6. How many Kl. of water can be kept in a vat that is $2\frac{1}{2}$ m. by 15 dm. by 50 cm.?

7. Two boys, having received 40 cents for some

work, divided it so that one boy received $\frac{3}{5}$ as much as the other. How much did each receive?

8. C failed in business, owing A \$3000, B \$2500, and D \$4500. His property was worth only \$6400. How much should each creditor receive?

9. A farmer bought two cows for \$80, paying $\frac{2}{3}$ as much for one as for the other. Find the cost of each.

10. Separate 2723 into three parts having the ratio of $\frac{1}{2}$ to 1 to 2.

657. *Oral*

1. Find the value of 2^4; 5^2; 3^3; 7^3; 5^3; 2^6; 12^2.

2. A number which is the product of equal factors is called what?

3. Find the value of $\sqrt{16}$; $\sqrt[4]{16}$; $\sqrt[3]{216}$; $\sqrt{400}$; $\sqrt[5]{32}$.

4. Finding one of the equal factors which produce a number is called what?

5. The legs of a right triangle are 3 ft. and 4 ft. What is the hypotenuse?

6. The hypotenuse of a right triangle is 10 ft., and one of the legs 8 ft. What is the other leg?

7. What are the two equal factors of 121?

8. Find one of the three equal factors of $\frac{1}{8}$.

9. One of the three equal factors of a number is 5. What is the number?

10. The entire surface of a cube is 24 sq. in. How long is each edge of the cube?

658. *Written*

1. Find the square root of 3,396,649.

2. The entire surface of a cube is 1350 sq. in. Find the volume of the cube.

3. $\sqrt{\frac{2}{7}\frac{1}{1}\frac{0}{3}\frac{2}{4}\frac{5}{0}} = ?$

4. The perimeter of a square is 1320 rd. Find its area in acres.

5. How many feet of fence are required to inclose a square field containing $2\frac{1}{2}$ A.?

6. A cylindrical oil tank, 24 ft. in diameter and 18 ft. high, will contain how many barrels of oil, allowing $4\frac{1}{4}$ cu. ft. for a barrel?

7. Find, to the nearest tenth of a foot, the depth of a cylindrical cistern whose capacity is 40 barrels, and the diameter of whose base is 6 ft.

8. *a.* Find the difference in time between two places in 79° 18′ and 103° 4′ west longitude, respectively.

b. When it is noon at the first place, what is the time at the second place?

9. When it is 11 A.M. at a place in 73° 7′ west longitude, what is the time at a place in 14° 53′ east longitude?

10. What is the longitude of a place in which the time is half-past one A.M., when it is midnight at a place whose longitude is 47° 17′ 15″ East?

659. 1. A coal company has $85,000 invested in a shaft mine. Assuming the cost of mining the coal and preparing it for market to be 76¢ per ton, the average price received to be $1.05, and the commission paid for selling to be 5¢ per ton, how many tons per year must the company take from this mine to yield a net income of 8% on the investment?

2. A mine owner bought coal at $2130 per acre and mined it. The vein averaged 5 ft. 6 in. in thickness and yielded

1000 tons of coal per acre for each foot of the thickness of the vein. If the net price received for the coal was 98¢ per ton, what was received for $7\frac{3}{5}$ acres of the coal?

3. A pane of plate glass was listed at \$96.40, with trade discounts of 75 and 5%, and a further discount of 2% for cash payment. What was the net cash price?

4. Make out and receipt a bill for $22\frac{1}{2}$ yd. of muslin at 14¢ per yard, $5\frac{3}{4}$ yd. of cambric at 12¢ a yard, and 20 handkerchiefs at \$3.60 per dozen.

5. A typist writes daily 130 folios of 10 lines each, averaging 10 words to a line and 7 letters to a word. Her typewriter has 42 keys, 5 of which are vowel keys. If the vowel keys are used three times as often as the other keys, how many vowels are written in a day?

6. When camphor gum is bought at 85¢ per pound and sold at 10¢ an ounce Avoirdupois, what is the rate per cent of profit?

7. A druggist who buys cocaine at the rate of \$5 per ounce of 480 gr. and sells it at the rate of 2 gr. for 5¢, gains what per cent?

8. The railroad company charges \$59.40 for the use of a freight car from Quincy, Mass., to Syracuse, N. Y., and is responsible for all damages to the freight carried. Mr. Harding, by releasing the company from liability for damage, secured a reduction of $33\frac{1}{3}$% from the regular freight rate. He then had his freight insured for \$2000, at a premium rate of $\frac{1}{8}$%. How much did he save on a carload of freight by this plan?

9. Simplify $\dfrac{\frac{3}{5} + 3\frac{3}{5} \div 4\frac{1}{5}}{4\frac{4}{5} \times \frac{2}{3}}$ and express the result as a decimal.

10. Find the sum which a bank would pay for a note for \$750, without interest, 90 da. before it was due, if its discount rate was 7% per annum.

660. 1. *Add:*

$289.52
79.68
81.73
786.39
496.38
809.99
78.63
61.92
5.48
689.73
738.93
604.52
900.68
99.83
86.49
808.70

NOTE. — Problems 2–6 are taken from an arithmetic published over one hundred years ago.

2. There are two numbers; the less number is 8761, the difference between the numbers is 597. What is the sum of the numbers?

3. What is the length of the road, which, being 33 ft. wide, contains an acre?

4. A bankrupt whose effects are $3948 can pay his creditors but 28 cents 5 mills on the dollar. What does he owe?

5. The river Po is 1000 feet broad and 10 feet deep, and it runs at the rate of 4 miles an hour. In what time will it discharge a cubic mile of water (reckoning 5000 feet to the mile) into the sea?

6. At the late census, taken A.D. 1800, the number of inhabitants in the New England states was as follows, viz.: New Hampshire, 183,858; Massachusetts, 422,845; Maine, 151,719; Rhode Island, 69,122; Connecticut, 151,002; Vermont, 154,465. What was the entire number?

7. Draw two straight lines having the ratio of 3 to 2.

8. What is the selling price of 48 yd. of cloth bought at 3*s.* 6*d.* per yard and sold at a gain of $21\frac{3}{7}\%$?

9. Estimating a bushel of coal to weigh 80 lb., find to the nearest tenth the number of cubic feet of space needed for the storage of one ton of coal.

10. Find the product of the common prime factors of 1395 and 1736.

661. 1. $4937 \times 398 = ?$

2. A note drawn for 90 da. without interest was discounted 24 da. after date, at 6 % per annum, yielding $553.84 proceeds. What was the face of the note?

3. *a.* How many kiloliters of water can be contained in a rectangular cistern 2.5 m. by 3.6 m. and 75 cm. deep?

b. What is the weight of this water in kilograms?

4. *a.* How many shares of preferred stock, paying $5\frac{1}{2}$ % dividends, must I buy to secure an annual income of $500.50?

b. What will the stock cost, at $124\frac{3}{8}$, brokerage $\frac{1}{8}$ %?

5. A barn roof is 58 ft. long and the slant height is 24 ft. on each side. Find the cost of the shingles for this roof at $5.00 per M, allowing 1000 shingles for 120 square feet.

6. When it is noon at Boston, 71° 4′ west longitude, what is the time at Rochester, 77° 51′ west longitude?

7. *a.* A six months' note for $900 without interest, dated Oct. 26, 1906, is discounted Feb. 21, 1907, at 6 %. What are the proceeds?

b. If the note were interest-bearing, what would be the proceeds?

8. A tract of land is 424 rods long and 324 rods wide. It cost $36919.80. What was the cost per acre?

9. Three loads of coal weighing respectively 3805 lb., 3965 lb., and 4730 lb., cost $38.75. What was the price per ton?

10. Find the square root of 160 correct to four decimal places.

APPENDIX

CUBE ROOT

The cube of a number composed of tens and units may be found as follows:

$24 = 20 + 4 = 2$ tens $+ 4$ units.

$24^3 = (20 + 4) \times (20 + 4) \times (20 + 4)$.

$$\begin{aligned}
20 + 4 &= 24 \\
20 + 4 &= 24 \\
\hline
(20 \times 4) + 4^2 &= 96 \\
20^2 + (20 \times 4) &= 480 \\
\hline
20^2 + 2 \times (20 \times 4) + 4^2 &= 576 \\
20 + 4 &= 24 \\
\hline
(20^2 \times 4) + 2 \times (20 \times 4^2) + 4^3 &= 2304 \\
20^3 + 2 \times (20^2 \times 4) + (20 \times 4^2) &= 11520 \\
\hline
20^3 + 3 \times (20^2 \times 4) + 3 \times (20 \times 4^2) + 4^3 &= 13824
\end{aligned}$$

From the operation we find that,

The cube of the tens	$20^3 = 8000$
3 times the square of tens multiplied by units . . .	$3 \times (20^2 \times 4) = 4800$
3 times the tens multiplied by the square of the units .	$3 \times (20 \times 4^2) = 960$
The cube of the units	$4^3 = 64$
	$8000 + 4800 + 960 + 64 = 13824$

Summary

The cube of a number composed of tens and units is equal to the cube of the tens plus 3 times the square of the tens multiplied by the units, plus 3 times the tens multiplied by the square of the units, plus the cube of the units.

By reversing the process, we may find the cube root.

1. What is the cube root of 13,824?

Solution. — Separating into periods of three figures each, beginning at units, we have 13'824. Since there are two periods in the power, there must be two figures in the root, tens and units.

The greatest cube of tens contained in 13824 is 8000, and its cube root is 20 (2 tens).

$$\begin{array}{rl} & 13'824 \mid 20+4 \\ \text{Tens}^3 = 20^3 = & \underline{8000} \\ 3 \times \text{tens}^2 = 3 \times 20^2 = 1200 & 5824 \\ 3 \times \text{tens} \times \text{units} = 3 \times 20 \times 4 = 240 & \\ \text{units}^2 = 4^2 = 16 & \\ 3 \times \text{tens}^2 + 3 \text{ tens} \times \text{units} + \text{units}^2 = \underline{1456} & \end{array}$$

$$(3 \times \text{tens}^2 + 3 \times \text{tens} \times \text{units} + \text{units}^2) \times \text{units} = 5824$$

Subtracting the cube of the tens, 8000, the remainder, 5824, consists of $3 \times (\text{tens}^2 \times \text{units}) + 3 \times (\text{tens} \times \text{units}^2) + \text{units}^3$. 5824, therefore, is composed of two factors, units being one of them, and $3 \times \text{tens}^2 + 3 \times \text{tens} \times \text{units} + \text{units}^2$, being the other. But the greater part of this factor is $3 \times \text{tens}^2$. By trial we divide 5824 by $3 \times \text{tens}^2$ (1200) to find the other factor (units), which is 4 if correct. Completing the divisor, we have $1200^2 + 3 \times (20 + 4) + 4^2 = 1456$, which, multiplied by the units, 4, gives the product, 5824, proving the correctness of the work. Therefore, the cube root is $20 + 4$, or 24.

To find the cube root by the aid of blocks.

Finding the cube root of a number is equivalent to finding the thickness of a cube, its volume being given.

The following formulas illustrate the principles that underlie operations in cube root.

NOTE. — For convenience, l, b, t, and v will represent length, breadth, thickness, and volume, respectively.

(1) $l \times b \times t = v$. (2) $v \div (l \times b) = t$. (3) $v \div (l \times t) = b$. (4) $v \div (b \times t) = l$.

2. What is the thickness of a cube whose volume is 13824 cubic feet?

PRODUCT OF LENGTH AND BREADTH	VOLUME	THICKNESS
$3 \times 20^2 = 1200$	13'824	20 ft.
$2 \times 20 \times 4 = 240$	8000	4 ft.
$4^2 = 16$	5824	24 ft.
1456	5824	

SOLUTION. — The greatest cube of even tens contained in 13824 cu. ft. is 8000 cu. ft. (Cube A, p. 394.) Its thickness, therefore, is 20 ft. Subtracting 8000 (A) from 13824 leaves a remainder of 5824 cu. ft., which are added in solids of equal thickness to three sides of A, as seen in Fig. 2. It now remains to find the thickness of the additions (b, c, d),

(e, f, g), and h, which have a uniform thickness. As the solids, b, c, d, form the greater part of the volume of the additions (5824 cu. ft.), and the length and breadth of each is 20 ft. (the length and breadth of A), by trial, using Formula 2, we find $5824 \div (3 \times 20^2) = 4$ ft., thickness of the additions, if correct. Knowing the thickness, which is also the breadth of e, f, g, h, we find the product of the length and breadth of e, f, $g = 3 \times 20 \times 4 = 240$ sq. ft.; and that of $h = 4^2 = 16$ sq. ft.; both of which added to 1200 sq. ft. = the product of the length and

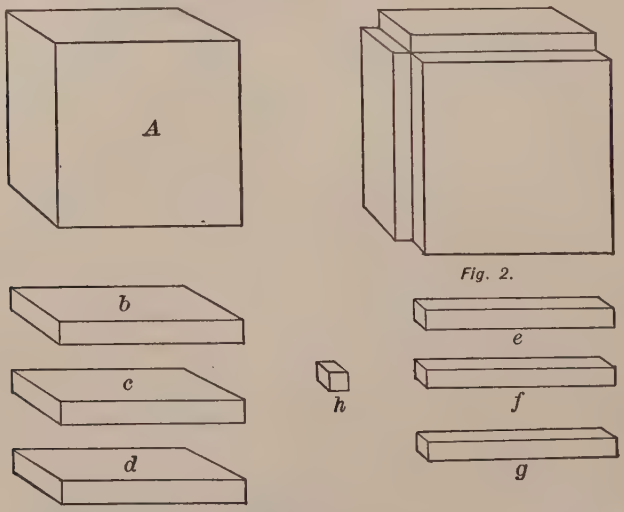


Fig. 2.

breadth of all the additions. This product, by Formula 1, multiplied by the thickness, 4 ft. = 5824 cu. ft., proving the correctness. Therefore,

The thickness of a cube whose volume is 13824 cu. ft. is 20 + 4 ft., or 24 ft.

The numbers in the middle column (Ex. 2) all indicate volume:

13824 = volume of original cube.
8000 = volume of Cube A.
5824 = volume of the additions (b, c, d), (e, f, g), and h.

The numbers in the left-hand column indicate product of length and breadth:

$1200 = l \times b$ of solids b, c, d.
$240 = l \times b$ of solids e, f, g.
$16 = l \times b$ of cube h.

The numbers in the right-hand column indicate thickness:

20 ft. = thickness of A.
4 ft. = thickness of all the additions.
24 ft. = thickness of original cube.

Short method.

Rule for finding the cube root:

Beginning at the decimal point, separate the number into periods of three figures each, thus: 16′581′.375.

Find the greatest cube in the left-hand period, and write its root at the right. Subtract the cube from the left-hand period, and bring down the next period for a dividend, thus:

16′581′.375 | 2
8
8581

To find the trial divisor, square the root already found with a cipher annexed, and multiply by 3, thus:

16′581′.375 | 2 — 20
8 — 20
Trial divisor, 1200) 8581 — 400
3
1200

To find the trial figure, find how many times the trial divisor is contained in the dividend, thus:

16′581′.375 | 25 — 20
8 — 20
Trial divisor, 1200) 8581 — 400
3
1200

To find the correction, multiply the former root by 3, annex the trial figure, and multiply by the trial figure, thus:

16′581′.375 | 25.5 — 2
8 — 3
1200 | 8581 — 65
325 — 5
Complete divisor, 1525 | 7625 — 325
187500 | 956375
3775
191275 | 956375

Continue thus, until all the periods are exhausted.

Note 1. — When there is a remainder after all the periods are exhausted, annex decimal periods, and continue the process as far as desired. The result will be the approximate root.

NOTE 2. — When a cipher occurs in the root, we annex two ciphers to the trial divisor, and bring down the next period.

NOTE 3. — The right-hand decimal period must have three places.

3. What is the cube root of 8.414975304?

OPERATION.

	8.414′975′304	2.034
	8	
120000	414975	
1809		
121809	365427	
6362700	49548304	
24376		
12387076	49548304	

Since 0 occurs in the root, annex 00 to the trial divisor, making 120000; bring down the next period.

NOTE. — To find the cube root of a common fraction, extract the root of each term separately. If both terms are not cubes, reduce to a decimal and then extract the root. The result will be the approximate root.

Find the cube root of:

4. 42875

5. 884736

6. 4492125

7. 77854483

8. 8.615125

9. 17.373979

10. 450827

11. 1879.080904

12. 32.890033664

13. 10077696

14. What is the cube root of $\frac{226981}{658503}$? $\frac{7}{16}$? $\frac{31}{1872}$? $39\frac{8}{27}$? $\frac{1}{64}$?

Extract the cube root to the third decimal place:

15. 14.323

16. 31982.4

17. .06324

18. .0015

19. 3

20. 7

21. What is the width of a cube whose volume is 91125 cubic inches?

22. A cubical cistern holds 50 barrels of water. How deep is it?

23. What is the entire surface of a cube whose edge is 9 ft.?

24. $\sqrt[3]{.006 \times 32.5} = ?$

SIMILAR SOLIDS

Solids having the same form without regard to size are **similar solids.** Any two cubes or any two spheres are similar solids. Solids are similar when their corresponding dimensions are proportional.

Similar solids are to each other as the cubes of their corresponding dimensions.

1. A globe is 3 inches in diameter, and another 6 inches in diameter. What is the ratio of their volumes?

EXPLANATION. — They are to each other as 3^3 to 6^3, or 27 : 216.

2. There are 64 cubic inches in a 4-inch cube. How many in an 8-inch cube?

3. Two similar solids contain 386 and 284 cubic inches, respectively. If the larger is 11 inches thick, how thick is the smaller?

4. If a man 6 ft. 2 in. tall weighs 215 pounds, what should be the weight of a man 5 ft. 10 in. tall of the same proportions?

5. The width of a bin is 4 ft. 6 in. How wide must a similar bin be to hold 4 times as much?

6. An oil tank 22 ft. in diameter holds 30,000 gallons.

a. How many gallons will a tank of the same shape and 88 feet in diameter hold?

b. What must be the diameter of a similar tank to hold 3750 gallons?

METHODS OF COMPUTING INTEREST

METHOD BY ALIQUOT PARTS

What is the interest on \$348 for 3 yr. 5 mo. 15 da. at 5 %?

\$34.80	Interest for 2 yr. at 5 % ($\frac{1}{10}$ of \$348)
17.40	Interest for 1 yr. at 5 % ($\frac{1}{2}$ of \$34.80)
5.80	Interest for 4 mo. at 5 % ($\frac{1}{3}$ of \$17.40)
1.45	Interest for 1 mo. at 5 % ($\frac{1}{4}$ of \$5.80)
.73	Interest for 15 da. at 5 % ($\frac{1}{2}$ of \$1.45)
\$60.18	Interest for 3 yr. 5 mo. 15 da. at 5 %. *Ans.*

If the time were 7 mo. 18 da., we should separate it as follows: ($\frac{1}{2}$ of 1 yr.) + ($\frac{1}{6}$ of 6 mo.) + ($\frac{1}{2}$ of 1 mo.) + ($\frac{1}{5}$ of 15 da.).

BANKERS' METHOD

This method is variously known as the *Six per cent Bankers', Sixty Day, Two Month,* or *Two Hundred Month* method. It is based on the fact that *any sum, on interest at 6 %, doubles in 200 months.* That is to say, *the simple interest for 200 months at 6 % is equal to the principal.*

The interest for 2 mo. is what part of the principal?
The interest for 6 da. is what part of the principal?

What is the interest on $476 for 2 mo. 19 da. at 5 %?

$4.76	Interest for 2 mo. at 6 % ($\frac{1}{100}$ of $476)
1.19	Interest for 15 da. at 6 % ($\frac{1}{4}$ of $4.76)
.24	Interest for 3 da. at 6 % ($\frac{1}{5}$ of $1.19)
.08	Interest for 1 da. at 6 % ($\frac{1}{3}$ of $.24)
$6.27	Interest for 2 mo. 19 da. at 6 %
1.045	Interest for 2 mo. 19 da. at 1 %
$5.225	Interest for 2 mo. 19 da. at 5 % *Ans.*

NOTE. — This method is especially useful in computing interest at 6%, for periods of 90 days or less, a common rate and time in bank transactions.

ORDINARY SIX PER CENT METHOD

What is the interest of $50.24 at 6 % for 2 yr. 8 mo. 18 da.?

The interest of $1 for 2 yr.	$= 2 \times \$.06$	$= \$.12$
for 8 mo.	$= 8 \times \$.00\frac{1}{2}$	$= .04$
for 18 da.	$= 18 \times \$.000\frac{1}{6}$	$= .003$
The interest of $1 for 2 yr. 8 mo. 18 da.		$= \$.163$
The interest of $50.24 is 50.24 times $.163		$= \$8.19$

TRUE DISCOUNT AND PRESENT WORTH

The **present worth** *of a debt due at a future time without interest is a sum which will amount to the debt if put at interest till that time.*

The debt is therefore the amount of the present worth for the given time.

The **true discount** *is the difference between the debt and its present worth. It is the interest of the present worth for the given time.*

1. What is the present worth and the true discount of a debt of $582.40, due in 8 months without interest, when money is worth 6%?

SOLUTION. — $1.04 = amount of $1 for 8 mo. at 6%.
STATEMENT OF RELATION. — $1.04 × present worth = $582.40.
$582.40 ÷ $1.04 = $560, present worth
$582.40 − $560 = $22.40, true discount } *Ans.*

Summary

To find the present worth, divide the face of the debt by the amount of $1 for the given time.

To find the true discount, subtract the present worth from the face of the debt.

2. What are the present worth and true discount of $400, due in one year, when money is worth 5%?

3. A father wills his two sons $3000 each, to be paid in three years from the time of his death. What is the present value of the legacies if money is worth 6%?

4. What is the present worth of $450, due in two years at 5%?

5. What is the present worth of $250.51, payable in 8 months, money being worth 6%?

6. Which is better, to buy flour for $5 cash, or for $5.25 on 6 months' time, when money can be borrowed at 5%?

7. Find the present worth of $750 for 6 months, money being worth 6%.

8. What is the present worth of $600, due in 1 year without interest, money being worth 6%?

9. Write the note which would be given for the above debt.

10. A man wishing to buy a house and lot has his choice between paying $5400 in cash, or $4000 in cash and $1700 in two years. With money at 6%, which is the most advantageous for him?

11. What is the present worth of a debt of $385.31, due in 5 months 15 days, at 6%?

12. Which would be better, and how much, to pay $4000 cash for a house, or $4374.93 in 3 yr. 6 mo., money being worth 7 % ?

13. I can sell my house for $2800 cash, or $3000 and wait 6 months without interest. I choose the latter. Do I gain or lose, and how much, money being worth 6 % ?

SURETYSHIP

Suretyship *is a contract whereby one party (usually a fidelity or surety company) binds itself to indemnify another party (usually a person or corporation employing some one in a position of trust or confidence) against loss by the dishonesty, willful neglect, or misconduct of an employee; or an agreement to indemnify one party to a contract against loss due to the failure of the other party to fulfill the contract.*

The instrument by which a contract of suretyship is made is called a **suretyship bond.**

Suretyship bonds are generally required of employees in banks; collectors and cashiers; treasurers of companies, societies, cities, villages, towns, counties, and other political divisions of the country; administrators of estates; guardians of infants; managers of business enterprises for others; and persons in many other positions of trust. Contractors in all sorts of undertakings are often required to furnish bonds for the proper fulfillment of their contracts.

Suretyship is generally classed as a branch of insurance; but it differs from ordinary insurance in that the surety receives its premium *for services rendered*, rather than *for risk assumed;* and that it does not *expect* losses when executing its bond. Losses occur occasionally from causes unforeseen by the surety, or arising after the execution of the bond.

The company, before issuing a suretyship bond, inquires into the character of the person applying for the bond, his habits, business standing, and reputation. If these are not satisfactory, the application is refused.

A suretyship bond always involves three parties:

a. The **principal**, *or party required to furnish the bond.* (The "Employee," in the bond given on page 401.)

b. The **surety**, *or party joining with the principal in an agreement to pay indemnity for loss.* (The Surety Company.)

c. The **obligee**, *or party to whom the indemnity is promised.* (The "Bank" in the bond given on page 401.)

The premium paid to the surety company is computed at a certain sum for each one thousand dollars of the bond.

The following form illustrates the essential parts of one kind of

SURETYSHIP BOND

American Surety Company

Amount, $2000 Premium, $8.00

*WE, John Doe, as principal, hereinafter called the "Employee," and the American Surety Company of New York, hereinafter called the "Surety Company," as surety (in consideration of the payment of an agreed premium to it, the said "Surety Company"), bind ourselves for the term commencing January 1, 1908, at 9 A.M., and ending January 1, 1909, at the like hour, to pay the Exchange National Bank of St. Louis, hereinafter called the "Employer," at the home office of the Surety Company in the City of New York, such direct pecuniary loss not exceeding two thousand dollars, as it may sustain of moneys, bullion, funds, bills of exchange, acceptances, notes, bonds, drafts, mortgages, or other valuable securities of similar nature, embezzled, wrongfully abstracted * * * in the course of his employment as Messenger of said employer.*

* * * * * * * * *

In witness whereof, said Employee, as principal, has hereunto set his hand and seal, and said American Surety Company of New York, as Surety, has caused the execution hereof by its President, and Assistant Secretary, and its seal to be hereunto affixed, at the City of New York, this 24th day of December, 1907.

JOHN DOE, Principal [L.S.]

AMERICAN SURETY COMPANY OF NEW YORK [L.S.]

RICHARD ROE, President

Attest: HERBERT HOOKWAY, Assistant Secretary

Oral

1. Who is the principal in the above bond? The surety? The obligee?

2. How much is the premium? Who pays it? Who receives it? What is the rate per $1000?

3. If John Doe remains in the employ of the bank as messenger for five years, how much will his bond cost him, during that time, at the same rate?

4. How many parties are there in a suretyship bond? Name them.

5. How many parties are there in an insurance contract? Name them.

Written

1. The treasurer of a bank gave a bond of $45,000. What did the premium amount to, the rate being $4 per $1000?

2. A firm in Chicago employed a man to manage a branch store in Cleveland, requiring him to give a bond for $7500. He had to pay a premium of $56.25 per year. What was the rate per $1000?

3. A cashier procured a bond, paying $5 per $1000. The premium amounted to $22.50. What was the amount of the bond?

4. A tax collector held office for four years, giving a new bond each year. He paid $56 in premiums during the four years on a bond of $3500. What was the rate per year on $1000?

5. A contractor furnishing supplies to a large manufacturing concern gave a bond of $12,000 for the faithful performance of his contract. What did he pay in premiums during six years, the rate being $\frac{1}{4}\%$ per year?

6. A paving contractor gave a five-year bond for the fulfillment of his contract, paying a premium of $\frac{1}{10}\%$ per year. What was the amount of the bond, if the premium amounted to $150?

COMPOUND PROPORTION

An equality between a compound and a simple ratio is a **compound proportion**; thus,

$$\left.\begin{matrix} 8:4 \\ 3:10 \end{matrix}\right\} :: 12:20 \text{ is a compound proportion.}$$

Find the fourth term.

$$\left.\begin{matrix}3:6\\4:8\end{matrix}\right\}::3:x$$

Solution. — First change to a simple proportion, we have,

$$3\times 4:6\times 8::3:x.$$

Then divide the product of the means by the given extreme, using cancellation. Thus,

$$\frac{\overset{2}{\cancel{6}}\times\cancel{8}\times\cancel{3}}{\cancel{3}\times\cancel{4}}=12. \quad \textit{Ans.}$$

1. If 5 men earn $72 in 8 days, how much can 10 men earn in 6 days?

Solution. — Since the answer is to be in dollars, place $72 for the third term, and arrange the terms of each couplet according as the answer should be greater or less than the third term if it depended on that couplet alone.

$$\left.\begin{matrix}5\text{ men}:10\text{ men}\\8\text{ days}:6\text{ days}\end{matrix}\right\}::\$72:(x)$$

Since 5 men earn $72, 10 men can earn more, so we place 10 men for the second and 5 men for the first; and since they earn $72 in 8 days, they will earn less in 6 days, so we place 6 days for the second term, and 8 days for the first. Dividing the product of the means by the extremes, we have,

$$\frac{\overset{9}{\cancel{72}}\times\overset{2}{\cancel{10}}\times 6}{\cancel{5}\times\cancel{8}}=\$108. \quad \textit{Ans.}$$

Summary

Consider the answer as the fourth term, and place the number that is like it for the third.

Arrange the couplets as if the answer depended on each couplet alone, as in simple proportion.

Divide the product of the means by the product of the extremes. Cancel when possible.

2. If four horses eat 10 bushels of oats in 5 days, how many bushels will be required to feed 5 horses for 2 days?

3. If 10 men working 8 hours a day can do a piece of work in 12 days, how many days would it take 6 men, working 10 hours a day, to do the same amount of work?

4. If it costs $84 to carpet a room 24 ft. long and 21 ft. wide with carpet 1 yard wide, how much will it cost to carpet a room 25 ft. long and 12 ft. wide with carpet 27 inches wide?

5. If a wheelman rides 144 miles in 3 days of 6 hours each, how many miles can he ride in 5 days of 9 hours each?

6. A section of street 33 ft. long and 20 ft. wide can be paved with 15,840 stones, each 9 inches long and 8 inches wide. How many stones 12 inches long and 10 inches wide will it take to pave a street 12 rods long and 16 ft. wide?

7. If 18 men chop 360 cords of wood in 12 days of 9 hours each, how many cords could 17 men chop in 13 days of 10 hours each?

8. If 50 men, working 10 hours a day for 11 days, can dig 25 rods of a canal 60 ft. wide, and 5 ft. deep, how many rods of a canal 90 ft. wide, and 7 ft. deep, can 140 men dig in 22 days of 8 hours each?

9. If 60 men can build a wall 150 ft. long, 64 ft. high, 2 ft. thick, in 8 days of 10 hours each, how many days of 8 hours each will 36 men require to build a wall 180 ft. long, 80 ft. high, $2\frac{1}{2}$ ft. thick?

10. How many men will it require to mow 48 acres in 3 days of 12 hours each, if 6 men mow 24 acres in 4 days of 9 hours each?

11. If 4 lb. 6 oz. of tea cost $\$2\frac{3}{16}$, what will 3 lb. 11 oz. cost?

12. If sufficient flour to fill 8 bags containing 98 lb. each can be produced from 16 bushels of wheat, how many bushels will be needed to fill 14 barrels of 196 lb. each?

13. My gas bill for the month of November is \$ 3.50 when I use 6 burners $3\frac{1}{2}$ hours each evening. How much ought it to be for the month of December, when I use 4 burners for 5 hours each evening?

14. How long a piece of cloth .4 m. wide can be made from 175 Kg. of wool, if 45 Kg. make a piece 25 m. long and .6 m. wide?

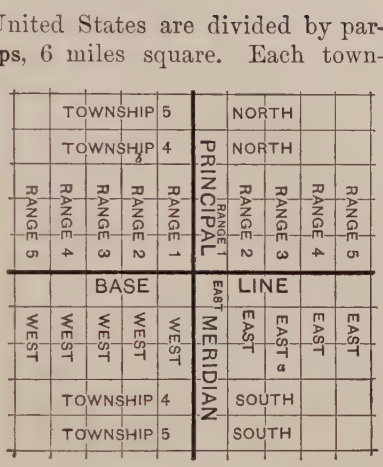
15. How many hours daily ought 30 men to labor to perform in 10 days a piece of work which is $\frac{2}{3}$ as great as a similar job which 25 men, working 12 hours per day, accomplished in 12 days?

16. If \$ 475 yield \$ 171 interest in 6 years, how long will it take \$ 960 to double itself at the same rate?

17. A bin 8 ft. long, 6 ft. wide, and $4\frac{1}{2}$ ft. deep will contain 270 bushels of wheat. How deep must another bin be built that is 12 ft. long and 9 ft. wide, to hold 405 bushels?

GOVERNMENT LANDS

The government lands of the United States are divided by parallels and meridians into **townships**, 6 miles square. Each township is divided into 36 square miles, or **sections**. Each section is subdivided into *half-sections* and *quarter-sections*.



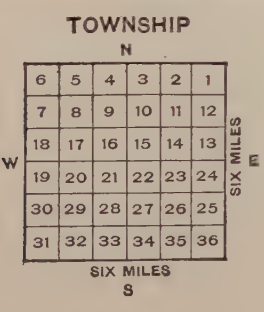
In surveying the public lands, lines 6 miles apart are run from east to west and from north to south, dividing the territory into square townships. An east and west line is established as a **base line**, and a north and south line as a **principal meridian**.

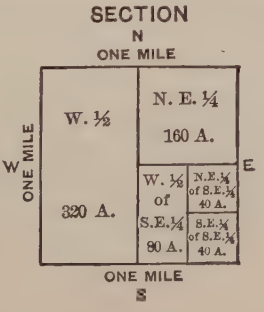
A line of townships running east and west is called a **tier**, and a line of townships running north and south is called a **range**.

Any township is designated by its number north or south of the base line, and its number east or west from the principal meridian.

Thus, a township that is in the 15th tier north of the base line, and in the 28th range east of the 4th principal meridian, is designated: T. 15 N. R. 28 E. 4th P.M.

There being 36 sections in a township, each section is designated by a number. The numbering begins at the N.E. corner, increasing toward the west and east, as shown in the accompanying diagram.





GREATEST COMMON DIVISOR BY CONTINUED DIVISION

PRINCIPLES

1. A divisor of a number will divide any multiple of that number.

2. A common divisor of two numbers will divide their sum and their difference.

1. Find the G. C. D. of 1395 and 1798.

$$
\begin{array}{r}
1 \\
1395\overline{)1798} \\
1395 \quad 3 \\
\hline
403\overline{)1395} \\
1209 \quad 2 \\
\hline
186\overline{)403} \\
372 \quad 6 \\
\hline
31\overline{)186} \\
186
\end{array}
$$

Any common divisor of 1395 and 1798 will divide their difference, or 403. Any divisor of 403 will divide 3 times 403, or 1209. Any common divisor of 1395 and 1209 will divide their difference, or 31. Therefore the G. C. D. cannot be greater than 31. By a similar use of the principles stated above, it may be shown that 31 will divide 186, 372, 403, 1209, 1395, and 1798. Hence 31 is the G. C. D. of 1395 and 1798.

Summary

To find the greatest common divisor of two numbers, divide the greater by the less, and the last divisor by the last remainder, continuing the process until there is no remainder. The divisor last used is the greatest common divisor required.

When more than two numbers are given, find the greatest common divisor of two of them; then of that greatest common divisor and one of the remaining numbers, and so on till all of the numbers have been used. The greatest common divisor last found is the greatest common divisor of all the given numbers.

Find the G. C. D. of

2. 672 and 960

3. 616 and 1012

4. 272 and 428

5. 1034 and 987

6. 1802 and 1431

7. 2989 and 1830

8. 2263 and 3604

9. 5494 and 4355

10. 1650 and 1920

11. 696, 1218, and 1160

12. 450, 720, and 810

13. 465, 434, and 341

14. 738, 553, and 1271

15. 1316, 517, and 1504

16. 1554, 2590, and 703

17. 649, 2065, and 2478

FARMERS' ESTIMATES

To find the number of bushels in a bin or granary,

Divide the number of cubic feet in the bin or granary by $1\frac{1}{4}$.

To find how large a bin will contain a given number of bushels,

Multiply the number of bushels by $1\frac{1}{4}$.
The result is the number of cubic feet in the required bin.

To find the number of gallons of water in a cistern or tank,

Multiply the number of cubic feet of water by $7\frac{1}{2}$.

To find how large a cistern will hold a given number of gallons,

Divide the number of gallons by $7\frac{1}{2}$.
The result will be the number of cubic feet in the required cistern.

To find how many bushels of shelled corn are equal to a given number of bushels of corn in the ear,

Divide the number of bushels of corn in the ear by 2.

The following table shows the number of **pounds in a legal bushel,** of different commodities, in various states:

	CAL.	CONN.	DEL.	ILL.	IND.	IOWA.	KY.	LA.	MASS.	MICH.	MINN.	MO.	N.J.	N.Y.	N.C.	OHIO.	OREGON.	PENN.	VT.	WASH.	WIS.
Wheat	60	56	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
Indian Corn, shelled	52	56	56	52	56	56	56	56	56	56	56	52	56	58	54	56	56	56	56	56	56
Oats	32	28		32	32	32	$33\frac{1}{3}$	32	30	32	32	35	30	32		32	34	32	32	36	32
Barley	50			48	48	48	48	32	46	48	48	48	48	48	48	48	46	47	46	45	48
Buckwheat	40	45		40	50	52	52		46	42	42	42	50	48	50		42	48	46	42	42
Rye	54	56		54	56	56	56	32	56	56	56	56	56	56		56	56	56	56	56	56
Clover Seed				60	60	60	60			60	60	60	64	60		60	60			60	60
Timothy Seed				45	45	45	45					45		45							46
Blue Grass Seed				14	14	14	14					14									

Beans, peas, and potatoes usually 60 lb.; in N.Y., beans 62 lb.

Coal, 80 lb., except Ind., 70 or 80, and Ky. 76 lb.

Salt: Ill., 50 lb. common, or 55 lb. fine,
N.J., 56 lb., Ind., Ky., and Iowa 50 lb.,
Penn., 80 lb. coarse, 70 lb. ground, or 62 lb. fine.

KINDS OF PAPER MONEY

The paper money of this country is of four kinds, viz.:

1. *United States Treasury Notes.*

These are promises of the United States to pay to the bearer, on demand, the sum named in the note. They are given and received in ordinary business transactions on a par with gold, because all people believe that the United States is able to fulfill its promises and will do so.

Treasury notes can be exchanged for gold at any time, but people prefer the notes for most purposes, because they are more convenient to carry and less liable to be lost. Why cannot notes of individuals be used for money?

2. *National Bank Notes.*

A national bank note is a promise by a national bank to pay to the bearer, on demand, a specified sum of money. Every national bank, in order to issue this kind of money, must own bonds of the United States at least equal in amount to the notes which it issues. These bonds, although owned by the bank, are held by the Treasurer of the United States.

If any national bank should fail, or refuse to pay its notes, the United States government would pay them and take its payment from the bonds in its possession. So that the credit of the United States is really what gives value to national bank notes.

3. *Gold Certificates.*

These are paper bills certifying that there is gold on deposit in the United States Treasury of a value corresponding to the denomination of the certificate, payable to the bearer of the certificate on demand.

The holder of the certificate may exchange it for gold at any time. The value of a gold certificate, therefore, depends on the fact that there is an amount of gold in the Treasury designed expressly for the payment of the certificate.

4. *Silver Certificates.*

These are similar to gold certificates, except that they are secured by silver instead of gold, on deposit in the treasury.

Ask your father to let you take some paper money to examine. See if you can tell to which class of paper money it belongs, and upon what its value depends.

2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5
2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10
2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25
2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30
2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35
2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40
2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45
2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50
2 × 11 = 22	3 × 11 = 33	4 × 11 = 44	5 × 11 = 55
2 × 12 = 24	3 × 12 = 36	4 × 12 = 48	5 × 12 = 60

6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9
6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18
6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27
6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36
6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45
6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54
6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63
6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72
6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81
6 × 10 = 60	7 × 10 = 70	8 × 10 = 80	9 × 10 = 90
6 × 11 = 66	7 × 11 = 77	8 × 11 = 88	9 × 11 = 99
6 × 12 = 72	7 × 12 = 84	8 × 12 = 96	9 × 12 = 108

10 × 1 = 10	11 × 1 = 11	12 × 1 = 12	ROMAN NUMERALS
10 × 2 = 20	11 × 2 = 22	12 × 2 = 24	
10 × 3 = 30	11 × 3 = 33	12 × 3 = 36	I = 1
10 × 4 = 40	11 × 4 = 44	12 × 4 = 48	V = 5
10 × 5 = 50	11 × 5 = 55	12 × 5 = 60	X = 10
10 × 6 = 60	11 × 6 = 66	12 × 6 = 72	L = 50
10 × 7 = 70	11 × 7 = 77	12 × 7 = 84	C = 100
10 × 8 = 80	11 × 8 = 88	12 × 8 = 96	D = 500
10 × 9 = 90	11 × 9 = 99	12 × 9 = 108	M = 1000
10 × 10 = 100	11 × 10 = 110	12 × 10 = 120	$\overline{M}$ = 1,000,000
10 × 11 = 110	11 × 11 = 121	12 × 11 = 132	
10 × 12 = 120	11 × 12 = 132	12 × 12 = 144	

Compound Interest Table

Periods	¾ Per Cent	1 Per Cent	1¼ Per Cent	1½ Per Cent	2 Per Cent	2½ Per Cent
1	1.007500	1.010000	1.012500	1.015000	1.020000	1.025000
2	1.015056	1.020100	1.025156	1.030225	1.040400	1.050625
3	1.022669	1.030301	1.037970	1.045678	1.061208	1.076891
4	1.030339	1.040604	1.050945	1.061364	1.082432	1.103813
5	1.038066	1.051010	1.064082	1.077284	1.104081	1.131408
6	1.045852	1.061520	1.077383	1.093443	1.126162	1.159693
7	1.053696	1.072135	1.090850	1.109845	1.148686	1.188686
8	1.061598	1.082856	1.104486	1.126493	1.171660	1.218403
9	1.069560	1.093685	1.118292	1.143390	1.195093	1.248863
10	1.077582	1.104622	1.132270	1.160541	1.218994	1.280085
11	1.085664	1.115668	1.146424	1.177949	1.243374	1.312087
12	1.093806	1.126825	1.160754	1.195618	1.268242	1.344889
13	1.103010	1.138093	1.175263	1.213552	1.293607	1.378511
14	1.110275	1.149474	1.189954	1.231756	1.319479	1.412774
15	1.118602	1.160968	1.204829	1.250232	1.345868	1.448298
16	1.126992	1.172578	1.219889	1.268985	1.372786	1.484506
17	1.135444	1.184304	1.235138	1.288020	1.400241	1.521618
18	1.143960	1.196147	1.250477	1.307341	1.428246	1.559659
19	1.152540	1.208108	1.266108	1.326951	1.456811	1.598650
20	1.161184	1.220190	1.281934	1.346855	1.485947	1.638616

Periods	3 Per Cent	3½ Per Cent	4 Per Cent	5 Per Cent	6 Per Cent	7 Per Cent
1	1.030000	1.035000	1.040000	1.050000	1.060000	1.070000
2	1.060900	1.071225	1.081600	1.102500	1.123600	1.144900
3	1.092727	1.108718	1.124864	1.157625	1.191016	1.225043
4	1.125509	1.147523	1.169859	1.215506	1.262477	1.310796
5	1.159274	1.187686	1.216653	1.276282	1.338226	1.402552
6	1.194052	1.229255	1.265319	1.340096	1.418519	1.500730
7	1.229874	1.272279	1.315932	1.407100	1.503630	1.605781
8	1.266770	1.316809	1.368569	1.477455	1.593848	1.718186
9	1.304773	1.362897	1.423312	1.551328	1.689479	1.838459
10	1.343916	1.410599	1.480244	1.628895	1.790848	1.967151
11	1.384234	1.459970	1.539454	1.710339	1.898299	2.104852
12	1.425761	1.511069	1.601032	1.795856	2.012197	2.252192
13	1.468534	1.563956	1.665074	1.885649	2.132928	2.409845
14	1.512590	1.618695	1.731676	1.979932	2.260904	2.578534
15	1.557967	1.675349	1.800944	2.078928	2.396558	2.759031
16	1.604706	1.733986	1.872981	2.182875	2.540352	2.952164
17	1.652848	1.794676	1.947901	2.292018	2.692773	3.158815
18	1.702433	1.857489	2.025817	2.406619	2.854339	3.379932
19	1.753506	1.922501	2.106849	2.526950	3.025600	3.616527
20	1.806111	1.989789	2.191123	2.653298	3.207136	3.869684

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23. 604.
24. 763.
25. 849.
26. $2125\frac{248}{329}$.
27. $684{,}876\frac{3}{7}$.
28. $28{,}166\frac{55}{172}$.
29. $13{,}185\frac{581}{1251}$.
30. $990\frac{202}{851}$.
31. 4076.
32. $3602\frac{164}{491}$.

Sec. 62

2. 552.5.
3. 2513.4.
4. 20.636.
5. 18.6465.
6. 25,683.
7. .96936.
8. 1.6625.
9. .50135.
10. .39508.
11. .14742.
12. .0088326.
13. 16.472.
14. .043185.
15. 178.8.
16. 3.41528.
17. 102.136.
18. 81.9504.
19. .0022638.
20. 7379.02.
21. .7826.
22. .4025.
23. 999.
24. 7.88544.
25. 1485.7605.

Sec. 64

2. 7.2.
3. 89.5.
4. 3.58.
5. 8.942.
6. .38.
7. 654.
8. 3.199.
9. 380.
10. 9100.
11. 6800.
12. 8100.
13. 2390.
14. 9.364.
15. .042.
16. 2531.
17. 346,999.
18. 3176.
19. 4.29.
20. 3.007.
21. .0087.

Sec. 65

1. 5.104+.
2. .226+.
3. .762+.
4. 36.476+.
5. 1460.551+.
6. 222.882+.
7. .330+.
8. .046+.
9. .076+.
10. .046+.
11. .005+.
12. .066+.
13. .112+.
14. .048+.
15. .070.

Sec. 70

1. 7.18.
2. 38.418.
3. 1172.63705.
4. 1111.245548
5. 1085.5.
6. 33.069.
7. 425.396211.
8. 2444.
9. 38217.7256.

Sec. 71

1. $(496.37 - 288.037) \times 183.75 \div 2.5 = 15{,}312.4755$.
2. $(\$.80 - \$.65) \times 48 = \$7.20$.
3. $18.902 \div (130 \times .02) = 7.27$.
4. $\$900 \div \{\$1080 - [\$180 + \$306 + \$369]\} = 4$.
5. 1779.7.
6. $(\$1200 + \$190 \times 3) \div 30 = \$59$.
7. $2151 \div (1 + 3 + 5) \times 3 = 717$.
8. $(2107.62 - 1009.006 - (342.807 + 231.96 + 324.7) = 199.147)$.
9. $20.45 \times 20.45 = 418.2025$.
10. ————.
11. ————.
12. $(6090 \times 148 + 111) \div 37 = 24{,}363$.
13. $[(345 + 3067 + 468 + 384) \div 2] \times \$.27 = \$575.64$.
14. ————.

Sec. 85

2. Comp.
3. Comp.
4. Comp.
5. Comp.
6. Prime.
7. Comp.
8. Comp.
9. Comp.
10. Prime.
11. Comp.
12. Comp.
13. Comp.
14. Comp.
15. Prime.
16. Comp.
17. Comp.
18. Prime.
19. Prime.
20. Comp.
21. Comp.

Sec. 86

2. 2, 2, 2, 2, 7.
3. 2, 2, 3, 5, 7.
4. 2, 2, 3, 5, 11.
5. 11, 101.
6. 5, 211.
7. 2, 3, 3, 257.
8. 5, 29.
9. 3, 43.
10. 5, 5, 5, 5.
11. 3, 3, 3, 3, 53.
12. 3, 5, 5, 19.
13. 2, 7, 101.
14. 3, 3, 7, 7, 7.
15. 23, 29.
16. 2, 5, 31.
17. 3, 7, 19.
18. 3, 3, 11, 13.
19. 11, 23.
20. 2, 29, 29.
21. 3, 11, 17.
22. 7, 11, 13.
23. 5, 5, 7, 7.
24. 2, 3, 3, 379.
25. 2, 3, 3, 3, 3, 3, 3, 5.

Sec. 88

1. 4788.
2. 2.
3. 18.
4. $5\frac{1}{3}$.
5. $3\frac{4}{7}$.
6. $1\frac{1}{3}$.
7. $7\frac{7}{9}$.
8. 36 bu.
9. 720.
10. 41.

Sec. 94

1. 540.
2. 72.
3. 720.
4. 378.
5. 720.
6. 360.
7. 420.
8. 1260.
9. 7560.
10. 4200.
11. 4410.
12. 576.
13. 360.
14. 240.
15. 210.
16. 1680.
17. 1260.
18. 300.
19. 4199.
20. 1260.
21. 2520.

Sec. 99

2. 42.
3. 30.
4. 56.
5. 36.
6. 20.
7. 16.
8. 12.
9. 24.
10. 10.
11. 5.
12. 9.
13. 11.
14. 63.
15. 9.
16. 13.
17. 16.
18. 14.
19. 5.
20. 63.
21. 2, 3, 3, 5.
22. 72.
23. (351).
24. 26, 39, 52.

Sec. 109

2. $\frac{36}{55}$.
3. $\frac{1}{3}$.
4. $\frac{9}{19}$.
5. $\frac{12}{13}$.
6. $\frac{1}{5}$.
7. $\frac{7}{118}$.
8. $\frac{1}{2}$.
9. $\frac{1}{3}$.
10. $\frac{1}{9}$.
11. $\frac{11}{34}$.
12. $\frac{7}{15}$.
13. $\frac{2}{5}$.
14. $\frac{1}{5}$.
15. $\frac{3}{4}$.
16. $\frac{1}{2}$.
17. $\frac{1}{7}$.
18. $\frac{7}{9}$.
19. $\frac{1}{7}$.
20. $\frac{11}{13}$.
21. $\frac{176}{221}$.
22. $\frac{13}{17}$.
23. $\frac{4}{7}$.
24. $\frac{4}{7}$.

Sec. 114

1. 49.
2. $18\frac{6}{25}$.
3. 28.
4. $26\frac{5}{9}$.
5. $21\frac{25}{32}$.
6. $19\frac{6}{41}$.
7. $80\frac{3}{5}$.
8. $21\frac{44}{45}$.
9. $12\frac{26}{67}$.
10. $30\frac{10}{43}$.
11. $40\frac{1}{6}$.
12. $402\frac{5}{16}$.
13. $23\frac{28}{37}$.
14. $175\frac{4}{7}$.
15. $24\frac{5}{12}$.
16. $18\frac{7}{9}$.
17. $27\frac{7}{12}$.
18. $10\frac{1}{4}$.
19. $20\frac{34}{37}$.
20. $90\frac{1}{2}$.
21. $100\frac{34}{79}$.
22. $165\frac{41}{124}$.

Sec. 115

2. $\frac{87}{5}$.
3. $\frac{63}{4}$.

4. $\frac{419}{14}$.
5. $\frac{330}{13}$.
6. $\frac{479}{8}$.
7. $\frac{858}{5}$.
8. $\frac{899}{8}$.
9. $\frac{393}{15}$.
10. $\frac{505}{11}$.
11. $\frac{514}{9}$.
12. $\frac{282}{9}$.
13. $\frac{390}{11}$.
14. $\frac{301}{14}$.
15. $\frac{495}{8}$.
16. $\frac{3751}{30}$.
17. $\frac{1278}{8}$.
18. $\frac{1007}{6}$.
19. $\frac{764}{31}$.
20. $\frac{2455}{44}$.
21. $\frac{1423}{11}$.
22. $\frac{6498}{30}$.
23. $\frac{3918}{18}$.
24. $\frac{1493}{6}$.
25. $\frac{7349}{16}$.
26. $\frac{21451}{134}$.
27. $\frac{6135}{16}$.
28. $\frac{39460}{61}$.
29. $\frac{26862}{48}$.
30. $\frac{2800}{5}$.
31. $\frac{4000}{16}$.
32. $\frac{202}{16}$.
33. $\frac{1872}{12}$.

Sec. 119

1. $\frac{45, 48, 50}{60}$.
2. $\frac{30, 27, 35}{45}$.
3. $\frac{35, 12, 20}{40}$.
4. $\frac{78, 63}{91}$.
5. $\frac{60, 56, 39, 18}{84}$.
6. $\frac{90, 96, 100, 75}{120}$.
7. $\frac{280, 35, 36, 24}{40}$.
8. $\frac{6, 3, 10, 9}{12}$.
9. $\frac{60, 135, 108, 80, 162}{180}$.
10. $\frac{20, 35, 36, 30, 32}{40}$.
11. $\frac{312, 75, 259}{455}$.
12. $\frac{48, 75, 30, 110}{120}$.
13. $\frac{170, 90, 85, 36}{204}$.
14. $\frac{104, 189, 91, 90}{234}$.
15. $\frac{2574, 3432, 3465, 143, 6552}{9009}$.

Sec. 121

3. $2\frac{97}{105}$.
4. $1\frac{41}{90}$.
5. $4\frac{71}{108}$.
6. $2\frac{41}{182}$.
7. $83\frac{233}{252}$.
8. $1\frac{89}{99}$.
9. $17\frac{67}{280}$.
10. $61\frac{7}{48}$.
11. $65\frac{37}{56}$.
12. $28\frac{61}{120}$.
13. $59\frac{5}{24}$.
14. $1381\frac{157}{187}$.
15. $1712\frac{29}{72}$ ft.

Sec. 123

3. $11\frac{11}{15}$.
4. $21\frac{5}{6}$.
5. $208\frac{31}{35}$.
6. $148\frac{14}{15}$.
7. $207\frac{1}{72}$.
8. $17\frac{5}{6}$.
9. $\frac{3}{56}$.
10. $\frac{7}{24}$.
11. $\frac{1}{20}$.
12. $2\frac{9}{14}$.
13. $\frac{13}{18}$.
14. $96\frac{13}{15}$.
15. $1\frac{13}{15}$.
16. $18\frac{37}{45}$.
17. $90\frac{11}{12}$.
18. $5\frac{18}{21}$.
19. $5\frac{7}{20}$.
20. $48\frac{1}{2}$.
21. $15\frac{53}{63}$.
22. $190\frac{5}{22}$.
23. $46\frac{5}{18}$.
24. $8\frac{37}{51}$.
25. $41\frac{28}{45}$.
26. $43\frac{19}{22}$.

Sec. 124

1. 250.
2. 280.
3. 80.
4. 756.
5. $2\frac{2}{9}$.
6. 5.
7. $174\frac{14}{39}$.
8. $48\frac{4}{7}$.
9. $58\frac{1}{9}$.
10. $94\frac{64}{65}$.
11. $11\frac{9}{10}$.
12. 7.

Sec. 128

2. $\frac{8}{5}$.
3. $\frac{7}{16}$.

4. $\frac{1}{3}$.
5. $\frac{1}{4}$.
6. $\frac{2}{19}$.
7. $\frac{5}{18}$.
8. $\frac{3}{5}$.
9. $\frac{7}{16}$.
10. $\frac{3}{8}$.
11. $\frac{3}{45}$.
12. 20.
13. 2.
14. $\frac{7}{9}$.
15. $7\frac{1}{3}$.
16. $9\frac{1}{3}$.
17. $\frac{49}{144}$.
18. $1\frac{1}{2}$.
19. $\frac{4}{45}$.

Sec. 129

1. $30\frac{3}{8}$.
2. 94.
3. 233.
4. $4108\frac{13}{15}$.
5. $1381\frac{3}{5}$.
6. $2\frac{9}{16}$.
7. $73\frac{2}{3}$.
8. $14\frac{16}{21}$.
9. $18\frac{89}{132}$.
10. $102\frac{2}{45}$.
11. $17\frac{1}{2}$.
12. $626\frac{2}{63}$. (a)
12. $101\frac{1}{2}$. (b)
13. 4032.
14. $115\frac{5}{6}$.
15. $2\frac{1}{5}$.
16. 1.
17. $16\frac{1}{2}$.
18. 154.

Sec. 130

2. $448\frac{8}{9}$.
3. $1252\frac{4}{5}$.
4. $9426\frac{3}{7}$.
5. $14946\frac{12}{13}$.
6. $57037\frac{3}{4}$.
7. $3547855\frac{5}{7}$.
8. 46170.
9. $10294\frac{1}{4}$.
10. $46197\frac{1}{2}$.
11. $8262\frac{3}{8}$.
12. 113108.
13. $108504\frac{3}{4}$.
14. 106624.
15. $423815\frac{3}{8}$.
16. $252616\frac{1}{2}$.

Sec. 132

3. 6.
4. $\frac{5}{6}$.
5. $\frac{7}{10}$.
6. $\frac{14}{27}$.
7. $\frac{7}{5}$.
8. $1\frac{1}{4}$.
9. $4\frac{1}{4}$.
10. $13\frac{3}{5}$.
11. $\frac{100}{253}$.
12. $\frac{1}{14}$.
13. $\frac{238}{1995}$.
14. $2\frac{1}{4}$.
15. $25\frac{5}{7}$.
16. 18.
17. $\frac{1}{18}$.
18. $\frac{1}{12}$.
19. $\frac{1}{2}$.
20. $4\frac{3}{4}$.
21. 4.
22. $1\frac{5}{21}$.
23. $1\frac{61}{887}$.
24. $4\frac{3}{14}$.
25. $\frac{5}{5}$.
26. $2\frac{47}{39}$.
27. $\frac{45}{154}$.
28. 4.
29. $\frac{21}{40}$. (a)
29. $\frac{26}{55}$. (b)
30. $\frac{364}{583}$. (a)
30. $\frac{35}{132}$. (b)

Sec. 135

4. 12.
5. $10\frac{5}{8}$.
6. $4\frac{7}{24}$.
7. $\frac{7}{135}$.
8. $1\frac{21}{64}$.
9. $\frac{10}{11}$.
10. $\frac{4}{5}$.
11. $1\frac{2}{13}$.
12. $\frac{9}{32}$.
13. $4\frac{5}{7}$.
14. \$361$\frac{1}{2}$.
15. $25\frac{1}{2}$ rd.
16. $11\frac{1}{2}$ tons.

Sec. 137

1. .42+.
2. .08+.
3. .94+.
4. 2.84+.
5. 4.04+.
6. 168.28.
7. 108.26+.
8. .46+.
9. 1.66+.
10. 56.25.
11. 2.61+.
12. .90+.

Sec. 141

1. $\frac{7}{25}$.
2. $\frac{1}{8}$.
3. $\frac{47}{200}$.
4. $\frac{3}{4}$.
5. $\frac{41}{250}$.
6. $\frac{41}{50}$.
7. $\frac{189}{500}$.
8. $\frac{17}{40}$.
9. $\frac{7}{8}$.
10. $\frac{3}{8}$.
11. $\frac{11}{20}$.
12. $\frac{1}{400}$.
13. $\frac{14}{25}$.
14. $\frac{17}{25}$.
15. $16\frac{3}{40}$.
16. $\frac{1}{80}$.
17. $\frac{27}{5000}$.
18. $\frac{1}{4}$.
19. $\frac{11}{80}$.
20. $\frac{7}{160}$.
21. $\frac{1}{8}$.
22. $\frac{5}{8}$.
23. $\frac{1}{16}$.
24. $\frac{229}{16000}$.
25. $\frac{3}{8000}$.
26. $\frac{6229}{50000}$.
27. $\frac{7}{8}$.
28. $\frac{2}{3}$.
29. $\frac{2189}{16000}$.
30. $116\frac{1}{4}$.
31. $2\frac{1}{3}$.
32. $\frac{11}{350}$.
33. $22\frac{5}{8}$.
34. $7\frac{7}{80}$.
35. $6\frac{27}{200}$.
36. $58\frac{1}{16}$.
37. $491\frac{1}{10}$.
38. $\frac{4}{55}$.
39. $\frac{19}{150}$.
40. $\frac{9}{70}$.
41. $\frac{1}{6}$.
42. $\frac{47}{240}$.
43. $\frac{13}{80}$.
44. $40\frac{57}{140}$.
45. $61\frac{37}{90}$.
46. $42\frac{11}{60}$.

47. $179\frac{1}{180}$.
48. $8\frac{1}{14}$.
49. $890\frac{407}{450}$.
50. $8\frac{1}{1025}$.

Sec. 143

1. .4.
2. .6.
3. .8.
4. .75.
5. .125.
6. .375.
7. .875.
8. $.466\frac{2}{3}$.
9. .3.
10. .15625.
11. 3.0625.
12. .0375.
13. .15.
14. .16.
15. 1.5.
16. .734375.
17. .5375.
18. .85.
19. .05625.
20. .448.
21. .059375.
22. .044.
23. .375.
24. 2.09375.
25. 13.35
26. 19.21875.
27. 12.36.
28. .022.
29. .0575.
30. .696.
31. 19.152.
32. .0016.
33. .056.
34. 12.0096.
35. 14.3625.
36. 9.0072.
37. 13.025.
38. 2.084.
39. 5.25.
40. .0071875.

Sec. 144

1. .666+.
2. .857+.
3. .272+.
4. .555+.
5. .444+.
6. .923+.
7. .774+.
8. .145+.
9. .764+.
10. 12.789+.
11. .012+.
12. 5.222+.
13. 8.266+.
14. 33.111+.
15. 681.363+.
16. 53.470+.
17. .521+.
18. .147+.
19. 62.344+.
20. .608+.
21. 1.309+.
22. 143.119+.
23. 216.126+.
24. 1.182+.

Sec. 148

1. *a*. $\$20\frac{3}{4}$.
   b. $\$41\frac{1}{3}$.
   c. $37.
   d. $\$93\frac{3}{4}$.
   e. $\$37\frac{1}{2}$.
   f. $40.
   g. $160.
   h. $\$10\frac{2}{3}$.
   i. $205.
   j. $574.
2. 450 yd.
3. 832 bu.
4. 96 knives.
5. 2600 lb.
6. 1078 articles.
7. 1664 books.
8. 1500 boxes.
9. 192 pr.
10. 640 bu.
11. 240 lb.

Sec. 152

1. 57,750.
2. 56,628.
3. .3628.
4. 380.82.
5. 670.
6. 88.41.
7. 108.
8. 77,232.
9. 1.39.
10. 254.
11. .382656.
12. 20.874.
13. 9.33.
14. 12,488.
15. .6705.
16. 237,303.
17. 452,547.
18. 1,511,300.
19. 7801.
20. 26,724.
21. 633.75.
22. 25.40.
23. 25.812.
24. .1904.
25. 190.4.

Sec. 160

1. Dr. $27.13.
2. Dr. $179.
3. Dr. $9.79.
4. Dr. $9.25.
5. Dr. $20.62

Sec. 162

1. 6020.602.
2. 3619.279.
3. 1128.305.
4. 1000.119.
5. 776.590.
6. 38,202.001.
7. 7654.99.
8. 6091.09.
9. 799,109.10.
10. 504.675.
11. 49,099.081.
12. 27,756.801.
13. 201.01.
14. 22,756.845.
15. 23,027.011.
16. 159.026.
17. 1481.04+.
18. 34,962.66.
19. 320.39.
20. 420.05.
21. 13.902.
22. 10,044.
23. 35875.
24. 35.04.
25. 55,737.
26. 1,202,500.
27. 24.592.
28. 105.175.
29. 30,292.
30. 518.52.
31. .40592.
32. 3932.28.

33. 11.753.
34. 620.9.
35. 338,450.
36. 1073.75.
37. .03.
38. 18266⅔.
39. 472.02.
40. 1642.5.
41. 900 bu.
42. $192,000.
43. MCMVIII.
44. $625.
45. $6050.
46. $250.
47. 5800 tons.
48. $\frac{6}{35}$ left. (*a*) $440. (*b*)
49. $37\frac{2}{15}\frac{5}{8}$.
50. 50.2 bu.
51. ——.
52. ——.
53. ——.

Sec. 163

4. *a*. $654.42.
   b. $443.03.
   c. $14.73.
   d. $45.
   e. $5.
   f. $73.13
   g. $832.
   h. $1.62.

Sec. 204

1. *a*. 2424 hr.
   b. 10,700 lb.
   c. 3 da. 9 hr. 40 min.
   d. 443,520 in.
   e. 4640 pt.
   f. 4 cu. yd. 3 cu. ft. 1671 cu. in.
   g. 5 da. 18 hr. 25 min. 42 sec.
   h. 25 tons.
   i. 359 lb. 4 oz.
   j. 4320 gr.
   k. £353 3*s*. 7*d*.
   l. 1331*d*.
   m. 175,819″
   n. 6,307,200 min.
   o. 3 mi. 2 ft. 9 in.
   p. 10 hhd.
   q. 1,887,840 min.
   r. 12 yr. 11 da.
   s. 748,687½ sq. ft.
   t. 233,280 cu. in.
   u. 24 A. 64 sq. rd.
   v. 17 cu. yd. 24 cu. ft. 145 cu. in.
2. $\frac{13}{66}$ mi.
3. 1180 gr.
4. 38,880 cu. in.
5. $\frac{1}{320}$ da.
6. 28 gr.
7. $\frac{27}{160}$ mi.
8. $\frac{3}{176}$ mi.
9. ⅛ gross.
10. 1468⅛ cu. in.
11. *a*. 5760 gr.
    b. 5760 gr.
    c. 43,560 sq. ft.
    d. 63,360 in.
    e. 480 gr.
    f. 480 gr.
12. *a*. 57¾ cu. in.
    b. 67.200625 cu. in.
    c. 480 gr.
    d. 437½ gr.
    e. $1\frac{31}{144}$ lb., Troy.
13. *a*. 1.244+ cu. ft.
    b. 5.181+ fr.
    c. $.952.
    d. $.243+.
    e. 4.725+ cu. in.
    f. 12.152+ lb., Troy.
    g. 4.296+ bu.
14. 10,500 tablets.
15. *a*. ——.
    b. ——.
    c. ——.
    d. ——.
    e. ——.
    f. ——.

Sec. 205

1. 117 ft. 4 in.
2. 112 A. 69 sq. rd.
3. 13 hr. 49 min. 17 sec.
4. 60 gal. 2 qt.
5. 197° 14′ 54″.
6. 53 T. 110 lb.
7. 40 yd. 2 ft.
8. 43 pk. 2 qt.
9. 70 cu. yd. 10 cu. ft.
10. 44 yr. 3 mo. 17 da.
11. 44 lb. 15 oz.
12. 26 bu. 2 pk.
13. 40 wk. 6 da.
14. 53 hr. 25 min. 50 sec.
15. 81 yr. 335 da.

Sec. 206

1. 8 yr. 1 mo. 26 da.
2. 15 yr. 5 mo. 4 da.
3. 4 yr. 6 mo.
4. 13 yr. 10 mo. 14 da.
5. 12 yr. 5 mo. 16 da.
6. 13 yr. 10 mo. 1 da.
7. 21 yr. 3 mo. 25 da.
8. 3 yr. 5 mo. 15 da.
9. 2 yr. 6 mo. 5 da.
10. 50 yr. 7 mo. 27 da.
11. 67 yr. 2 mo. 25 da.
12. 55 yr. 2 mo. 1 da.

13. 9 yr. 7mo. 28 da.
14. 95 yr. 9 mo. 24 da.
15. 23 yr. 4 mo. 23 da.
16. 73 yr. 5 mo. 12 da.
17. 8 yr. 9 mo. 4 da.
18. 4 yr. 11 mo. 20 da.
19. 4 hr. 1 min. 44 sec.
20. 162° 20′ 40″.
21. 13 hr. 19 min. 38 sec.
22. 13 ft. 8 in.
23. 8 gal. 2 qt.
24. 12° 50′.
25. 10 bu. 2 pk.
26. 115 A. 140 sq. rd.
27. 20 lb. 9 oz.
28. 30 ft. 5 in.
29. 6′ 45″.
30. 2 min. 59 sec.
31. 3 mo. 18 da.
32. ——.

Sec. 207

2. 295 da.
3. 23 da.
4. 329 da.
5. 29 da.
6. 132 da.
7. 150 da.
8. 332 da.

Sec. 208

2. 51 lb. 8 oz.
3. 110 gal. 1 qt.
4. 44 ft. 6 in.
5. 147 A. 120 sq. rd.
6. 33 hr. 52 min. 30 sec.
7. 38° 21′ 30″.
8. 10 hr. 34 min. 15 sec.
9. 45 hr. 4 min. 45 sec.
10. 3 da. 7 hr. 19 min. 45 sec.
11. 4 hr. 33 min. 15 sec.
12. 2° 45′ 55″.
13. 2° 41′ 20″.
14. 1° 9′ 13″.
15. 5′ 56″.
16. 2′ 39″.
17. 1° 48′ 2″.
18. 56′ 1″.
19. 2° 32′ 4″.
20. 30″.
21. 2° 46′ 48″.
22. 4′ 44″.
23. 2° 40′ 10″.
24. 34′ 43″.
25. 3′ 56″.
26. 1° 8′.
27. 4′ 6″.
28. 45′ 16″.

Sec. 218

1. $10\frac{1}{2}$ A.
2. $3227.15.
3. $\frac{9}{44}$ A.
4. $3\frac{1}{3}$ rd. 55 ft.

Sec. 221

1. $\frac{448}{1089}$ sq. rd.
2. .375 A. or $\frac{3}{8}$ A.
3. $\frac{15}{242}$ sq. rd.
4. $6\frac{1}{2}$ rd.
5. 24 ft.

Sec. 226

1. $\frac{2}{9}$ A.
2. $71.82.
3. $\frac{15}{16}$ sq. yd.

Sec. 233

1. *a*. $6\frac{3}{4}$ cd.
   b. $3\frac{1}{8}$ cd.
   c. $12\frac{1}{2}$ cd.
   d. $18\frac{3}{4}$ cd.
   e. $1\frac{7}{8}$ cd.
2. *a*. 4 ft.
   b. 3 ft.
   c. 2 ft.
   d. 1 ft.
   e. $1\frac{1}{2}$ ft.
   f. $1\frac{1}{4}$ ft.
3. 44 ft.
4. *a*. 4 ft.
   b. $6\frac{2}{5}$ ft.
   c. 3 ft.

Sec. 235

1. $288.
2. $682.50.
3. $186.20.
4. $224.25.
5. $68.06.
6. $84.00.
7. $158.33\frac{1}{3}$.

Sec. 237

1. 7 strips.
2. $61\frac{2}{3}$ yd.
3. *a*. $72.93.
   b. $26.80.
4. $ 22.78.
5. $166\frac{2}{3}$ yd.
6. $77.00.
7. ——.

Sec. 239

1. *a*. 56 ft.; 9 ft.
   b. 504 sq. ft.
   c. 192 sq. ft.
   d. 696 sq. ft.
   e. $77\frac{1}{3}$ sq. yd.
   f. $25.12.
2. *a*. $57.14.
   b. $144.
3. $74.80.
4. *a*. ——.
   b. ——.
   c. ——.

Sec. 241

1. *a.* $20.62.
   b. $16.02.
2. $45.30.
3. *a.* ——.
   b. ——.

Sec. 245

1. $174.41.
2. $10.40.
3. $2.88.
4. $34.56.
5. $37.28.
6. 1.416⅔ M.
7. 1344 bd. ft.
8. 2400 bd. ft.
9. *a.* $158.40.
   b. $27.00.
   c. $34.56.

Sec. 247

1. *a.* 6 shingles.
   b. 600 shingles.
   c. 2160 shingles.
2. $54.675.
3. $49.795.
4. $25.92.
5. $54.00.
6. $6.00.

Sec. 249

1. 5430⅚ gal.
2. *a.* 7.48 gal.
   b. .80+ bu.
3. 9.31 gal.
4. *a.* 1000 bu.
   b. 1244.5 cu. ft.
   c. 900.5 cu. ft.
   d. 582.5 cu. ft.
5. $19.20.
6. 4.57+ bbl.
7. 47.26 bu.
8. 27⅜ bbl.
9. 4,752,000 gal.
10. *a.* 8316 cu. in.
    b. 693 cu. in.
    c. 3 gal.
11. Too large. 32⅜ cu. in.
12. 8.355 lb.
13. *a.* 112.20 gal.
    b. 937.5 lb.
14. 12.66 bbl.
15. 73.7 bu.
16. 14.2 in. **17.** 7.5″.
18. *a.* 5″.
    b. 7.87+″.
    c. 192 gal.
    d. 577.50 lb.
19. 32.057+ bbl.

Sec. 251

1.	3.64.	**8.**	2.70.
2.	2.61.	**9.**	2.84.
3.	3.14.	**10.**	1.16.
4.	3.38.	**11.**	1.33.
5.	3.13.	**12.**	1.51.
6.	4.76.	**13.**	3.10.
7.	2.39.	**14.**	1.28.

15. 86,579,553.
16. 34,200 mi.
17. *a.* 212 yr. 120 da.
    b. 6369 yr. 315 da.
18. *a.* 73,180,800 col.
    b. 340 A.
19. 821,184.

Twenty-two thousand, four hundred sixty-four.

Forty-nine thousand, nine hundred twenty.

Seven hundred forty-eight thousand eight hundred.

20. 8.9901.
21. $121.50. (*a*)
    $5.87. (*b*)
    $60.00. (*c*)
    $187.37. (*d*)
22. *a.* $171.12.
    b. 6.045 tons.
23. *a.* 470.0625 sq. in.
    b. 30.634+ sheets.
24. 3, 19, 79.
25. 529; 403; 143; 1943; 407.
26. $\frac{19}{41}$.
27. $\frac{11}{140}$.
28. *a.* 2899.
    b. 2364.
    c. 113,293.
    d. 756.25.
    e. 3,936,636.
    f. 288,420,000.
    g. 20,425.
    h. 1224.5.
    i. 146,600.
    j. 828,700.
29. *a.* 1578.96.
    b. 77,810.
    c. 283.14.
    d. 52,518.
    e. 5624.52.
    f. 2998.45.
    g. 30,592.
    h. 279,126.
    i. 68,901.
    j. 5.4205.
30. $41\frac{4}{21}$.
31. 7560.
32. ——.

33. $53\frac{1}{3}$ A.
34. 31,556,926.
35. $293\frac{1}{3}$ fathoms.
36. $73.73.
37. ——.
38. $36.
39. *a*. 48 cu. ft.
b. $3.49.
c. 18,671.37 gal.
d. 16 ft.

Sec. 253

1. $225.
2. 40 pupils.
3. .75.
4. .90.
5. *a*. $26,000.
b. .50.
c. $.16\frac{2}{3}$.
6. $3020.
7. .05.
8. $.12\frac{1}{2}$.
9. .30.
10. *a*. $780.
b. $936.
c. $1040.
11. 100.
12. $8.
13. .15.
14. $2790.
15. $41.80.
16. *a*. $\frac{17}{20}$.
b. $183.
c. $30.
17. *a*. $.12\frac{1}{2}$.
b. .25.
c. $.62\frac{1}{2}$.

Sec. 259

1. *a*. .07.
b. .06.
c. .02.
d. .12.
e. .78.
f. .0625.
g. .125.
h. .1575.
i. .375.
j. .04625.
k. 1.01.
l. 1.10.
m. 2.50.
n. 2.00.
o. 1.275.
p. .005.
q. .0075.
r. .004.
s. .00625.
t. .009.
2. 1746.
3. 2000.
4. 20%.
5. 21.
6. 618 tons.
7. $4\frac{52}{125}$%.
8. $89\frac{2}{7}$%.
9. 941.84.
10. 30,000.
11. 86.75+%.
12. 32,800.
13. 15%.
14. 25%.
15. $8.54\frac{6}{11}$.
16. $58\frac{7}{16}$%.
17. $\frac{1}{4}$%.
18. 95%.
19. $2675.
20. $10,890.
21. $37.50.
22. 40 %.
23. Equal.
24. *a*. 40 %.
b. 250 %.
25. 18 A.
26. $183,872.
27. 7000 men.
28. $10,500.
29. 200 boys.

Sec. 262

1. *a*. 112 bu.
b. 618 T.
c. 210 gal.
d. 106 qt.
e. $75.
f. 750.
g. 75,000.
h. 624.
2. 236.40.
3. $37\frac{1}{2}$ %.
4. $266\frac{2}{3}$ %.
5. 500 ; $437\frac{1}{2}$.
6. .5, .005 ; .6, .006 ; .125, .00125 ; .375, .00375 ; .75, .0075 ; .00875 ; .001 ; .00625.
7. $21\frac{3}{52}$ %.
8. $53\frac{11}{18}$%; $38\frac{6}{13}$%; $7\frac{9}{13}$%.
9. $87\frac{1}{2}$ %.
10. $1750.
11. 90 da.
12. 50 yd.
13. 20%.
14. 20%.
15. *a*. $1500.
b. $4\frac{4}{5}$%.
c. $528.
16. $84.25.
17. $2.50 ; $2.38.
18. 100,000.

19. *a.* $1750.
b. $21.
c. $1300.
20. $8100.

Sec. 265

1. *a.* $50.
b. $33\frac{1}{3}\%$.
2. $306.
3. $200.
4. *a.* $5.50.
b. $75.00.
5. $1993.25.
6. *a.* $100.
b. $70.
c. 5000 lb.
7. *a.* $.27\frac{3}{11}$.
b. $5\frac{5}{11}\%$.
c. $3.19.
8. 20¢.
9. $2550; ——.
10. $38\frac{8}{9}\%$.
11. $5\frac{1}{2}$¢ per lb.
12. $55\frac{5}{9}\%$.
13. $2880.
14. $2000.
15. $231.
16. Nothing.
17. *a.* Lost $5.
b. 4%.
18. *a.* $28,080.
b. $18,720.
c. $936.
d. $9864.
e. $27\frac{2}{5}\%$.
19. *a.* $33\frac{1}{3}\%$.
b. 21¢.
c. 2400 ears.
20. *a.* $4200.
b. $3000.
c. $780.
d. 26%.
e. $3500.

Sec. 276

1. $200.81.
2. $2239.40.
3. *a.* $1260.
b. $134,166\frac{2}{3}$ lb.
4. 40%.
5. $300,000.
6. *a.* $3380.
b. $51.20.
c. $16.40.
7. *a.* $1152.25.
b. $4544.
8. *a.* $14,300.
b. $28,481.82.
9. *a.* $396.
b. $10.24.
c. $2300.
10. $120.
11. $8000.
12. *a.* $1875.
b. $150.
13. $53,000.
14. $7000.
15. $3494.15.
16. $2000.

Sec. 283

1. $2268.70.
2. $70.
3. $300.
4. *a.* $194.75.
b. $251.37.
c. $566.98.
d. $227.81.
e. $151.
f. $218.03.
5. $34.30.
6. *a.* 8%.
b. $12.60.
c. $53\frac{1}{3}\%$.
7. *a.* 72¢.
b. 80¢.
8. $354.
9. *a.* $5066.66\frac{2}{3}$.
b. $4000.
10. *a.* $500.
b. $664.58\frac{1}{3}$.
11. $105.10.
12. $492.48.
13. $44.80 loss.
14.
a. 1st, $833.33\frac{1}{3}$;
2d, $1000.
b. $86.66\frac{2}{3}$.
15. Same.
16. The 2d, $1\frac{4}{5}\%$;
or $3.60.
17. $327.60.
18. *a.* $14\frac{1}{2}\%$.
b. $16\frac{7}{8}\%$.
c. $19\frac{1}{4}\%$.
d. $23\frac{1}{2}\%$.
e. 25%.
f. 24%.
g. 28%.
h. $31\frac{3}{5}\%$.
i. $28\frac{3}{4}\%$.
j. $32\frac{1}{2}\%$.
k. $35\frac{7}{8}\%$.
l. $33\frac{1}{3}\%$.
m. 37%.
n. $40\frac{3}{20}\%$.
o. $36\frac{2}{3}\%$.
p. 40%.
q. 43%.
r. 43%.
s. 46%.
t. $48\frac{7}{10}\%$.
u. 52%.
v. $54\frac{2}{5}\%$.
w. $50\frac{1}{2}\%$.
x. $52\frac{1}{2}\%$.
y. 55%.
z. $57\frac{1}{4}\%$.
19. $200.
20. $200.
21. 10%.
22. 2%.
23. $200.
24. *a.* $35.
b. $15\frac{5}{18}\%$.
c. $30.
d. $2000.
25. *a.* $19.80.
b. $40\frac{20}{27}\%$.

Sec. 290

1. *a.* $22.75.
b. $91.
c. $23.10.
d. $5.77\frac{1}{2}$.
2. $42.
3. $7.65.
4. *a.* $3\frac{3}{4}\%$.
b. $33.75.
5. $16.
6. $4750.
7. $5500.
8. $7600.
9. *a.* $21.
b. $2800.
c. $3500.
10. $25,750.
11. *a.* $45.
b. $67.50.
c. $457.50.

12. *a.* $3775.30.
b. $2600.
c. $3775.30.
13. *a.* $2575.
b. $2575.
14. $\frac{9}{10}$ %.
15. $10,054.80.
b. $768.075.
c. $251.37.
16. $5000.
17. $5595.
18. $830.20.
19. $1.75.

Sec. 299

2. *a.* $90.
b. $33.75.
c. $35.
d. $12.50.
e. $19.26.
f. $19.58.
g. $43.74.
h. $61.60.

Sec. 301

2. *a.* $3.50.
b. $2.81.
c. $14.145.
d. $46.67.
e. $340.625.
f. $129.60.
g. $37.54.
h. $18.60.
i. $12.47.
j. $21.82.
k. $45.84.
l. $4114.92.
m. $30.89.
n. $456.30.
o. $10.53.

Sec. 303

2. *a.* $273.
b. $5505.
c. $376.32.
d. $1108.93.
e. $890.75.
f. $4397.90.
g. $663.
h. $1444.88.
i. $177.13.
3. $744.10.
4. ——.
5. ——.

Sec. 304

2. *a.* $12.13.
b. $16.07.
c. $9.58.
d. $6.32.
e. $11.19.
3. *a.* $10.56.
b. $4.21.
c. $13.16.
d. $14.97.
e. $21.90.
4. *a.* $504.40.
b. $4041.33.
c. $464.64.
d. $511.08.
5. $489.60.

Sec. 307

2. *a.* $123.29.
b. $7.62.
c. $1.12.
d. $57.54.
e. $.65.
3. $503.90.
4. $15.
5. $333.35.
6. $1119.09.
7. $132.84.

Sec. 309

2. 6 %.
3. 8 %.
4. $4\frac{1}{2}$ %.
5. 6 %.
6. $7\frac{1}{7}$ %.
7. 6 %.
8. 8 %.
9. $4\frac{1}{2}$ %.

Sec. 310

2. 2 yr.
3. 7 yr.
4. $2\frac{1}{2}$ yr.
5. $4\frac{1}{4}$ yr.
6. 1 yr. 4 mo. 8 da.
7. 1 yr. 9 mo. 21 da.
8. 1 yr. 4 mo. 8 da.
9. 1 yr. 1 mo. 6 da.
10. 3 yr. 7 mo. 6 da.
11. 2 mo. 24 da.
12. 1 yr. 2 mo.
13. 3 mo. 5 da.
14. 1 yr. 25 da.
15. 1 yr. 1 mo. 15 da

Sec. 311

2. $800.
3. $3000.
4. $300,000.
5. $240.
6. $879.60.
7. $1080.
8. $4800.

Sec. 312

1. $550.
2. $385.

3. $400.
4. $2000.
5. $479.22.
6. *a*. $1600.
   b. $4000.
7. $1225.58.

Sec. 313

1. $29.14.
2. $2846.25.
3. $148.83; $2498.83.
4. $3.96 int.
   $295.96 amt.
5. 4 yr. 7 mo. 6 da.
6. $12,580.
7. 8 %.
8. 3 yr. 7 mo. 6 da.
9. 16 yr. 8 mo.; $1000.
10. 4 %; $2.50.
11. $6\frac{6}{19}$ %.
12. $3509.33; $70.67.
13. $4000; 7 mo. 6 da.
14. $8\frac{31}{213}$ %.
15. $480.
16. $623.31.
17. $750.
18. 7 %.
19. 144 da.
20. $361.55.
21. $.11\frac{1}{4}$.
22. 3 yr. 9 mo. 10 da.
23. $30,000.

Sec. 316

2. $130.83.
3. $761.58.
4. $61.52.
5. 137.92.
6. 56.31.
7. $179.40.
8. $82.55.

Sec. 345

1. $267.50.
2. $507.50.
3. $1039.33.
4. $101.70.
5. $430.74.
6. $724.43.
7. *a*. Oct. 2/06.
   b. $977.36.
8. $1212.50.
9. ——.
10. $260.
11. $609.
12. Rate 8 %.
13. $253.08.
14. ——.
15. ——.

Sec. 349

1. $647.53.
2. $683.68.
3. $622.14.
4. $1408.45.
5. 418.09.
6. $310.72.
7. $54.38.
8. $408.04.
9. 478.12.
10. $1477.17.
11. $160.18.
12. $1102.86.

Sec. 351

1. $230.68.
2. $70.47.
3. $870.12.
4. $400.21.
5. $123.13.

Sec. 353

1. Jan., 41.15°.
2. Feb., 43.75°.
3. Mar., 51.60°.
4. Apr., 63.2°.
5. May, 72.75°.
6. June, 80.7°.
7. July, 85.35°.
8. Aug., 83.65°.
9. Sept., 77°.
10. Oct., 66.1°.
11. Nov., 53.15°.
12. Dec., 44.65°.
13. 56.83.
14. 69.67.
15. 63.25.
16. 56.83.
17. 51.33.
18. 57.25.
19. 63.25.
20. 63.67.
21. 74.92.
22. 60.
23. 61.58.
24. 77.92.
25. 54.50.
26. 76.08.
27. 53.58.
28. 61.08.
29. 64.83.
30. 60.75.
31. 58.08.
32. 86.33.
33. ——.
34. *a*. 993,564,200.
   b. 4,258,375.
   c. 39,076,092.
35. *a*. 1590.52.
   b. 78.672.
   c. 4,778,718.

36. 7, 29, 37.
37. $\frac{3}{7}$.
38. 3276.
39. 63.
40. .492.
41. ——.
42. $7\frac{1}{2}$ A.
43. $\frac{6}{80}$.
44. 1° 13′ 10″.
45. 115 hr. 2 min.
46. $55.50.
47. $164.96.
48. 9 ft.
49. $400.
50. 600 qt.
51. $21\frac{3}{7}$ %.
52. 42%.
53. *a*. $25\frac{25}{31}$%.
b. $70\frac{30}{31}$%.
c. $3\frac{7}{31}$%.
54. $497.42.
55. *a*. $531.07.
b. $.89 (favor debtor).
56. $20.
57. The same.
58. The latter.
59. *a*. $5000.
b. $2500.
c. $9.
60. 10%.

Sec. 376

1. $709.20.
2. $644.58.
3. $91.25.
4. $297.79.
5. $9546.
6. $796.93.
7. $970.45.
8. *a*. ——.
b. $891.77.
9. $999.77.
10. $855.36.
11. $696.85.
12. $35.31.
13. $300.
14. $600.
15. $480.
16. ——.
17. $576.
18. $4.08.
19. $1400.
20. $405.90.
21. $551.84.
22. $580.
23. $890.34.
24. $1551.95.
25. $750.

Sec. 379

1. $459.13.
2. $502.08.
3. $7.75.
4. $600.

Sec. 390

1. .015.
2. $73,703.40.
3. $20,000,000.

Sec. 392

1. *a*. $1,853,007.44.
b. $1,606,779.44.
c. .0164.
d. $328.
e. $72.74.
f. $18,000.
g. $76.13.
2. $3000.
3. *a*. .015.
b. $270.
c. $2500.
4. $31.50.
5. .02.
6. $127.56.
7. .01203.
8. *a*. 1960 ft.
b. $1.02.
c. $118.32.

Sec. 402

5. *a*. $18.75.
b. $8200.
c. ——.
6. $\frac{1}{15}$%.
7. $\frac{1}{12}$%.
8. $\frac{1}{8}$%.

Sec. 411

1. $.35.
2. *a*. $4.67.
b. $494.83.
3. $2.
4. *a*. ——.
b. $18.
c. $1780.50.
5. $789.
6. $1429.80.
7. $2000.

Sec. 431

1. *a*. $1217.50.
b. £900.
2. *a*. $399.71.
b. 3569.02 francs.
3. $4.89\frac{1}{2}$.
4. $5.17\frac{3}{4}$ francs.
5. $7.05.
6. $232.26.
7. $1912.50.
8. 8400 marks.

9. *a.* $2600.
   b. £300.
   c. $2047.835.
   d. 976.56.
   e. 800 marks.
   f. $4000.
   g. $4015.04.
   h. £450.
   i. 1032.5 francs.
   j. $212.80.
10. 2057.5 lire.

Sec. 438

6. 6.3071 Km.; 630,710 cm.
7. 1220.47 in.
8. 27,685,329 mm.
9. 32,808⅓ ft.
10. 70,000.006 m.
11. 7500 cm.
12. .0086 Mm.; 86,000 mm.
13. 75,006.2 m.; 7,500,620 cm.; 7.50062 Mm.
14. 3 Mm. 7 Hm. 6 Dm. 9 m. 5 dm. 4 cm. 3 mm.
15. 621.36+ miles.
20. *a.* 28,800 m.
    b. 28.8 Km.
21. 7.4564+ miles.
22. 63.36 Km.

Sec. 441

1. 745,000 sq. cm.
2. *a.* .002408 sq. m.
   b. 75,300 sq. mm.
   c. 984.769302 sq. Km.
   d. 2480 sq. cm.
   e. 48,007,300 sq. m.
3. 2.3750 sq. m.
4. 30 sq. m.
5. 68.992 francs.
6. 9.375 sq. m.
7. 44.8 sq. m.
8. 2050 cm.
9. $29.12.
10. 576 sq. dm.
12. 10,000 sq. dm.

Sec. 444

1. *a.* 9380 sq. m.
   b. 93.8 ares.
   c. .938 hectares.
   d. 93.80 sq. Dm.
   e. .938 sq. Hm.
   f. 93,800,000 sq. cm.
2. *a.* 160,750 sq. cm.
   b. 16,075,000 sq. mm.
   c. .000016075 sq. Km.
3. 2.471 A.

Sec. 447

6. *a.* 16.2 cu. m.
   b. 16.2 steres.
   c. 16,200,000,000 cu. mm.
7. *a.* 1,800,000 cu. cm.
   b. 1800 cu. dm.
8. 37.5 steres.

Sec. 451

5. .0015467 Kl.
6. 1234.6 dl.
7. 1000 l.
8. 27,200 l.; 272 Hl.; 27.2 Kl.
9. 5⅝ dm.
10. 1000 l.

Sec. 453

9. 74,200,000 cg.
10. .0000543 Mg.
11. 15432 gr.
12. 2.204+ lb.
13. 220.4 lb.
14. 1,000,000 gr.
16. 14,400 Kg.

Sec. 466

1. $573.75.
2. $1344.70.
3. $490
4. $5485.08.
5. $86.13
6. $16.20.
7. $2568.44.
8. $481.25.
9. $273.
10. $110.
11. $720.
12. $3.36.
13. $404.25.
14. $93.75.
15. $255.26.
16. *a.* $415.
    b. $194.58.
17. *a.* 2629 marks.
    b. $626.
    c. 70.5 lb.
    d. $406.62.
18. *a.* 2337 crowns.
    b. $474.
    c. $161.49.
    d. $641.46.
19. $4580.76.
20. $1012.36.
21. $85.80.
22. $.30.

23. 3000 francs.
24. $106.58.
25. $58.39.

Sec. 474

1. 7.
2. 6.
3. 7.
4. 4.
5. 1032.
6. 1650.
7. 400.
8. 54.
9. 90 yd.
10. 130.
11. 5.
12. 3.
13. 40.
14. $77.
15. $1\frac{8}{9}$.
16. $50.

Sec. 475

7. John, $12.80; Henry, $25.60.
8. 3541; 4806.
9. $7; $14; $21.
10. 68 ft.
11. 44 ft.
12. A, $895; B, $2685; C, $5370.
13. *a*. Son, 21 Kg.; man, 84 Kg.
    b. Son, 46.296 lb.; man, 185.184 lb.
14. 168.
15. 198.
16. $90.
17. $6.68.
18. $204.
19. 90.
20. 24.
21. 1st, 96; 2d, 24; 3d, 8; 4th, 48.
22. 1st, 483; 2d, 613; 3d, 562.
23. 1st, $1.32; 2d, $4.60; 3d, $5.97.
24. 3 bu. onions; 6 bu. turnips; 25 bu. apples; 38 bu. potatoes.
25. 13 girls; 5 boys.
26. $16.

Sec. 477

1. *a*. 42.
   b. 48,510.
2. .03125.
3. $1{,}164{,}179\frac{7}{67}$ gal.
4. $14\frac{1858}{833358}$ %.
5. $90\frac{8}{85}$ lb.
6. 3456 gal.
7. $93.90.
8. $1.50.
9. 71 A., 73 sq. rd.
10. 70° 48′ 34″.
11. 2° 21′ 26″.
12. $8\frac{3}{4}$ cd.
13. $194.52.
14. $39.83.
15. $91.39.
16. 20 ft.
17. 121%.
18. $3600.
19. $5.25.
20. *a*. $7\frac{23}{53}$ %.
    b. $92\frac{24}{53}$ %.
    c. 200 lb.
21. *a*. $805.55.
    b. $15\frac{251}{800}$ %.
    c. Diminished $1\frac{1}{10}$ %.
22. 3.75 steres.
23. 6009 mm.
24. 15,000 l.
25. 375 Kg.
26. $693.98.
27. $12\frac{1}{2}$ yd.
28. $9.84.
29. $500.
30. $596.50.
31. $350.
32. 6%.
33. $29.52.
34. 72%.
35. $5500.
36. *a*. .0125.
    b. $93.75.
    c. $19.44.
    d. $7640.
37. $6\frac{1}{4}$%.
38. $3749.
39. *a*. ——.
    b. $666.27.
40. $824.40.
41. *a*. $2080.
    b. $13,333.33.

Sec. 491

1. *a*. $2343\frac{3}{4}$.
   b. $13{,}781\frac{1}{4}$.
   c. $37{,}537\frac{1}{2}$.
   d. $3125.
2. *a*. $17{,}718\frac{3}{4}$.
   b. $3633\frac{3}{4}$.
   c. $27,525.
   d. $42,250.
3. $12{,}162\frac{1}{2}$.
4. 90 shares.
5. 90 shares.
6. 300 shares.
7. $284.75.
8. $10,800.

9. 99 shares.
10. ——.
11. $9275.
12. $8593¾.
13. 75 shares.
14. 685 shares.
15. *a.* $28,618¾.
b. $28,571¼.
16. *a.* $112½.
b. 112½.
17. *a.* $20,000.
b. 200 shares.
c. $19,550.
18. *a.* $87,500.
b. $70,000.
c. $5600.
d. 6⅔%.
19. *a.* $131.25.
b. 400 shares.
20. $265,491.
21. *a.* 6%.
b. 7½%.
22. $275.
23. 83⅓.
24. *a.* $6000.
b. 33⅓%.

Sec. 500

1. *a.* $41,600.
b. $800.
c. $20,000.
d. $400.
e. $34,000.
f. $680.
2. *a.* $25,800.
b. $860.
c. $70,500.
d. $2820.
3. *a.* $90,000.
b. $17,550.
c. $877.50.
d. $12,000.
e. $540.
f. $3150.
g. $54,000.
4. *a.* $191,070.
b. $198,000.
c. $6930.
d. $200,000.
5. $51,000.
6. ——.
7. ——.

Sec. 516

3. 5.
4. 8.
5. 40.
6. 6.
7. 77.
8. 85.
9. 110.
10. 190.
11. 35.
12. $87½.
13. $100.
14. 127⅐ bu.
15. 1.
16. $6.
17. 1332 hr.
18. 2⅝.

Sec. 518

2. 35 tons.
3. $1519.
4. 28 min.
5. $5.41½.
6. $36.
7. 574 Km.
8. $10.45.
9. 522 mi.
10. 18 : 27 :: 26 : 39.
11. 16⅜ bu.
12. 6 9/16 gal.
13. 305 da.
14. 323 ft.
15. $9.75.
16. 11 bu.
17. $8320.
18. 50 hr.
19. *a.* $842.93.
b. 728 marks.
20. *a.* $124.98.
b. 4398.75 francs.
21. $11,616.
22. $450.
23. $3600.
24. 4%.
25. 100 days.
26. 34⅔ yd.
27. 31⅛ mi.
28. 33 men.
29. 36 hens.

Sec. 520

3. 39, 52.
4. *a.* 550, 650.
b. 738, 2583, 369.
c. $284, $355, $71, $213.
d. 289, 578, 867, 1445.
e. 88 bu., 110 bu., 154 bu., 66 bu.
f. 96 mi., 336 mi., 48 mi., 96 mi., 48 mi.
g. $720, $640, $560, $480, $240.
h. 43, 86, 258, 17⅕, 25⅘.

i. 9700, 8300.
j. $28\frac{1}{12}$, $112\frac{1}{3}$, $196\frac{7}{12}$.

5. Joe, $10; Harry, $15.
6. Olsen, $96; sons, $48 each.
7. 9 peach; 18 pear; 72 apple.
8. 14,000 gr. licorice.
   4200 gr. ipecac.
   2800 gr. ammonia.

Sec. 522

1. A, $400; B, $500; C, $600.
2. Mead, $600; Wilson, $900.
3. Jones, $750; Smith, $1250.
4. A, $312.50; B, $500; C, $387,50.
5. A, $6\frac{2}{3}$; B, $8; C, $9\frac{1}{3}$.
6. A, $300; B, $360; C, $240.
7. $1800; $2700.
8. A, $6400; B, $9600.
9. $120; $200; $280.
10. A, $3333\frac{1}{3}$; B, $2000.
11. $120.
12. *a.* $8125.
    b. Diminished $125.

Sec. 523

2. A, $3000; B, $2100.
3. 1st, $576; 2d, $540; 3d, $480.
4. A, $1928.38; C, $1355.90; H, $3615.72.
5. A, $1250; B, $2490.
6. A, $2000; B, $4000; C, $1800.
7. A, $95; B, $133.
8. A, $1373.684+; B, $1526.315+;
9. A, $1800; B, $900; C, $1440; D, $360.
10. $30,000.

Sec. 525

1. Harness, $20; carriage, $100; horse $200.
2. $30,125.
3. *a.* 400 shares.
   b. $1800.
4. $20,000.
5. $4680.
6. $1\frac{15}{16}$.
7. $54,717.
8. $2142\frac{8}{7}$.
9. *a.* $20,000.
   b. $25,000.
10. *a.* $10,000.
    b. $50,000.
11. $8.33\frac{1}{3}$.
12. The latter, $66.69 better.
13. *a.* ——.
    b. ——.
14. 7%.
15. $13.50.
16. A, $4750; B, $6650; C, $7600.
17. $46\frac{2}{3}$ cu. yd.
18. *a.* 1355.2 lb. carbon.
    b. 28 lb. moisture.
    c. 655.2 lb. combustible matter.
19. *a.* $583.20.
    b. $10\frac{90}{91}$%.
20. $17,500.
21. *a.* £175.
    b. $851.55.
    c. $370.31.
    d. $1.25+.
    e. $1.75.
22. A, $3500; B, $2625; C, $1750; D, $9625.
23. A, $250,000; B, $150,000; C, $50,000.
24. $27\frac{1}{2}$ mo.
25. 49 men.

Sec. 533

1. 225.
2. 1089.
3. 9604.
4. 7569.
5. 5832.
6. 576.
7. 371,293.
8. 1,259,712.
9. 640.09.
10. 16.4836.
11. .697225.
12. 66.430125.
13. $\frac{13456}{53361}$.
14. 19.683.
15. $\frac{31125}{161051}$.
16. 19.4481.
17. $\frac{7784}{136389}$.
18. .00001225.
19. $228\frac{8}{81}$.
20. $5513\frac{26}{27}$.
21. .000512.

22. 1.1449.
23. 4.6225.
24. .000,000,020,736.
25. .5776.
26. $14{,}408\frac{1}{27}$.
27. $\frac{7776}{16807}$.
28. $1815\frac{106}{125}$.
29. 1,000,000,000.
30. $\frac{1}{729}$.

Sec. 549

1. 82.
2. 53.
3. 45.
4. 62.
5. 74.
6. 39.
7. 81.
8. 91.
9. 61.
10. 57.
11. 38.
12. 86.
13. 95.
14. 47.
15. 79.
16. 37.
17. 46.
18. 24.
19. 35.
20. 19.
21. 32.
22. 29.
23. 18.
24. 26.
25. 21.
26. 63 ft.
27. 40 rd.
28. 66 ft. × 33 ft.
29. 84.
30. 36 rd. by 36 rd.
31. *a*. 23 rd. by 23 rd.
    b. 69 rd. by 69 rd.
32. 76, 76.
33. 79.

Sec. 551

1. 94.
2. 765.
3. 371.
4. 283.
5. 319.
6. 245.
7. 204.
8. 907.
9. 406.
10. 309.
11. 485.
12. 4023.
13. 2307.
14. 909.
15. 888.
16. 890.
17. 5500.
18. 8030.
19. 6600.
20. 7006.

Sec. 554

1. *a*. .25.
   b. .35.
   c. .94.
   d. .16.
   e. .089.
   f. .304.
   g. .0075.
   h. 12.6.
   i. 5.39.
   j. 4.93.
   k. 78.3.
   l. .011.
2. *a*. .50+.
   b. .70+.
   c. 3.60+.
   d. 7.90+.
   e. 6.70+.
   f. .86+.
   g. 1.73+.
   h. 8.20+.
   i. 13.12+.
   j. 2.02+.
   k. 5.59+.
   l. .12+.

Sec. 557

1. $\frac{16}{29}$.
2. $\frac{36}{71}$.
3. $\frac{65}{87}$.
4. $\frac{43}{53}$.
5. $\frac{67}{94}$.
6. $\frac{992}{111}$.
7. $\frac{455}{181}$.
8. $2\frac{25}{8}$.
9. 1.718+.
10. $1\frac{3}{16}$.
11. $2\frac{12}{35}$.
12. .765+.
13. 4.055+.
14. .654+.
15. .825+.
16. 5.011+.
17. 2.677+.
18. 2.291+.
19. 3.132+.
20. .745+.
21. .143+.
22. 1.217+.
23. $\frac{53}{120}$.
24. 2077.
25. 7.
26. .749+.
27. 43.829+.
28. 61.
29. 180.

30. 20.784+.
31. 180.
32. 378.
33. 8.
34. $\frac{1}{5}$.
35. $3\frac{3}{13}$.

Sec. 559

1. 60.
2. 10.
3. 21.
4. 35.
5. 22.
6. 33.
7. 28.
8. .36.
9. 4.5.
10. .49.
11. 42.
12. 28.
13. 24.
14. 20.
15. 75.
16. 30.
17. 90.
18. $\frac{13}{75}$.
19. 220.
20. 105.

Sec. 566

1. A, 30′;
   B, 65′;
   C, 39′;
   D, 27′;
   E, 84′;
   F, 30′;
   G, 143′;
   H, 33 rd.;
   I, 41.01 yd.;
   J, 45 mi.
2. 40 rd.
3. 51 rd.; 51 rd.; 68 rd.
4. 322 ft.
5. *a*. 13 in.
   b. 18.38 in.
6. 268 ft.
7. 25.45″.
8. *a*. 3249 sq. cm.
   b. 806 mm.
9. *a*. 15″.
   b. 30″.
   c. 33.54″+.
10. 8.4 m. from A;
    5.6 m. from C.
11. 20′ 3″.
12. 32 ft.
13. 26 ft.
14. 7.708+ft.
16. 8.246+in.
17. 20.602 ft.
18. 40 rd.
19. 217.278+ft.
20. 20 rd.
21. $52\frac{1}{2}$ in.
22. 8.602+in.
23. 33.509+rd.

Sec. 574

1. 234 sq. in.
2. $337\frac{1}{2}$ sq. ft.
3. $4\frac{1}{16}$ A.
4. $11\frac{1}{2}$ bd. ft.
5. *a*. 221 ares.
   b. 2.21 Ha.
6. 1470 sq. ft.
7. 27 cm.
8. *a*. 59′.
   b. 27′; 32′.

Sec. 580

1. 157.08″.
2. 29′.
3. 2500 mi.
4. 25132.8 mi.
5. *a*. 5131.28 ft.
   b. 1564.017+m.
   c. 720.28+rotations.
6. 314.16 ft.
7. 376.992 ft.

Sec. 582

1. 1256.64 sq. in.
2. 8171.3016 sq. m.
3. 1963.50 sq. cm.
4. 19.6350 sq. ft.
5. 7.0686 sq. in.
6. .7854 sq. mi.
7. 12,076.3104 sq. rd.
8. 3848.46 sq. cm.
9. 50.2656 sq. ft.
10. 2827.44 sq. Km.
11. 78.54 sq. yd.
12. 12.5664 sq. mi.
13. 7854 sq. ft.
14. *a*. $\frac{119}{600}$ A.
    b. .36+A.
15. *a*. .7854 sq. mi.
    b. 12.5664 sq. mi.
    c. 3.1416 sq. mi.
    d. 28.2744 sq. mi.
    e. 7.0686 sq. mi.
    f. 19.635 sq. mi.
    g. 50.2656 sq. mi.
16. *a*. 29.
    b. 91.1064.
17. *a*. 47.124.
    b. 176.715.
18. 9 ft.
19. 12.5664 m.
20. .35 m.

Sec. 591

1. $11\frac{1}{7}$ sq. ft.
2. $64\frac{1}{2}$ sq. ft.
3. 270 cu. ft.
4. $9\frac{1}{2}$ cu. ft.
5. $15.58\frac{1}{3}$ cu. ft.
6. *a*. $1\frac{1}{2}$ sq. ft.
   b. 144 cu. in.
7. 18 cm. × 18 cm.

Sec. 594

1. $7\frac{5}{6}$ sq. ft.
2. 26.18 sq. ft.
3. 125.664 sq. ft.
4. 15.708 sq. ft.

Sec. 596

1. 485.3772 sq. ft.
2. 69.1152 sq. ft.
3. 402.1248 sq. in.
4. 785.4 sq. m.
5. 1884.96 sq. in.

Sec. 598

1. *a*. 157.08 cu. in.
   b. 9.4248 cu. ft.
   c. 502,656 cu. m.
   d. 5471.0964 cu. in.
   e. 65.45 cu. ft.
   f. 282.744 cu. m.
   g. .2618 cu. yd.
   h. 15859.48 cu. ft.
   i. 589.05 cu. in.
   j. 11.34903 cu. m.
2. 3.0294 gal.
3. 9.59805 cu. in. more.
4. 47.0016 bbl.
5. 1.7671+ cu. ft.
6. 88.3575 cu. ft.
7. 3392.928 cu. ft.
8. 3.408+ cu. ft.
9. *a*. 395.8416 gal.
   b. 1.389+ ft.

Sec. 605

1. *a*. 13 ft.
   b. 204.204 sq. ft.
   c. 282.744 sq. ft.
2. *a*. $2\frac{1}{2}$ ft.
   b. 6 ft.
   c. 51.05+ sq. ft.
   d. 70.68+ sq. ft.

Sec. 608

1. $8\frac{1}{6}$ cu. ft.
2. $8.72\frac{2}{3}$ cu. ft.
3. .128282 cu. m.

Sec. 616

1. $562\frac{1}{2}$ sq. ft.
2. *a*. $7\frac{1}{2}$ ft.
   b. 135 sq. ft.
   c. 162 cu. ft.
3. *a*. 66.592 sq. in.
   b. 32 cu. in.
4. 800 sq. ft.
5. 93.5 cu. m.

Sec. 622

1. 31,416 sq. ft.
2. 201,062,400 sq. mi.
3. 452.3904 sq. Km.
4. 490.875 sq. in.
5. 14,522,046 sq. mi.

Sec. 624

1. *a*. 4,188,800 cu. ft.
   b. 4,188,800 cu. m.
   c. 65,450 cu. cm.
   d. 8181.25 cu. in.
   e. 4.1888 cu. ft.
2. 4,188,800,000 cu. mi.
3. 268,083,200,000 cu. mi.
4. *a*. 2094.4 cu. in.
   b. 6283.2 cu. in.
   c. 4188.8 cu. in.
   d. 2 times.
   e. 3 times; $1\frac{1}{2}$ times.

Sec. 627

1. *a*. 22 ft.
   b. 12 rd.
   c. 22 mi.
2. $31\frac{1}{5}$ ft.
3. $17\frac{1}{2}$ rd.
4. 45 ft.
5. 59 ft.
6. 144 ft.

Sec. 629

1. $51\frac{3}{4}$ sq. in.
2. $88\frac{19}{32}$ sq. rd.
3. 1:4.
4. 1:2.
5. 21 sq. ft.
6. 26.12+ rd.
7. 320 sq. ft.
8. 153 rd.
9. $937\frac{1}{2}$ sq. in.

Sec. 635

3. .87°, 36′, 59″.
4. .75°, 19′, 15″.
5. 19°, 41′.
6. 9°, 20′.
7. 30°, 12′.
8. 120°, 35′.
9. 3 hr., 9 min., 18 sec.
10. 1 hr., 14 min., 44 sec.
11. 36 min., 3 sec.
12. 48 min., 30 sec.
13. 3 min., 53 sec.
14. 7 hr., 34 min., $\frac{2}{3}$ sec.
15. 7 hr., 8 min., $6\frac{2}{3}$ sec.
16. 121°, 50′, 35″.
17. 15°, 27′, 15″.
18. 10 hr., 40 min., $59\frac{1}{3}$ sec.
19. 1 hr., 54 min., 40 sec.

20. 153°, 36′, 45″.
21. 3 hr., 5 min., 12 sec.
22. 15°, 33′, 40″.
23. 3 min., 13 sec.
24. $7\frac{4}{15}$ sec.
25. 105°, 12′, 30″.
26. 2 hr., 49 min., $16\frac{1}{3}$ sec.
27. 11 hr., 23 min., 40 sec.
28. 59 min.
29. 7 hr., 7 min., 48 sec.
30. 6 hr., 1 min., 25 sec.
31. *a*. 12 : 49 A.M.
    b. 50 min., 48 sec. past 8 P.M.
32. *a*. 11 : 30 : 30 P.M.
    b. 8 : 55 : 40 A.M.
33. 12 : 13 : 17 P.M.
34. 52°, 59′, 45″ W.
35. *a*. 33′, 45″.
    b. $28\frac{1}{8}$ mi.
36. *a*. 576 mi.
    b. 18,000 mi.

Sec. 638

1. 7 : 45 : 11 A.M.
2. 9 min., $33\frac{1}{5}$ sec.
3. 5 : 50 : $17\frac{4}{15}$ P.M.

Sec. 640

1. 328,144.
2. 34,378.
3. $1696.50.
4. $22,241.26.
5. 899,903.19.
6. 1. 2992.
   2. 22,440.
   3. 74,800.
7. 97.
8. 62,413.
9. 4869.
10. 17, 13, 11, 5, 3.

Sec. 642

1. 137.
2. 1386.
3. 181, 654.
4. 36.
5. $\frac{7}{24}$.
6. $\frac{11}{43}$.
7. *a*. $535.
   b. $1605.
8. 6 yd.
9. $\frac{3}{11}$.
10. $1.50.

Sec. 644

1. $2220.86.
2. $2720.
3. 5788.
4. 1200 bu.
5. 3, 3, 3, 5, 2, 2, 2, 2, 2, 2, 2.
6. $\frac{63}{64}$.
7. $\frac{16}{77}$.
8. $\frac{3}{4}$.
9. 180 rd., 4 yd., 2 ft., 8 in.
10. $75.

Sec. 646

1. 91.48.
2. $251\frac{5}{4}$ sq. ft.
3. 120 ft.
4. $43.75.
5. $84.36.
6. 715 gal., 3 qt., 1 pt.
7. 19 T., 250 lb.
8. $34.38.
9. 3 T., 6 cwt., 6 lb.
10. $49\frac{1}{2}$¢.

Sec. 648

1. $8.10.
2. 84.4 bu.
3. 3374.9 lb.
4. $10\frac{1}{2}$ in.
5. ——.
6. ——.
7. 6.12\frac{1}{2}$.
8. 204 da.
9. 6336.
10. .1328125.

Sec. 650

1. 1212.311.
2. $933.50.
3. 60¢.
4. 2%.
5. $6.05.
6. $600.
7. 14.53\frac{1}{2}$.
8. $2850.
9. $3758.96.
10. *a*. $27.
    b. $18\frac{3}{4}$%.

Sec. 652

1. $13,409,301.08.
2. $21.88.
3. ——.
4. 579.56\frac{1}{2}$.
5. .0075.
6. $19.09.
7. $970.78.
8. $250.
9. 2 yr., 5 mo., 24 da.
10. $638.30.

Sec. 654

1. 78,766.
2. 157.5 Hl.
3. 10,000 Kg.
4. $171.85.
5. 87 ; 193 ; 215.
6. 1600 marks.

7. $231.42.
8. $22,044.75.
9. *a*. $300.
   b. 600 shares.
10. $16,500.

Sec. 656

1. $5502.47.
2. $10,592.
3. $1500.
4. 1 yr., 5 mo.
5. 54 ft.
6. 1.875 Kl.
7. 25¢; 15¢.
8. *a*. $1920.
   b. $1600.
   c. $2880.
9. $48; $32.
10. 389; 778; 1556.

Sec. 658

1. 1843.
2. 3375 cu. in.
3. .542+.
4. $680\frac{5}{8}$ A.
5. 1320 ft.
6. 1916.0064 bbl.
7. 5.9 ft.
8. *a*. 1 hr., 35 min., $59\frac{1}{15}$ sec.
   b. 10 : 24 : $\frac{14}{15}$ A.M.
9. 4 : 48 : 4 P.M.
10. 69°, 47′, 15″.

Sec. 659

1. 28,333$\frac{1}{3}$ T.
2. $40,964.
3. $22.44.
4. $9.84.
5. 26,250 vowels.
6. $88\frac{4}{17}$%.
7. 140%.
8. $17.30.
9. .4553$\frac{4}{7}$.
10. $736.88.

Sec. 660

1. $6618.60.
2. 18,119.
3. 80 rd.
4. $13,852.63.
5. 26 da., 1 hr.
6. 1,133,011.
7. ——.
8. £10, 4*s*.
9. 31.1 cu. ft.
10. 31.

Sec. 661

1. 1,964,926.
2. $560.
3. *a*. 6.75 Kl.
   b. 6750 Kg.
4. *a*. 91 shares.
   b. $11,329.50.
5. $116.
6. 11 : 32 : 52 A.M.
7. *a*. $890.40.
   b. $917.11.
8. $43.
9. $6.20.
10. 12.6491+.

APPENDIX

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4. 35.
5. 96.
6. 165.
7. 427.
8. 2.05.
9. 2.59.
10. 76.67+.
11. 12.34.
12. 3.203+.
13. 216.
14. $\frac{61}{87}$; .76+; .25+; 3.39+; $\frac{1}{4}$.
15. 2.428+.
16. 31.742.
17. .398+.
18. .114+.
19. 1.442+.
20. 1.912+.
21. 45 in.
22. 71.3 in.
23. 54 sq. yd.
24. .579+.

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2. 512 cu. in.
3. 9.9+ in. thick.
4. 181.9 lb.
5. 7.13 ft.
6. *a*. 1,920,000 gal.
   b. 11 ft.

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2. $380.95; $19.05.
3. $2542.37.
4. $409.09.
5. $240.88.
6. Better cash, $.121.
7. $728.16.
8. $566.04.
9. ——.
10. Gain, $117.86 by paying cash.

11. $374.997.
12. $486 better on time.
13. Gain $112.62.

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1. $180.
2. 7.50.
3. $4500.
4. $4.
5. $180.
6. $30,000.

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2. 5 bu.
3. 16 da.
4. $66\frac{2}{3}$ dollars.
5. 360 mi.
6. $45{,}619\frac{1}{5}$ stones.
7. $409\frac{7}{27}$ cords.
8. $53\frac{1}{3}$ rd.
9. $31\frac{1}{4}$ da.
10. 12 men.
11. $1.84.
12. 56 bu.
13. $3.44.
14. $145\frac{5}{6}$ m.
15. 8 hr.
16. 16 yr., 8 mo.
17. 3 ft.

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2. 96.
3. 44.
4. 4.
5. 47.
6. 53.
7. 61.
8. 1.
9. 67.
10. 30.
11. 58.
12. 90.
13. 31.
14. 1.
15. 47.
16. 37.
17. 59.